

## JEE MATHS

### Topic - Method of Differentiation

- Q.1 If  $g$  is the inverse of  $f$  &  $f'(x) = \frac{1}{1+x^5}$  then  $g'(x) =$
- (A)  $1 + [g(x)]^5$  (B)  $\frac{1}{1 + [g(x)]^5}$  (C)  $-\frac{1}{1 + [g(x)]^5}$  (D) none
- Q.2 If  $y = \tan^{-1} \left( \frac{\ln \frac{e}{x^2}}{\ln ex^2} \right) + \tan^{-1} \frac{3 + 2 \ln x}{1 - 6 \ln x}$  then  $\frac{d^2y}{dx^2} =$
- (A) 2 (B) 1 (C) 0 (D) -1
- Q.3 If  $y = f\left(\frac{3x+4}{5x+6}\right)$  &  $f'(x) = \tan x^2$  then  $\frac{dy}{dx} =$
- (A)  $\tan x^3$  (B)  $-2 \tan \left[ \frac{3x+4}{5x+6} \right]^2 \cdot \frac{1}{(5x+6)^2}$
- (C)  $f\left(\frac{3 \tan x^2 + 4}{5 \tan x^2 + 6}\right) \tan x^2$  (D) none
- Q.4 If  $y = \sin^{-1} \left( x\sqrt{1-x} + \sqrt{x} \sqrt{1-x^2} \right)$  &  $\frac{dy}{dx} = \frac{1}{2\sqrt{x(1-x)}} + p$ , then  $p =$
- (A) 0 (B)  $\sin^{-1} x$  (C)  $\sin^{-1} \sqrt{x}$  (D) none of these
- Q.5 If  $y = f\left(\frac{2x-1}{x^2+1}\right)$  &  $f'(x) = \sin x$  then  $\frac{dy}{dx} =$
- (A)  $\frac{1+x-x^2}{(1+x^2)^2} \sin \left( \frac{2x-1}{x^2+1} \right)$  (B)  $\frac{2(1+x-x^2)}{(1+x^2)^2} \sin \left( \frac{2x-1}{x^2+1} \right)$
- (C)  $\frac{1-x+x^2}{(1+x^2)^2} \sin \left( \frac{2x-1}{x^2+1} \right)$  (D) none
- Q.6 Let  $g$  is the inverse function of  $f$  &  $f'(x) = \frac{x^{10}}{(1+x^2)}$ . If  $g(2) = a$  then  $g'(2)$  is equal to
- (A)  $\frac{5}{2^{10}}$  (B)  $\frac{1+a^2}{a^{10}}$  (C)  $\frac{a^{10}}{1+a^2}$  (D)  $\frac{1+a^{10}}{a^2}$

Q.7 If  $\sin(xy) + \cos(xy) = 0$  then  $\frac{dy}{dx} =$

- (A)  $\frac{y}{x}$  (B)  $-\frac{y}{x}$  (C)  $-\frac{x}{y}$  (D)  $\frac{x}{y}$

Q.8 If  $y = \sin^{-1} \frac{2x}{1+x^2}$  then  $\left. \frac{dy}{dx} \right|_{x=-2}$  is :

- (A)  $\frac{2}{5}$  (B)  $\frac{2}{\sqrt{5}}$  (C)  $-\frac{2}{5}$  (D) none

Q.9 The derivative of  $\sec^{-1} \left( \frac{1}{2x^2-1} \right)$  w.r.t.  $\sqrt{1-x^2}$  at  $x = \frac{1}{2}$  is :

- (A) 4 (B) 1/4 (C) 1 (D) none

Q.10 If  $y^2 = P(x)$ , is a polynomial of degree 3, then  $2 \left( \frac{d}{dx} \right) \left( y^3 \cdot \frac{d^2y}{dx^2} \right)$  equals :

- (A)  $P'''(x) + P'(x)$  (B)  $P''(x) \cdot P'''(x)$  (C)  $P(x) \cdot P'''(x)$  (D) a constant

Q.11 Let  $f(x)$  be a quadratic expression which is positive for all real  $x$ . If  $g(x) = f(x) + f'(x) + f''(x)$ , then for any real  $x$ , which one is correct.

- (A)  $g(x) < 0$  (B)  $g(x) > 0$  (C)  $g(x) = 0$  (D)  $g(x) \geq 0$

Q.12 If  $x^p \cdot y^q = (x+y)^{p+q}$  then  $\frac{dy}{dx}$  is :

- (A) independent of  $p$  but dependent on  $q$  (B) dependent on  $p$  but independent of  $q$   
(C) dependent on both  $p$  &  $q$  (D) independent of  $p$  &  $q$  both.

Q.13 Let  $f(x) = \begin{cases} g(x) \cdot \cos \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$  where  $g(x)$  is an even function differentiable at  $x = 0$ , passing through the origin. Then  $f'(0)$  :

- (A) is equal to 1 (B) is equal to 0 (C) is equal to 2 (D) does not exist

Q.14 If  $y = \frac{1}{1+x^{n-m}+x^{p-m}} + \frac{1}{1+x^{m-n}+x^{p-n}} + \frac{1}{1+x^{m-p}+x^{n-p}}$  then  $\frac{dy}{dx}$  at  $e^{mnp}$  is equal to:

- (A)  $e^{mnp}$  (B)  $e^{mn/p}$  (C)  $e^{np/m}$  (D) none

Q.15  $\lim_{x \rightarrow 0} \frac{\log_{\sin^2 x} \cos x}{\log_{\sin^2 \frac{x}{2}} \cos \frac{x}{2}}$  has the value equal to

- (A) 1 (B) 2 (C) 4 (D) none of these

Q.16 If  $f$  is differentiable in  $(0, 6)$  &  $f'(4) = 5$  then  $\lim_{x \rightarrow 2} \frac{f(4) - f(x^2)}{2-x} =$

- (A) 5 (B) 5/4 (C) 10 (D) 20

Q.17 Let  $l = \lim_{x \rightarrow 0^+} x^m (\ln x)^n$  where  $m, n \in \mathbb{N}$  then :

- (A)  $l$  is independent of  $m$  and  $n$  (B)  $l$  is independent of  $m$  and depends on  $m$   
 (C)  $l$  is independent of  $n$  and dependent on  $m$  (D)  $l$  is dependent on both  $m$  and  $n$

Q.18 Let  $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$ . Then  $\lim_{x \rightarrow 0} \frac{f'(x)}{x} =$

- (A) 2 (B) -2 (C) -1 (D) 1

Q.19 Let  $f(x) = \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$  then  $f'\left(\frac{\pi}{2}\right) =$

- (A) 0 (B) -12 (C) 4 (D) 12

Q.20 People living at Mars, instead of the usual definition of derivative  $Df(x)$ , define a new kind of derivative,  $D^*f(x)$  by the formula

$$D^*f(x) = \lim_{h \rightarrow 0} \frac{f^2(x+h) - f^2(x)}{h} \text{ where } f^2(x) \text{ means } [f(x)]^2. \text{ If } f(x) = x/\ln x \text{ then}$$

$D^*f(x)|_{x=e}$  has the value

- (A)  $e$  (B)  $2e$  (C)  $4e$  (D) none

Q.21 If  $f(4) = g(4) = 2$ ;  $f'(4) = 9$ ;  $g'(4) = 6$  then  $\lim_{x \rightarrow 4} \frac{\sqrt{f(x)} - \sqrt{g(x)}}{\sqrt{x} - 2}$  is equal to :

- (A)  $3\sqrt{2}$  (B)  $\frac{3}{\sqrt{2}}$  (C) 0 (D) none

Q.22 If  $f(x)$  is a differentiable function of  $x$  then  $\lim_{h \rightarrow 0} \frac{f(x+3h) - f(x-2h)}{h} =$

- (A)  $f'(x)$  (B)  $5f'(x)$  (C) 0 (D) none

Q.23 If  $y = x + e^x$  then  $\frac{d^2x}{dy^2}$  is :

- (A)  $e^x$  (B)  $-\frac{e^x}{(1+e^x)^3}$  (C)  $-\frac{e^x}{(1+e^x)^2}$  (D)  $\frac{-1}{(1+e^x)^3}$

Q.24 If  $x^2y + y^3 = 2$  then the value of  $\frac{d^2y}{dx^2}$  at the point  $(1, 1)$  is :

- (A)  $-\frac{3}{4}$  (B)  $-\frac{3}{8}$  (C)  $-\frac{5}{12}$  (D) none

Q.25 If  $f(a) = 2$ ,  $f'(a) = 1$ ,  $g(a) = -1$ ,  $g'(a) = 2$  then the value of  $\lim_{x \rightarrow a} \frac{g(x) \cdot f(a) - g(a) \cdot f(x)}{x - a}$  is:

- (A) -5 (B)  $1/5$  (C) 5 (D) none

- Q.26 If  $f$  is twice differentiable such that  $f''(x) = -f(x)$ ,  $f'(x) = g(x)$   
 $h'(x) = [f(x)]^2 + [g(x)]^2$  and  
 $h(0) = 2$ ,  $h(1) = 4$

then the equation  $y = h(x)$  represents :

- (A) a curve of degree 2 (B) a curve passing through the origin  
 (C) a straight line with slope 2 (D) a straight line with y intercept equal to  $-2$ .

- Q.27 The derivative of the function,  $f(x) = \cos^{-1} \left\{ \frac{1}{\sqrt{13}} (2 \cos x - 3 \sin x) \right\} + \sin^{-1} \left\{ \frac{1}{\sqrt{13}} (2 \cos x + 3 \sin x) \right\}$

w.r.t.  $\sqrt{1+x^2}$  at  $x = \frac{3}{4}$  is :

- (A)  $\frac{3}{2}$  (B)  $\frac{5}{2}$  (C)  $\frac{10}{3}$  (D) 0

- Q.28 Let  $f(x)$  be a polynomial in  $x$ . Then the second derivative of  $f(e^x)$ , is :

- (A)  $f''(e^x) \cdot e^x + f'(e^x)$  (B)  $f''(e^x) \cdot e^{2x} + f'(e^x) \cdot e^{2x}$   
 (C)  $f''(e^x) e^{2x}$  (D)  $f''(e^x) \cdot e^{2x} + f'(e^x) \cdot e^x$

- Q.29 The solution set of  $f'(x) > g'(x)$ , where  $f(x) = \frac{1}{2} (5^{2x+1})$  &  $g(x) = 5^x + 4x (\ln 5)$  is :

- (A)  $x > 1$  (B)  $0 < x < 1$  (C)  $x \leq 0$  (D)  $x > 0$

- Q.30 If  $y = \sin^{-1} \frac{x^2-1}{x^2+1} + \sec^{-1} \frac{x^2+1}{x^2-1}$ ,  $|x| > 1$  then  $\frac{dy}{dx}$  is equal to :

- (A)  $\frac{x}{x^4-1}$  (B)  $\frac{x^2}{x^4-1}$  (C) 0 (D) 1

## Answer Key

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	C	B	D	B	B	B	C	A	C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	D	B	D	C	D	A	B	C	C
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	A	B	B	B	C	C	C	D	D	C