A Division of Aggarwal Educare
Aggarwal CLASSES

JEE (MAIN)

TEST SERIES

SUBJECT : PHYSICS,CHEMISTRY, MATHEMATICS ***TEST CODE : TSJMT220***

ANSWER PAPER

TIME : 3 HRS ***MARKS : 300***

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark(-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or "un- darken" a darkened bubble.

Part A - PHYSICS

Q.1 **The equation of state of some geses can be expressed as $\left(P+\frac{a}{V^2}\right)=\frac{R\theta}{V}$, where P is pressure, V volume, θ absolute temperature and a and b are constants. The dimensional formula of a is**

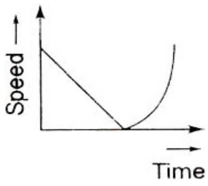
(a) $[ML^5T^{-2}]$ **(b) $[M^{-1}L^5T^{-2}]$** **(c) $[ML^{-1}T^{-2}]$** **(d) $[ML^{-5}T^{-2}]$**

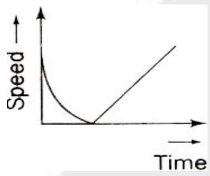
Ans: (a)

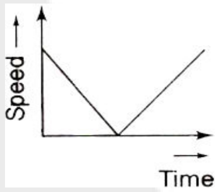
Sol: By the principle of dimenstion homogenity $[P]=\left[\frac{a}{V^2}\right]$

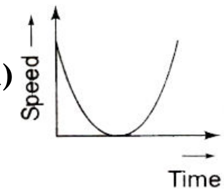
$\Rightarrow \quad [a]=[P]\times[V^2] \quad =[ML^{-1}T^{-2}][L^6]=[ML^5T^{-2}]$

Q.2 **A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its flight if the air resistance is not ignored ?**

(a) 

(b) 

(c) 

(d) 

Ans: (c)

Sol: In first half of motion, the acceleration is uniform and velocity gradually decreases, so slope wil be negative, but for next half, acceleration is positive, So slope will be positive. Thus, graph (c) is correct. Not ignoring air resistance means upwards motion will have acceleration $(a+g)$ and the downward motion will have $(g-a)$.

Q.3 **The speed of a projectile at the highest point becomes $1/\sqrt{2}$ time its initial speed. The horizontal range of the projectile will be**

(a) $\frac{u^2}{g}$ **(b) $\frac{u^2}{2g}$** **(c) $\frac{u^2}{3g}$** **(d) $\frac{u^2}{4g}$**

Ans: (a)

Sol: Velocity at the heighest point is given by $u\cos\theta=u/\sqrt{2}$ (given). $\theta=45^o$

$$R=\frac{u^2\sin 2\theta}{g}=\frac{u^2\sin(2\times45^o)}{g}=\frac{u^2}{g}$$

Q.4 **A car A moves due north at a speed of 40 km/h. While another car B moves due east at a speed of 30 km/h Find the velocity of car B relative to car B (both in magnitude and direction)**

(a) 40 km/h; at an angle $\tan^{-1}\left(\frac{3}{5}\right)$ east of south

(b) 50 km/h; at an angle $\tan^{-1}\left(\frac{3}{5}\right)$ east of south

(c) 40 km/h; at an angle $\tan^{-1}\left(\frac{3}{4}\right)$ east of south

(d) 50 km/h; at an angle $\tan^{-1}\left(\frac{3}{4}\right)$ east of south.

Ans: (d)

Sol: $\vec{v}_A = 40\hat{j},\ \vec{v}_B = 30\hat{i}$

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = 30\hat{i} - 40\hat{j}$$

$$|\vec{v}_{B/A}| = \sqrt{30^2 + 40^2} = 50 \text{ km/h}$$

Q.5 When forces F_1, F_2, and F_3 are acting on a particle of mass m sucha that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed, then the acceleration of the particle is

(a) F_1/m **(b) F_2F_3/mF_1** **(c) $(F_2 - F_3)/m$** **(d) F_2/m**

Ans: (a)

Sol: For the equilibrium of the body, F_1 must act opposite to F_2 and F_3 and must be equal and opposite to the resultant of F_2 and F_3. So on removing F_1.

$a =$ (resultant of F_2 and F_3) $m = F_1 / m$

Q.6 A heavy uniform chain lies on a horizontal table top. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum fraction of the length of the chain that can hang over on edge of the table is

(a) 20% **(b) 25%** **(c) 35%** **(d) 15%**

Ans: (a)

Sol: From the expression

$$l' = \left(\frac{\mu}{\mu+1}\right) l = \left(\frac{0.25}{0.25+1}\right) l \qquad [\text{as } \mu = 0.25]$$

$$\Rightarrow \quad l' = \frac{0.25}{1.25} l \quad = \frac{l}{5} = 20\%$$ of the lenght of the chain.

Q.7 The radii of two soap bubbles are R_1 and R_2. respectively. The ratio of masses of air in them will be

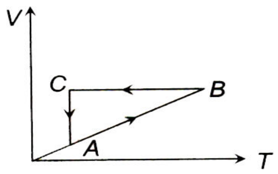
(a) $\frac{R_1^3}{R_2^3}$ **(b) $\frac{R_2^3}{R_1^3}$** **(c) $\left(\frac{P + \frac{4T}{R_1}}{P + \frac{4T}{R_2}}\right)\frac{R_1^3}{R_2^3}$** **(d) $\left(\frac{P + \frac{4T}{R_2}}{P + \frac{4T}{R_1}}\right)\frac{R_2^3}{R_1^3}$**

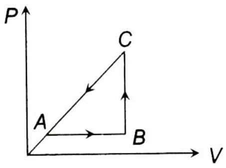
Ans: (c)

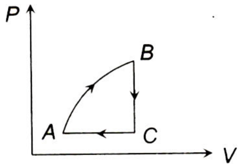
Sol: From $PV = \mu RT$, at a given temperature, the ratio masses of air

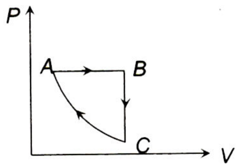
$$\frac{\mu_1}{\mu_2} = \frac{P_1V_1}{P_1V_2} = \frac{\left(P + \frac{4T}{R_1}\right)\frac{4}{3}\pi R_1^3}{\left(P + \frac{4T}{R_2}\right)\frac{4}{3}\pi R_2^3} = \frac{\left(P + \frac{4T}{R_1}\right)}{\left(P + \frac{4T}{R_2}\right)}\frac{R_1^3}{R_2^3}$$

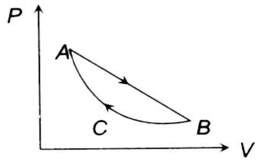
Q.8 A cycle process ABCA is shown in figure Process on the $P - V$ diagram is



(a) 

(b) 

(c) 

(d) 

Ans: (c)

Sol: From the given VT diagram

In process AB, $V \propto T$

$\Rightarrow$ Pressure is constant (as quantitiy of the gas remains same)

In process BC, $V =$ constant and in process CA, $T =$ constant.

$\therefore$ These processes are correctly represented on PV diagram by graph (c).

Q.9 A thermally insulated vessel contains and ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and its suddenly brought to rest. Assuming no heat is lost to the surrounding, its temperature increases by

(a) $\frac{(\gamma-1)}{2(\gamma+1)R} Mv^2 K$ **(b)** $\frac{(\gamma-1)}{2\gamma} Mv^2 K$ **(c)** $\frac{\gamma Mv^2}{2R} K$ **(d)** $\frac{(\gamma-1)}{2R} Mv^2 K$

Ans: (d)

Sol: $\frac{1}{2} mv^2 = 1 \cdot C_v \, \Delta T$

$$\Rightarrow \quad \frac{1}{2} Mv^2 = \frac{R}{\gamma - 1} \cdot \Delta T \qquad \Rightarrow \qquad \Delta T = \frac{Mv^2(\gamma-1)}{2R} = \frac{(\gamma-1)Mv^2}{2R}$$

Q.10 A point mass oscillates along the x-axis according to the law $x = x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$. If acceleration of the particle is written $a = A(\omega t + \delta)$, then

(a) $A = x_0\omega^2,\ \delta = \frac{\pi}{4}$ **(b)** $A = x_0\omega^2,\ \delta = -\frac{\pi}{4}$

(c) $A = x_0\omega^2,\ \delta = \frac{3\pi}{4}$ **(d)** $A = x_0\omega^2,\ \delta = -\frac{\pi}{4}$

Ans: (c)

Sol: $x = x_0 \cos\left(\omega t - \frac{\pi}{4}\right) \quad \Rightarrow \quad v = \frac{dx}{dt} = -x_0\omega \sin\left(\omega t - \frac{\pi}{4}\right)$

$\therefore$ Acceleration, $a = \frac{dv}{dt} = -x_0\omega^2 \cos\left(\omega t - \frac{\pi}{4}\right) = x_0\omega^2 \cos\left(\pi + \omega t - \frac{\pi}{4}\right)$

Now comparing it with $a = A\cos(\omega t + \delta)$, we have $A = x_0\omega^2$ and $\delta = \frac{3\pi}{4}$

Q.11 There is destructive interference between the two waves of wavelength λ coming from

two different paths at a point. To get maximum sound or constructuve interference at that point, the path of one wave is to be increased by

(a) $\lambda / 4$ **(b)** $\lambda / 2$ **(c)** $3\lambda / 4$ **(d)** λ

Ans: (b)

Sol: Destructive interference means the path difference is $(2n-1)\ \lambda / 2$. If is increased by $\lambda / 2$ then

New path difference $= (2n-1)\frac{\lambda}{2} + \frac{\lambda}{2} = n\lambda,$ which is the codition of constructive interference.

Q.12 Two point charges $-Q$ and $2Q$ are separated by a distance R. The neutral point will be obtained at,

(a) A distance of $\frac{R}{(\sqrt{2}-1)}$ from $-Q$ charge and lies between the charges.

(b) A distance of $\frac{R}{(\sqrt{2}-1)}$ from $-Q$ charge on the left side of it.

(c) A distance of $\frac{R}{(\sqrt{2}-1)}$ from $2Q$ charge on the right side of it

(d) A point on the line which passes perpendicularly through the centre of the line joining $-Q$ and $2Q$ charges.

Ans: (b)

Sol: As we already discussed neutral point will be obtained on the side of charg's which is smaller in magnitude, i.e. it will obtained on the left side of $-Q$ charge and at a distance.

$$l = \frac{R}{\sqrt{\frac{2Q}{Q}} - 1} \quad \Rightarrow \quad l = \frac{R}{(2-1)}$$

Q.13 The potential at point x (measured in μm) due to some charges situated on the x-axis is given by $V(x) = 200 / (x^2 - 4)$ Volt. The electric field E at $x = 4\,\mu$m is given by

(a) $5/3\,V/\mu$m and in the positive x-direction

(b) 10/9V/μm and in the negative x-direction

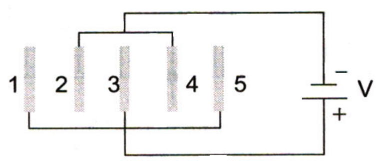
(c) 10/9V/μm and in the positive x-direction

(d) $5/3\,V/\mu$m and in the negative direction.

Ans: (b)

Sol: $$E = -\frac{dV}{dx} = -20\frac{d}{dx}\left(\frac{1}{x^2-4}\right) = \frac{-40x}{(x^2-4)^2} = \frac{-40\times 4}{(16-4)^2} = \frac{-10}{9}$$

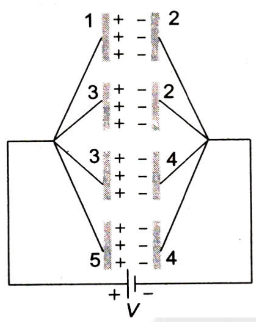
Q.14 Five similar condenser plates, each of area A are placed at equal distance d apart and are connected to a source of emf E as shown in figure. The charge on the plates 1 and 4 will be



(a) $\frac{\varepsilon_0 A}{d}, \frac{-2\varepsilon_0 A}{d}$ **(b)** $\frac{\varepsilon_0 AV}{d}, \frac{-2\varepsilon_0 AV}{d}$ **(c)** $\frac{\varepsilon_0 AV}{d}, \frac{-3\varepsilon_0 AV}{d}$ **(d)** $\frac{\varepsilon_0 AV}{d}, \frac{-4\varepsilon_0 AV}{d}$

Ans: (b)

Sol: Here five plates are given, even number of plates are connected together while odd number of plates are connected together so, four capacitors are formed and they are in parallel combination hence redrawing the figure as shown below



Capacitance of each capacitor $C = \frac{\varepsilon_0 A}{d}$

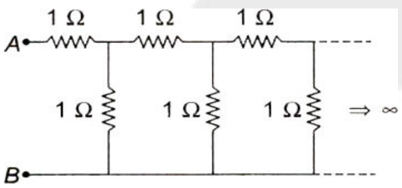
Potential difference acrosss each capacitor is V.

So charge on each capacitor, $Q = \frac{\varepsilon_0 A}{d} V$

Charge on plate (1) is $+\frac{\varepsilon_0 AV}{d}$.

Charge on plate 4 is $-\frac{\varepsilon_0 AV}{d} \times 2 = \frac{-2\varepsilon_0 AV}{d}$

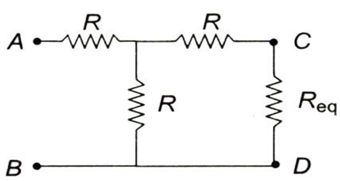
Q.15 The equivalent resistance between points *A* and *B* of an infinite network of resistance, each of 1Ω connceted as shown in figure is



(a) Infinite **(b)** 2Ω **(c)** $\frac{1+\sqrt{5}}{2}\Omega$ **(d) Zero**

Ans: (c)

Sol: Suppose the effective resistance between A and B is R_{req}. Since the network consists of infinite cell. If we exclude one cell from the chain, remaining network have infinite cells, i.e. effective resistance between C and D will also be R_{req}



So now, $R_{req} = R_0 + (R \parallel R_{req}) \quad = R + \frac{RR_{req}}{R + R_{req}} \Rightarrow R_{eq} = \frac{1}{2}[1+\sqrt{5}]$

Q.16 A particle of charge q and mass m starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and $\vec{B} = B_0\hat{i}$ with a velocity $\vec{v} = v_0\hat{j}$. The speed of the particle will become $2v_0$ after a time

(a) $t = \frac{2mv_0}{qE}$ **(b)** $t = \frac{2Bq}{mv_0}$ **(c)** $t = \frac{\sqrt{3}Bq}{mv_0}$ **(d)** $t = \frac{\sqrt{3}\,mv_0}{qE}$

Ans: (d)

Sol: $\vec{E}$ is parallel to $\vec{B}$ and $\vec{V}$ is perpendicular to both. Therefore, path of the partilcle is a heliex with increasing pitch. Speed of particle at any time t is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad ...(i)$$

Here $v_y^2 + v_z^2 = v_0^2$ and $v = 2v_0$

Substituting the values in (i), we get , $t = \frac{\sqrt{3}mv_0}{qE}$

Q.17 The plane of dip circle is set in the geographic meridian and the apparent dip is θ_1. It is then set in a vartical plane perpendicular to the geographic meridian. Now the apparent dip is θ_2. The angle of declination α at that place is

(a) $\tan\alpha\sqrt{\tan\theta_1 \tan\theta_2}$ **(b)** $\tan\alpha = \sqrt{(\tan\theta_1)^2 + (\tan\theta_2)^2}$

(c) $\tan\alpha = \frac{\tan\theta_1}{\tan\theta_2}$ **(d)** $\tan\alpha = \frac{\tan\theta_2}{\tan\theta_1}$

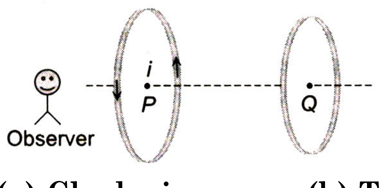
Ans: (c)

Sol: $\tan\theta_1 = \frac{\tan\delta}{\cos\alpha}$ or $\cos\alpha = \frac{\tan\delta}{\tan\theta_1}$ ...(i)

Again $\tan\theta_2 = \frac{\tan\delta}{\sin\alpha}$ or $\sin\alpha\frac{\tan\theta_1}{\tan\theta_2}$ ...(ii)

Gives (i) and (ii) $\tan\alpha\frac{\tan\theta_1}{\tan\theta_2}$

Q.18 Two coils P and Q are lying a little distance apart coaxially. If an anticlockwise current i is suddenly set up in the coil P then the direction of current induced in coil Q will be



(a) Clockwise **(b) Towards north** **(c) Towards south** **(d) Anticlockwise**

Ans: (a)

Sol: Since current setup in the coil P is anticlockwise which increases the dot's linked with coil Q, Hence induced current in coil Q will be clockwise.

Q.19 A transformer is used to light 140 W, 24 V lamp from 240 V ac mains. If the current in the mains is 0.7 A, then the efficiency of transformer is

(a) 63.8% **(b) 84%** **(c) 83.3%** **(d) 48%**

Ans: (c)

Sol: $P_{out} = V_s i_s = 140W, V_s = 24V, V_p = 240V, i_p = 0.7$

$$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{P_{out}}{V_i i_p} \times 100 \qquad = \frac{140}{240 \times 0.7} \times 100 = 83.3\%.$$

Q.20 A parallel plate capacitor consists of two circular plates each of radius 2 cm, separated by a distance of 0.1 mm. If voltage across the plates is varying at the rate of 5×10^{13} V/s, Then the value of displacement current is

(a) 5.50 A **(b) $5.56 \times 10^2 A$** **(c) $5.56 \times 10^3 A$** **(d) $2.28 \times 10^4 A$**

Ans: (c)

Sol: $I_d = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{A}{d} \frac{dV}{dr}$

$$= \frac{8.86 \times 10^{-12} \times 3.14 \times (2 \times 10^{-2})^2}{0.1 \times 10^{-3}} \times 5 \times 10^{13}$$

$$= 5.56 \times 10^3 A$$

Q.21 A 2-V battery, a -15Ω resistor, and a potentiometer of 100 cm length, all are connected in series . If the resistance of potentiometer wire is 5-Ω, then the potential gradient of the potentiometer wire is______?

Sol: By using $x = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L}$

$$\Rightarrow \qquad x = \frac{2}{(5 + 15 + 0)} \times \frac{5}{1} = 0.5V/m = 0.005\ V/cm$$

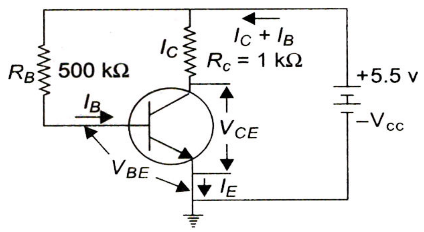
Q.22 A double slit arrangement produces interference fringes for sodium light (λ = 589 nm) that have an angular separation of 3.50×10^{-3} radian, For what wavelength would the angular separation be 10% greater ?

Sol: Angular separation = λ / d

For angular separation to be 10% greater λ should be 10% greater.

New wavelength is $\left(589 + \frac{589}{10}\right)nm$ or $(589 + 58.9)nm$ i.e. 647.9 nm i.e. 648 nm.

Q.23 In the circuit shown in the figure, the base current IB is 10 μA and the collector is 5.2 mA. The value of V_{BE} is______?



Sol: $V_{BE} = V_{CC} - I_B R_B = 5.5 - 10 \times 10^{-6} \times 5 \times 10^{5} = 0.5V$

Q.24 An ice box used for keeping eatables has a total wall area of 1 m^2 and a wall thickness of 5.0 cm. The thermal conductivity of the ice box is $K = 0.01$ J/m°C. It is filled with ice at 0°C along with eatables on a day when the temperature is 30°C. The latent heat of fusion of ice is 334×10^3 J/kg. The amount of ice melted on one day is ______?

(1 day = 86,400 s)

Sol: Quantity of heat transferred through wall will be utilized in the melting of ice.

$$Q = \frac{KA\ \Delta\theta_t}{\Delta x} = mL$$

$\therefore$ Amount of ice method, $m = \frac{KA\Delta\theta t}{\Delta x L}$

$$\therefore \quad m = \frac{0.01 \times 1 \times (30-0) \times 68400}{5 \times 10^{-2} \times 334 \times 10^{3}} = 1.552 \text{ kg or } 1552 \text{ g}$$

Q.25 A screw gauge gives the following reading when used to mesure the diameter of a wire. Main scale reading: 0 mm Circular scale reading: 52 divisions Given that 1 mm on main scale corresponds to 100 divisions of the circular scale , the diameter of wire from the above data is _______?

Sol: Least count of screw gauge

$\frac{1}{100}$ mm=0.52 mm

Diameter = Divisions on circular scale × Least count + Main scale rading

$$= 52 \times \frac{1}{100} + 0$$

$= 0.52$ mm

$= 0.052$ cm

Part - B - CHEMISTRY

Q.26 The oxidation number of sulphur in Sg, S_2, F_2, H_2S respectively. are :
(a) 0, +1, and -2 (b) +2, +1, and -2 (c) 0, +1, and +2 (d) -2, +1 and -2

Ans: (a)

Sol: (i) Oxidation state of element in its free state is zero
(ii) Sum of oxidation states of all atoms in compound is zero

Oxidation number of S in $S_g = 0$; Oxidation number of S in $S_2F_2 = +1$; Oxidation number of S in $H_2S = -2$.

Q.27 Uncertainty in position is twice the uncertainty in momentum. Uncertainty in velocity is :

(a) $\sqrt{\frac{h}{\pi}}$ **(b)** $\frac{1}{2m}\sqrt{\frac{h}{\pi}}$ **(c)** $\frac{1}{2m}\sqrt{h}$ **(d)** $\frac{h}{4\pi}$

Ans: (c)

Sol: $\Delta x = 2\Delta p$ $\quad \Delta x \cdot \Delta p \cdot m = \frac{h}{2} = \frac{h}{4\pi}$

$2\Delta p \cdot m \cdot \Delta V = \frac{h}{2}$ $\quad (\Delta V)^2 = \frac{h}{4m^2}$ or $\quad \Delta V = \frac{\sqrt{h}}{m}$

Q.28 The common features among the species CN^-, CO, and NO^+ are
(a) Bond order three and isoelectronic
(b) Bond order three and week field ligands
(c) Bond order two and π - acceptors
(d) Isoelectronic and week field ligands

Ans: (a)

Sol: Each of the species has 14 electrons, so they are isoelectronic and show bond order 3.

Bond order $= \frac{1}{2}[N_p - N_a] = \frac{1}{2}[10-4] = \frac{6}{2} = 3.$

Q.29 $N_2 + 3H_2 \longrightarrow 2NH_3$, 1 mol N_2 and 4 mol H_2 are taken in a 15-L flask at 27°C. After complete conversion of N_2 into NH_3, 5 L of H_2O is added. Pressure set up in the flask is :

(a) $\frac{3 \times 0.0821 \times 300}{15}$ **atm** **(b)** $\frac{2 \times 0.0821 \times 300}{10}$ **atm**

(c) $\frac{1 \times 0.0821 \times 300}{15}$ **atm** **(d)** $\frac{1 \times 0.0821 \times 300}{10}$ **atm**

Ans: (d)

Sol: $\underset{1}{N_2} + \underset{4}{3H_2} \longrightarrow 3NH_3$

H_2 left $= 1$ mol NH_3 formed $= 2$ mol But it is dissolved in H_2O.
Pressure is due to H_2 only in 10 L effective volume (5 L occupied by H_2O).

Q.30 In harber's process of ammonia manufacture : $N_{2(g)} + 3H_{2(g)} \rightarrow 2NH_{3(g)}$; $H^0_{25^\circ C} = -92.2$ kJ

Molecule	**$N_{2(g)}$**	**$H_{2(g)}$**	**$NH_{3(g)}$**
Cp (J/K mol)	**29.1**	**28.8**	**35.1**

In Cp is independent of temperature, then reaction at 100°C as compared to the of 25°C will be

(a) more endothermic **(b) less endothermic**
(c) more exothermic **(d) less exothermic**

Ans: (c)

Sol: $\Delta H^o = -92.2$ $\quad \Delta C_p = 2C_p(NH_3, g) - C_p(N_2, g) - 3C_p(H_2)$

$= 2 \times 35.1 - 29.1 - 3 \times 28.8$

$= -45.3\ J/K$

$\Delta^o_{100^oC} = H^o_{25^oC} + \Delta C_p(\Delta T) \qquad -92.2 - \frac{45.3 \times 75}{1000}$

$H^o_{100^oC} = -92.2 - 1.3975$ kJ/mol

Then the reaction at 100°C will be more exothermic compared to that at 25°C at

Q.31 PCl_5 dissociation in a closed container is given as :

$PCl_{5(g)} \rightleftharpoons PCl_{3(g)} + Cl_{2(g)}$

If the total pressure at equilibrium of the reaction mixture is *P* and the degree of dissociation of PCl_5 is α, the partial pressure of PCl_3 will be :

(a) $P \times \left[\frac{\alpha}{\alpha+1}\right]$ **(b)** $P \times \left[\frac{2\alpha}{1-\alpha}\right]$ **(c)** $P \times \left[\frac{\alpha}{\alpha-1}\right]$ **(d)** $P \times \left[\frac{\alpha}{1-\alpha}\right]$

Ans: (a)

Sol: $PCl_{5(g)} \rightleftharpoons PCl_{3(g)} + Cl_{2(g)}$

$1 \qquad 0 \qquad 0$

$1-\alpha \qquad \alpha \qquad \alpha$

$P'_{PCl_3} = P \times X_{PCl_3} = \left[\frac{P \cdot \alpha}{1+\alpha}\right]$

Q.32 For a sparingly souble salt A_pB_q, the relationship of its solubility product (L_s) with its solubility (S) is :

(a) $L_s = S^{p+q} \cdot p^p \cdot q^q$ **(b)** $L_s = S^{p+q} \cdot p^q \cdot q^p$ **(c)** $L_s = S^{pq} \cdot p^p \cdot q^q$ **(d)** $L_s = S^{pq} \cdot (p \cdot q)^{p+q}$

Ans: (a)

Sol: $A_pB_q \rightleftharpoons pA^{1+} + qB^{p-}$

$L_s = [A^{q+}]^p[B^{p-}]^q = (p \times s)^p (q \times s)^q$

$= S^{p+q} \cdot p^p \cdot q^q.$

Q.33 A compound has the emprical formula $C_{10}H_8Fe$. A solution of 0.26 g of the compound in 11.2 g of benezene (C_6H_6) boils at 80.26°C. The boiling point of benzene is 80.10°C and K_b is 2.53°C/ molal. What is the molecular formula of the compound ?

(a) $C_{30}H_{24}Fe_3$ **(b)** $C_{10}H_8Fe$ **(c)** C_5H_4Fe **(d)** $C_{20}H_{16}Fe_2$

Ans: (d)

Sol: $\Delta T_b = 80.26 \qquad 0.16 = 2.53 \times \frac{0.26/M}{11.20} \times 1000;\ M \approx 367$

This is almost equal to molar mass of $C_{20}H_{16}Fe_2$.

Q.34 For the cell $Ti|Ti||Cu^{2+}|Cu, E_{cell}$ at 25°C is 0.83 V. The EMF of the cell can be increasded by :

(a) increasing $[Cu^{2+}]$ **(b)** increasing $[Ti^+]$

(c) decreasing $[Cu^{2+}]$ **(d)** increasing temperature to 35°C

Ans: (a)

Sol: The reaction at both the electrodes are

At anode : $2Ti \longrightarrow 2Ti^+ + 2e^-$

At cathode ; $Cu^{2+} + 2e^- \longrightarrow Cu$

Net cell reaction : $2Ti + Ci^{2+} \longrightarrow 2TI^+ + Cu$

Applying Nernst equation gives,

$$E_{cell} = E^o_{Cu^{2+}|Cu} - E^o_{TI^+|Tl} - \frac{RT}{2F} In \frac{[Tl^+]}{Cu^{2+}}$$

Thus, it is evident from the equation that E_{cell} can be increased either by increasing [Cu^{2+}] or by decreasing [Ti^+] or by decreasing the temperature.

Q.35 Under the influence of an electric field, the particle in a sol migrate towards cathode. The coagulation of the same sol is studied using NaCl, Na_2SO_4 and Na_3SO_4 solution. Their coagulating values will be maximum for :

(a) NaCl (b) Na_2SO_4 (c) Na_3PO_4 (d) Same for all

Ans: (a)

Sol: The sol particles migrate towards cathods. So they are positively charged. Hence, anions would be effective in coagulation. The greater is the valence of effective ion, the smaller will be its coagulating value.

Q.36 An organic compound (A) has the molecular formula C_3H_6O. It undergoes iodoform test. When saturated with dil. HCl, it gives (B) of molecular formula $C_9H_{14}O$. A and B respectively are :

(a) propanal and mesitylene

(b) propanone and mesityl oxide

(c) propanone and 2,6-dimethyl-2,5-hyptadien -4-one

(d) propanone and mesitylene oxide

Ans: (c)

Sol: The compound A is propanone which gives the iodoform test and have formula C_3H_6O. 2,6-Dimethyl-2, 5-hyptadien -4-one is compound B having carbon atoms three times the number of carbon atoms in propanone.

Q.37 Among the following which one can have a meso form ?

(a) $CH_3 - CHOH - CH(Cl) - C_2H_5$

(b) $CH_3 - CHOH - CHOH - CH_3$

(c) $CH_3 - CH_2 - CHOH - CHOH - CH_3$

(d) $HOCH_2 - CHCl - CH_3$

Ans: (b)

Sol: For meso form, compound should have at least two similar asymmetic carbon atoms.

Q.38 Propan-1 ol can be prepared from propane by alcohol :

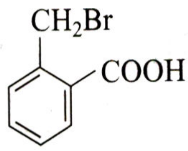
(a) H_2O / H_2SO_4 (b) $Hg(OAc)_2/H_2O$ followed by $NaBH_4$

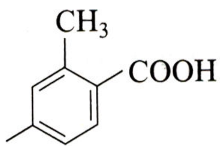
(c) B_2H_6 followed by B_2H_6 (d) CH_3CO_2/ H_2SO_4

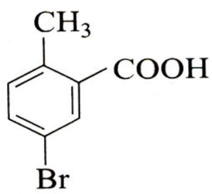
Ans: (c)

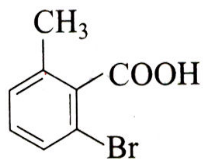
Sol: $6CH_3 - CH = CH_2 + B_2H_6 \xrightarrow{H_2O_2} CH_3 - CH_2 - CH_2OH$

Q.39 o-Touic acid on reaction with Br_2/Fe gives :

(a) 

(b) 

(c) 

(d) 

Ans: (c)

Sol: -CH_3 group is ortho and para directing, while -COOH group has metadirecting influence. Keeping in view the combined effecte, (c) is the correct answer.

Q.40 Empirical formula of a compound is CH_2O and its vapor density is 30. Molecular formula of the compound is :

(a) $C_3H_6O_3$ (b) $C_2H_4O_2$ (c) C_2H_4O (d) CH_2O

Ans: (b)

Sol: Empirical formula $= CH_2O$

Empirical formula mass $= 12 + 2 + 16 = 30$

Molecular mass $= 2 \times$ Vapor density $= 2 \times 30 = 60$

$$n = \frac{\text{Molecular mass}}{\text{Emperical mass}} = \frac{60}{30} = 2$$

$$= (CH_2O_2 = C_2H_4O_2)$$

Q.41 A colorless water-soluble solid X on heating gives equimolar quantities of Y and Z. Y gives dense white fumes with HCl and Z does so with NH_3. Y gives brown precipitate with Nessler's reagent and Z gives white preecipitate with nitrates of Ag^+, Pb^{2+}, and Hg^+, X is

(a) NH_4Cl (b) NH_4NO_3 (c) NH_4NO_2 (d) $FeSO_4$

Ans: (a)

Sol: $NH_4Cl \xrightarrow{\Delta} \underset{Y}{NH_3} + \underset{Z}{HCl}$

$$\underset{Y}{NH_3} + \underset{\text{Nessler's reagent}}{2K_2[Hg]_4} + 3KOH \longrightarrow \underset{\substack{\text{brown ppt} \\ \text{iodide of Milion's base}}}{H_2\,NHgO \cdot Hgl} + 7KI + H_2O$$

Q.42 Setting of plaster of Paris is :

(a) oxidation with atmospheric oxygen

(b) combination with atmospheric CO_2

(c) dehydration

(d) hydration to yield another hydrate

Ans: (d)

Sol: Setting of plaster of Paris is an exothermic process

$$CaSO_4 \cdot \frac{1}{2}H_2O \xrightarrow{H_2O} \underset{\text{Orthor hom bic}}{CaSO_4 \cdot 2H_2O} \xrightarrow{\text{Hardening}} \underset{\substack{\text{Mono orthor hom bic} \\ \text{Gypsum}}}{CaSO_4 \cdot 2H_2O}$$

The setting is due to the formation of another hydrate.

Q.43 The basic character of the transition metal monoxides follows the order :

(a) TiO > VO > CrO > FeO (b) VO > CrO > TiO > FeO

(c) CrO > VO > FeO > TiO (d) TiO > FeO > VO > CrO

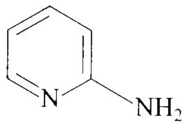
Ans: (a)

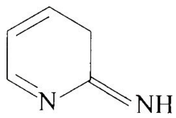
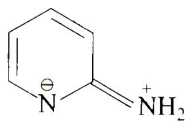
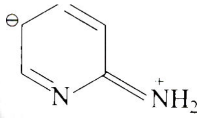
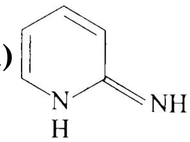
Sol: Basic character of oxide decreases from left to right in a period of periodic table.

Q.44 **Formation of $Ni(CO_4)$ and its subsequent decomposition into Ni and CO (recycled) make basis of Mond's process : $Ni + 4CO \xrightarrow{T_1} Ni(CO_4) \xrightarrow{T_2} Ni + 4CO$**
T_1 and T_2 are
(a) 100°C, 50°C **(b) 50°C, 100°C** **(c) 50°C, 230°C** **(d) 230°C, 50°C**

Ans: (c)
Sol: ------------------

Q.45 **The proper tautomeric structure for 2-aminopyridine (X) is :**


(X)

(a)  **(b)**  **(c)**  **(d)** 

Ans: (a)
Sol: ------------

Q.46 $xA + yB \to zC$. **If** $-\frac{d[A]}{dt} = -\frac{d[B]}{dt} = 1.5\frac{d[C]}{dt}$**, then x, y, and z are :**

Sol: $xA + yB \to zC$ $\quad \frac{-d}{dt}[A] = \frac{-d}{dt}[B] = 1.5\frac{-d}{dt}[C]$

$$\Rightarrow \quad \frac{-1}{3}\frac{-d}{dt}[A] = \frac{-1}{3}\frac{-d}{dt}[B] = \frac{1}{2}\frac{-d}{dt}[C]$$

$x = 3$ $\quad y = 3$ $\quad z = 2$

Q.47 **The density of a pure substance "A" whose atoms pack in cubic close pack arrangement is 1 g/cc. If B atoms can occupy tetrahedral void and if all the tetrahedral voids are occupied by B atom, what is the density of resulting solid in g/cc [Atomic mass $A = 30$ g/mol and atomic mass $(B) = 50$ g/mol]**

Sol: Let volume for fcc unit cell be V

$$\rho_A = \frac{4 \times M_A}{N_A \cdot V} \qquad \rho_B = \frac{8 \times M_B}{N_A \cdot V}$$

$$\frac{\rho_A}{\rho_B} = \frac{M_A}{2M_B} = \frac{30}{2 \times 50} = 0.3$$

$$\rho_B = \frac{\rho_A}{0.3} = 3.3$$

Total density $\rho_A + \rho_B = 4.33$ g/cc

Q.48 **What volume of liquid A has the same mass as 80.0 cm^3 of liquid B ?**

Sol: 193.0 cm^3

Q.49 **Oxidation states of vanadium in**
$V \to V^{2+} + 2e$,
$V^{2+} \to V^{3+} + e$,

are 2 and 3 respectively. The oxidation states of vanadium in this following reaction
$V^{3+} + H_2O \rightarrow VO^{2+} + 2H^+ + e^-$?

Sol: VO^{2+}
$x + 1(-2) = +2$
$x - 2 = 2$
$x = +4.$

Q.50 **A 25.0 mL sample of 0.1050 M H_2SO_4 is titrated with NaOH solution of unknown concentration. The phenolphthalein indicator end point was reached when 17.23 mL of NaOH solution had been added. What is the concentration of the NaOH ?**

Sol: 0.3047 M

Part - C - MATHEMATICS

Q.51 **The 120 permutations of MAHES are arranged in dictionary order, as if each were an ordinary five-letter word. The last letter of the 86th word in the list is**
(a) A **(b) H** **(c) S** **(d) E**

Ans: (d)

Sol: The first $24 = 4!$ words begin with A, the next 24 begin with E, and the next 24 begin with H. So, the 86th begins with M and it is the $86 - 72 = 14th$ such word. The first 6 words that begin with M begin with MA and the next 6 begin with ME. So the desired word begins with MH and it is the second such word. The first word that begins with MH is MHAES the second is MHASE. Thus E is the letter we seek.

Q.52 **$\log_7 \log_7 \sqrt{7\sqrt{(7\sqrt{7})}}$ is equal to**

(a) $3\log_2 7$ **(b) $3\log_7 2$** **(c) $1 - 3\log_7 2$** **(d) $1 - 3\log_7 7$**

Ans: (c)

Sol: $\log_7 \log_7 7^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}}$

$$= \log_7\left(\frac{1}{7} + \frac{1}{4} + \frac{1}{8}\right) = \log_7 = \left(\frac{7}{8}\right)$$

$$= 1 - \log_7 8 = 1 - 3\log_7 2$$

Q.53 **The area of the quadrilateral *ABCD* whose vertices are respectively $A(1, 1)$, $B(7, -3)$, $C(12, 2)$ and $(7, 21)$ is**
(a) 100 sq. units **(b) 125 sq. units** **(c) 132 sq. units** **(d) none of these**

Ans: (c)
Sol: Here given points are cyclic order then

$$Area = \frac{1}{2}\begin{Vmatrix} 1 & 1 \\ 7 & -3 \\ 12 & 2 \\ 7 & 21 \\ 1 & 1 \end{Vmatrix}$$

$= \frac{1}{2}|(-3-7)+(14+36)+(252-14)+(7-21)| = 132$ sq units.

Q.54 If non-zero numbers a, b, c are in HP, then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. The point is

(a) $(-1, -2)$ (b) $(-1, 2)$ (c) $\left(1, -\frac{1}{2}\right)$ (d) $(1, -2)$

Ans: (d)

Sol: a, b, c are in HP

$\Rightarrow \frac{2}{b}=\frac{1}{a}+\frac{1}{c}$ $\Rightarrow \frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ passes thus, $(1, -2)$

Q.55 If a line is drawn through a fixed point $P(\alpha, \beta)$ to cut the circle $x^2+y^2=a^2$ at A and B, then $PA \cdot PB =$

(a) $\alpha^2+\beta^2$ (b) $\alpha^2+\beta^2-a^2$ (c) a^2 (d) $\alpha^2+\beta^2+a^2$

Ans: (b)

Sol: $PA \cdot PB = PT^2 =$ (Lenth of tangent)^2

$= (\sqrt{\alpha^2+\beta^2-a^2})^2 = \alpha^2+\beta^2-a^2$

Q.56 The curve described parametrically by $x = t^2+r+1,\ y = t^2-t+1$ represents

(a) a pair of straight lines (b) an ellipse

(c) a parabola (d) a hyperbola

Ans: (c)

Sol: $\frac{x+y}{2} = t^2+1, \frac{x-y}{2} = t$

Eliminating t, $(x+y) = (x-y)^2+4$. Since 2[nd] degree term forms a perfect square it represents a parabola.

Q.57 If any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intersept equal lenghts ℓ on the axes, then $\ell =$

(a) a^2+b^2 (b) $\sqrt{a^2+b^2}$ (c) $(a^2+b^2)^2$ (d) none of these

Ans: (b)

Sol: The equation of tangent to the given ellipse at point $P(a\cos\theta, b\sin\theta)$ is $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

Intercept of line on the axes are $\frac{a}{\cos\theta}$ and $\frac{b}{\sin\theta}$.

Given that

$\therefore \quad \frac{a}{\cos\theta} = \frac{b}{\sin\theta} = l$

$\Rightarrow \quad \cos\theta = \frac{a}{l}$ and $\sin\theta = \frac{b}{l}$

$\Rightarrow \quad \cos^2\theta + \sin^2\theta = \frac{a^2}{l^2} + \frac{b^2}{l^2} \qquad \Rightarrow \; l^2 = a^2 + b^2$

$\Rightarrow \quad l = \sqrt{a^2 + b^2}$

Q.58 Two straight lines pass through the fixed points $(\pm a, 0)$ and have slopes whose products is $p > 0$. Then the locus of the points of intersection of the lines is

(a) ellipse (b) hyperbola (c) parabola (d) circle

Ans: (b)

Sol: Let equation of the line be $y = m_1(x-a)$ $y = m_2(x-a)$

$\Rightarrow \quad m_1 m_2 = p$

$\Rightarrow \quad y^2 = m_1 m_2 (x^2 - a^2) = p(x^2 - a^2)$

Hence locus of the points of intersection is $y^2 = p(x^2 - a^2)\, px^2 - y^2 = pa^2$ Which is a hyperbola.

Q.59 If $f(x+2y, x-2y) = xy$, then $f(x, y)$ equal

(a) $(x^2 - y^2)/8$ (b) $(x^2 - y^2)/4$ (c) $(x^2 + y^2)/4$ (d) $(x^2 - y^2)/2$

Ans: (a)

Sol: We have $f(x+2y, x-2y) = xy$

Let $x + 2y = u$ and $x - 2y = v$

Then $x = \frac{u+v}{2}$ and $y = \frac{u-v}{4}$

Substituting the value of x and y in (i) we obtain

$f(u,v) = \frac{u^2 - v^2}{8} \qquad f(x, y) = \frac{x^2 - y^2}{8}$

Q.60 The inverse of the function $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 2$ is given by

(a) $\log_e\left(\frac{x-2}{x-1}\right)^{\frac{1}{2}}$ (b) $\log_e\left(\frac{x-1}{3-x}\right)^{\frac{1}{2}}$ (c) $\log_e\left(\frac{x}{2-x}\right)^{\frac{1}{2}}$ (d) $\log_e\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$

Ans: (b)

Sol: Let $y = f(x) = \frac{e^{2x} - 1}{e^{2x} + 1} + 2$.

$\Rightarrow \quad y - 2 = \frac{e^{2x} - 1}{e^{2x} + 1} \qquad \Rightarrow (y-2)e^{2x} + y - 2 = e^{2x} - 1$

$\Rightarrow \quad (y-3)e^{2x} = 1-y \qquad \Rightarrow e^{2x} = \dfrac{y-1}{3-y}$

$\Rightarrow \quad 2x = \log \dfrac{y-1}{3-1} \qquad \Rightarrow x = \dfrac{1}{2}\log\left(\dfrac{y-1}{3-y}\right)$

$\Rightarrow \quad f^{-1}(x) = \log_e \left(\dfrac{x-1}{3-x}\right)^{\frac{1}{2}}$

Q.61 **If** $f(x) = \begin{cases} \sin x, & x \neq n\pi n n \in Z \\ 0, & \text{otherwise} \end{cases}$ **and** $g(x) = \begin{cases} x^2+1, & x \neq 0, 2 \\ 4, & x = 0 \\ 5, & x = 2 \end{cases}$ **then** $\lim_{x\to 0} g\{f(x)\} =$

(a) 1 **(b) 0** **(c)** $\dfrac{1}{2}$ **(d)** $\dfrac{1}{4}$

Ans: (a)

Sol: $\lim_{x\to 0} g(f(x)) = \lim_{x\to 0} [f(x)^2] + 1 = \lim_{x\to 0} (\sin^2 + x + 1) = 1$

Q.62 **The expression** $y^2 \dfrac{d^2y}{dx^2}$ **on the ellipse** $3x^2 + 4y^2 = 12$ **is equal to**

(a) $\dfrac{9}{4}$ **(b)** $-\dfrac{9}{4}$ **(c)** $\dfrac{4}{9}$ **(d)** $-\dfrac{4}{9}$

Ans: (b)

Sol: Differentiating implicite F^n we have $6x + yy' = 0$

$\Rightarrow \quad 3 + 4(y')^2 + 4yy' = 0$

Hence $3 + \dfrac{9x^2}{4y^2} + 4yy'' = 0$

Multilplying by y^2, we get

$3y^2 + \dfrac{9x^2}{4} + 4y^3 \dfrac{d^2y}{dx^2} = 0 \qquad \dfrac{3y^2}{4} + \dfrac{9x^2}{16} + y^3 y''' = 0$

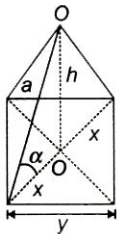
$\dfrac{3}{16}(3x^2 + 4y^2) + y^3 y'' = 0 \qquad$ But $3x^2 + 4y^2 = 12$ and hence $y^3 y''' = -\dfrac{9}{4}$ at every point on the ellipse.

Q.63 **The lateral edge of a regular rectangular pyramid is "*a*" cm long. The lateral edge makes and angle** α **with the plane of the base. The value of of** α **for which the volume of the pyramid is greatest is**

(a) $\dfrac{\pi}{4}$ **(b)** $\sin^{-1}\sqrt{\dfrac{2}{3}}$ **(c)** $\cot^{-1}\sqrt{2}$ **(d)** $\dfrac{\pi}{3}$

Ans: (c)

Sol:



$h = a \sin\alpha$ and $x = a\cos\alpha$; $x^2 + h^2 = a^2$

$V = \frac{1}{3}\, y^2\, h = \frac{1}{3}\, 2x^2\, h$ (Note $4x^2 = 2y^2 \Rightarrow y^2 = 2x^2$)

$V(\alpha) = \frac{2}{3}\, a^2\cos^2\alpha \cdot a\sin\alpha = \frac{2}{3}\, a^2 \sin\alpha\cos^2\alpha$

Now, $V'(\alpha) = 0 \Rightarrow \tan\alpha = \frac{1}{\sqrt{2}}$; $V_{max} = \frac{4\sqrt{3}\,a^3}{27}$

Q.64 $\int \frac{2x}{(1-x^2)\sqrt{x^4-1}}\,dx$ **is equal to**

(a) $\sqrt{\frac{x^2+1}{x^2-1}} + c$ **(b)** $\sqrt{\frac{x^2-1}{x^2+1}} + c$ **(c)** $\sqrt{x^4-1} + c$ **(d) none of these**

Ans: (a)

Sol: $\int \frac{2x}{(1-x^2)\sqrt{x^4-1}}\,dx = \int \frac{-2x}{(x^2-1)^{3/2}\sqrt{x^2+1}}$

Put $\sqrt{\frac{x^2+1}{x^2-1}} = z$

$\therefore \quad \frac{1}{2}\left(\frac{x^2-1}{x^2+1}\right)^{1/2} \cdot \frac{(x^2-1)\cdot 2x - (x^2+1)2x}{(x^2-1)^2}\,dx = dz$

$\Rightarrow \quad \frac{\sqrt{x^2-1}}{\sqrt{x^2+1}} \cdot \frac{-2x}{(x^2-1)^2}\,dx = dz$

$\Rightarrow \quad \frac{-2x}{(x^2-1)^{3/2}\sqrt{x^2+1}}\,dx = dz$

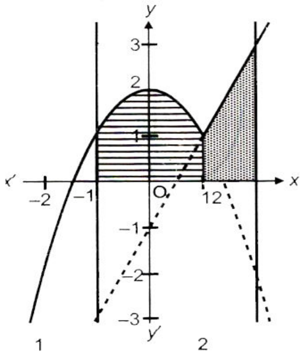
$\therefore$ Given integral $\int dz = z + c = \sqrt{\frac{x^2+1}{x^2-1}} + c$

Q.65 **The area of the closed figure bounded by** $x = -1$ **and** $x = 2$ **and** $y = \begin{cases} -x^2+2, & x \le 1 \\ 2x-1, & x > 1 \end{cases}$ **and the abscissa axis is**

(a) $\frac{16}{3}$ **sq units** **(b)** $\frac{10}{3}$ **sq units** **(c)** $\frac{13}{3}$ **sq units** **(d)** $\frac{7}{3}$ **sq units**

Ans: (a)

Sol:



$$A = \int_{-1}^{1} (-x^2 + 2)dx + \int_{1}^{2} (2x - 1)dx$$

$$= \left(-\frac{x^3}{3} + 2x\right)_{-1}^{1} + (x^2 - x)_1^2$$

$$= \frac{16}{3} \text{ sq units}$$

Q.66 An object falling from rest in the air is subjects not only to the gravitational force but also to the air resistance. Assume that the air resistance is proportional to the velocity with constant of proportionality as k > 0. and acts in a direction opposite to motion $(g = 9.8 \text{ m/sec}^2)\cdot$ Then velocity cannot exceed

(a) 9.8/k m/sec **(b) 9.8/k m/sec** **(c) $\frac{k}{9.8}$ m/sec** **(d) none of these**

Ans: (a)

Sol: Let $V(t)$ be the velocity of the object at time t.

Given $\frac{dV}{dt} = 9.8 - kV \Rightarrow \frac{dV}{9.8 - kV} = dt\cdot$

Integrating, we get $\log(9.8 - kV) = -kt + C \Rightarrow V(0) = 0$ Constant $= 9.8$.

Thus, $9.8 - kV = 9.8\, e^{kt} \Rightarrow kV = 9.8(1 - e^{-kt}) \Rightarrow V(t) = \frac{9.8}{k}(1 - e^{-kt}) < \frac{9.8}{k}$

For all t. Hence $V(t)$ cannot exceed $\frac{9.8}{k}\cdot$

Q.67 The variable "x" satisfying the equation

$|\sin x \times \cos x| + \sqrt{2 + \tan^2 x + \cot^2 x} = \sqrt{3}$, belongs to the interval

(a) $\left[0, \frac{\pi}{3}\right]$ **(b) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$** **(c) $\left[\frac{3\pi}{4}, \pi\right)$** **(d) non-existant**

Ans: (d)

Sol: $|\sin x \ \cos x| + |\tan x + \cot x| = \sqrt{3}$

$\Rightarrow \quad |\sin x \cos x| + \frac{1}{|\sin x \cos x|} = \sqrt{3}$ but $|\sin x \cos x| + \frac{1}{|\sin x \cos x|} \geq 2$

Hence, no solution.

Q.68 **The equation $2\cos^2\frac{x}{2}\sin^2 x = x^2 + x^{-2};\ 0 < x \le \frac{\pi}{2}$ has**

(a) no real solution **(b) one real solution**
(c) more then one solution **(d) none of these**

Ans: (a)

Sol: The given equation is $2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + \frac{1}{x^2}$ where $0 < x \le \frac{\pi}{2}$

LHS $= 2\cos^2\left(\frac{x}{2}\right)\sin^2 x = (1+\cos x)\sin^2 x$

$\because$ $1 + \cos x < 2$ and $\sin^2 x \le 1$ for $0 < x < \frac{\pi}{2}$

$\therefore$ $(1+\cos x)\sin^2 x < 2$ and RHS $= x^2 + \frac{1}{x^2} \ge 2$

$\therefore$ For $0 < x \le \frac{\pi}{2}$

Q.69 **Which of the following is the solution set of the equation $2\cos^{-1} x = \cot^{-1}\left(\frac{2x^2-1}{2x\sqrt{1-x^2}}\right)$?**

(a) (0, 1) **(b) (-1, 1)-{0}** **(c) (-1, 0)** **(d) [-1, 1]**

Ans: (a)

Sol: $2\cos^{-1} x = \cot^{-1}\left(\frac{2x^2-1}{2x\sqrt{1-x^2}}\right)$

Put $x = \cos\theta$ LHS $= 2\theta;\ 0 \le \theta \le \pi$ and $-1 \le x \le 1$ ...(i)

RHS $= \cot^{-1}\left(\frac{\cos 2\theta}{2\cos\theta|\sin\theta|}\right) = \cot^{-1}(\cot 2\theta) = 2\theta$

If $0 < 2\theta < \pi$ ...(ii)

From (i) and (ii), $0 < \theta < \pi/2$

$\therefore$ $x \in (0, 1)$

Q.70 **If x, y, z are drawn perpendicular to a, b, and c, then the value of $\frac{bx}{c} + \frac{cy}{a} + \frac{az}{b}$ will be**

(a) $\frac{a^2+b^2+c^2}{2R}$ **(b) $\frac{a^2+b^2+c^2}{R}$** **(c) $\frac{a^2+b^2+c^2}{4R}$** **(d) $\frac{2(a^2+b^2+c^2)}{2R}$**

Ans: (a)

Sol: Let the area of triangle be Δ. then according to the equation $\Delta = \frac{1}{2}ax = \frac{1}{2}by = \frac{1}{2}cz$

$\therefore$ $\frac{bx}{c} + \frac{cy}{a} + \frac{az}{b}$

$= \frac{b}{c}\left(\frac{2\Delta}{a}\right) + \frac{c}{a}\left(\frac{2\Delta}{b}\right) + \frac{a}{b}\left(\frac{2\Delta}{c}\right) \quad = \frac{2\Delta(b^2+c^2+a^2)}{abc}$

$$= \frac{2(a^2+b^2+c^2)}{abc} \cdot \frac{abc}{4R} \qquad = \frac{a^2+b^2+c^2}{2R}$$

Q.71 The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^{11}$ is _____?

Sol: $(1+x)^{11}(1+x^2)^{11}$

$$= (1+{}^{11}C_1x+{}^{11}C_2x^2+{}^{11}C_3x^3+{}^{11}C_4x^4+....)(1+{}^{11}C_1(x^2)+{}^{11}C_2(x^2)^2+....)$$

Coefficient of x^4 is

$${}^{11}C_2\cdot 1+{}^{11}C_1\cdot{}^{11}C_2+{}^{11}C_4 = 990$$

Q.72 $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos x} =$

Sol: $I = \int_{-\pi/2}^{\pi/2} \frac{dx}{1+\cos x}$ $\therefore$ $\frac{1}{1+\cos x}$ is an even function

$$\therefore I = 2\int_0^{\pi/2} \frac{1}{1+\cos x}dx = 2\int_0^{\pi/2} \frac{dx}{2\cos^2\frac{x}{2}}$$

$$= \int_0^{\pi/2} \sec^2\frac{x}{2}dx = 2\left[\tan\frac{x}{2}\right]_0^{\pi/2} = 2\left(\tan\frac{\pi}{4}-\tan 0\right) = 2.$$

Q.73 If $\frac{dy}{dx} = y+3 > 0$ and y (0) = 2, then y(In 2) is equal to ?

Sol: We have $\frac{dy}{dx}$=y+3

$\frac{1}{y+3}dy = dx$

In |(y + 3)| = x + In c, where In c is a constant of integration (y + 30) = c e^x

Initially when x = 0, y = 2 and c = 5.

Finally the required solution is y + 3 = 5 e^x

y(In 2) = $5e^{In2}$ - 3 = 10 - 3 = 7

Q.74 If A = $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 2 & 0 \\ 1 & 3 & 0 \end{vmatrix}$ then the value of $|adj\ A|$ is equal to ?

Sol: 1

Q.75 The value of $\int_{-1}^{1}|x+1|dx$ is ______?

Sol: $\int_{-1}^{1}|x+1|dx = \int_{-1}^{1}|x+1|dx = \left(\frac{x^2}{2}+x\right)_{-1}^{1}$ =2.

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ROUGH WORK

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