A Division of Aggarwal Educare

Aggarwal CLASSES ™

JEE (MAIN)

TEST PAPER

SUBJECT : PHYSICS,CHEMISTRY, MATHEMATICS ***TEST CODE : TSJMT219***

ANSWER PAPER

TIME : 3 HRS ***MARKS : 300***

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark(-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or “un- darken” a darkened bubble.

Part A - PHYSICS

Q.1 **The potential energy of a 1-kg particle free to move along the x-axis is given by $U(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)J$. The total mechanical speed of the particle is 2 J. Then, the maximum speed (in m/s) is**

(a) $1/\sqrt{2}$ **(b)** 2 **(c)** $3/\sqrt{2}$ **(d)** $\sqrt{2}$

Ans: (c)

Sol: $V = \frac{x^4}{4} - \frac{x^2}{2} \Rightarrow \frac{dV}{dx} = x^3 - x = 0 \Rightarrow x = 0, 1, -1$

V is minimum at $x = \pm 1$

$$V_{\min} = V_{x=\pm 1} = \frac{(\pm 1)^4}{4} - \frac{(\pm 1)^2}{2} = -\frac{1}{4}J$$

KE should be maximum at these points

$$KE_{\max} + V_{\min} = \text{Total mechanical energy}$$

$$\Rightarrow \quad KE_{\max} - \frac{1}{4} = 2 \qquad \Rightarrow \quad \frac{1}{2} = mv^2{}_{\max} = 2 + \frac{1}{4} = \frac{9}{4}$$

$$\Rightarrow \quad \frac{1}{2} \times 1 \times v_{\max}^2 = \frac{9}{4} \Rightarrow v_{\max} = \frac{3}{\sqrt{2}} m/s$$

Q.2 **Statement-1 : Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.**
Statement-2 : Principle of conservation of momentum holds true for all kinds of collision.

(a) Statements-1 is true, Statements-2 is true; Statements-2 is correct explanation of Statements-1
(b) Statements-1 true, Statements-2 is true; Statements-2 is not the correct explantion of Statements-1
(c) Statements-1 is false, Statements-2 is true
(d) Statements-1 is true, Statements-2 is false.

Ans: (a)

Sol:

If it is a completely inelastic collision, then

$$m_1v_1 + m_2v_2 = m_1v + m_2v$$

$$v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$KE = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$$

As $\overrightarrow{p_1}$ and $\overrightarrow{p_2}$ both simultaneously cannot be zero. therefore, total KE cannot be lost.

Q.3 **A solid sphere rolls down in inclined plane and its velocity at the bottom is v_1. Then the same sphere slides down the plane (without friction) and let its velocity at the bottom**

be v_2. Which of the following relations is correct ?

(a) $v_1 = v_2$ **(b)** $v_1 = \frac{5}{7} v_2$ **(c)** $v_1 = \frac{7}{5} v_2$ **(d) None of these**

Ans: (d)

Sol: When slid sphere roll down and inclined plane the velocity at bottom

$$v_1 = \sqrt{\frac{10}{7} gh}$$

But if there is not friction, then it slides on inclined plane and the velocity at bottom

$$v_2 = \sqrt{2gh} \qquad \therefore \qquad \frac{v_1}{v_2} = \sqrt{\frac{5}{7}}$$

Q.4 **A projectile is projected with velocity kv_e in vertically upward direction from the ground into the space, (v_e is the escape velocity and $k < 1$). If air resistance is considered to be negligible, then the maximum height from the centre of Earth to which it can go will be (R = radius of Earth)**

(a) $\frac{R}{k^2 + 1}$ **(b)** $\frac{R}{k^2 - 1}$ **(c)** $\frac{R}{1 - k^2}$ **(d)** $\frac{R}{k + 1}$

Ans: (c)

Sol: From the law of conservation of energy, difference in potential energy between ground and maximum height is equal to kinetic energy at the point of projection.

$$\therefore \quad \frac{mgh}{1 + h/R} = \frac{1}{2} m (kv_e)^2$$

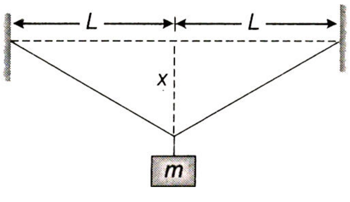
$$= \frac{1}{2} mk^2 v_e^2$$

$$= \frac{1}{2} mk^2 (\sqrt{2gR})^2 \quad v_e = \sqrt{2gR}$$

By solving height from the surface of Earth $h = \frac{Rk^2}{1 - k^2}$

So height from the center of Earth $r = R + h = R + \frac{Rk^2}{1 - k^2} = \frac{R}{1 - k^2}$

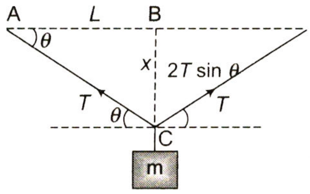
Q.5 **A steel wire of diameter d, area of cross section A and length 2L, is clamped firmly at two points A and B which are 2L meters apart and in the same plane. A body of mass m is hung from the middle point of wire each that the middle point sags by x, lower from original position in figure, If Young's modulus is Y, then m is given by**



(a) $\frac{1}{2} \frac{YAx^2}{gL^2}$ **(b)** $\frac{1}{2} \frac{YAL^2}{gx^2}$ **(c)** $\frac{1}{2} \frac{YAx^3}{gL^3}$ **(d)** $\frac{1}{2} \frac{YAL^2}{gx^2}$

Ans: (c)

Sol: Let the tension in the string is T and for the equilibrium of mass m.



$$2T\sin\theta = mg \Rightarrow T = \frac{mg}{2\sin\theta} = \frac{mgL}{2x}$$ [as θ is small, then $\sin x / L$]

Increament in the length.

$l = AC - AB$ $\quad = \sqrt{L^2 + x^2} - L$

$= (L^2 + x^2)1/2 - L$

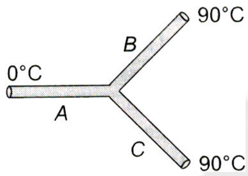
$$= L\left[\left(1+\frac{x^2}{L^2}\right)^{1/2}\right] = L\left[1+\frac{1}{2}\frac{x^2}{L^2} - 1\right] = \frac{x^2}{2L}$$

As Young's modulus, $Y = \frac{T}{A}\frac{L}{l} \Rightarrow T = \frac{YAl}{L}$ ...(i)

Substituting the value of T and l in (i), we get,

$$\frac{mgL}{2x} = \frac{YA}{L}\cdot\frac{x^2}{2L} \qquad \therefore \qquad m = \frac{YAx^3}{gL^3}$$

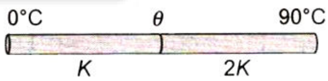
Q.6 Three rods made of the same material and having the same cross section have been joined as shown in figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C, respectively. The temperature of the junction of the three rods will be



(a) 45°C **(b) 60°C** **(c) 30°C** **(d) 20°C**

Ans: (b)

Sol: Let the conductivity of each rod is K. By considering that rods B and C are in parallel. effective thermal conductivity of B and C will be 2K



Now, with the help of given formula, temperature of interface

$$\theta = \frac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2} = \frac{K\times 0 + 2K\times 90}{K + 2K} = \frac{180}{3} = 60^oC$$

Q.7 Certain amount of an ideal gas is contained in a closed vessel. The vessel is moving with a constant velocity v. The molecular mass of gas is M. The rise in temperature of the gas when the vessel is suddenly stopped is

(a) $\frac{Mv^2}{2R(\gamma+1)}$ **(b)** $\frac{Mv^2(\gamma-1)}{2R}$ **(c)** $\frac{mv^2}{2R(\gamma+1)}$ **(d)** $\frac{mv^2}{2R(\gamma+1)}$

Ans: (b)

Sol: If is the total mass of the gas, then its kinetic energy $=\frac{1}{2}mv^2$

When the vessel is suddenly stopped, then total kinetic energy will increase the temperature of the gas (because will be aiiabatice), i.e.

$$\frac{1}{2}mv^2 = \mu C_v \Delta T = \frac{m}{M} C_v \Delta T \qquad \left[as\ C_v = \frac{R}{\gamma-1}\right]$$

$$\Rightarrow \quad \frac{m}{M}\frac{R}{\gamma-1}\Delta T = \frac{1}{2}mv^2$$

$$\Rightarrow \quad \Delta T = \frac{Mv^2(\gamma-1)}{2R}$$

Q.8 At 100°C, the volume of 1 kg of water is 10^{-3} m^3 and volume of 1 kg of steam at normal pressure is 1.671 m^3. The latent heat of steam is 2.3×10^6 j/kg and normal pressure is 10^5 N/m^2. If 5 kg of water at 100°C is converted into steam, the increase in the internal energy of water in this process will be

(a) 8.35×10^5 J (b) 10.66×10^6 J (c) 11.5×10^6 J (d) Zero

Ans: (b)

Sol: Heat required to convert 5 kg of water into steam.

$$\Delta Q = mL = 5 \times 2.3 \times 10^6 = 11.5 \times 10^6\ J$$

Work done in expanding volume.

$$\Delta W = P\Delta V = 5 \times 10^5\ [1.671 - 10^{-3}] = 0.835 \times 10^6\ J$$

Now by first law of thermodynamics, $\Delta U = \Delta Q - \Delta W$

$$\Rightarrow \quad \Delta U = 11.5 \times 10^6 - 0.835 \times 10^6 = 10.66 \times 10^6\ J$$

Q.9 A block is placed on a frictionless horizontal table. The mass of the block is *m* and and springs of force constant k_1, k_2 are attached on either side of it, if the block is displaced a little and left to oscillate, then the angular frequency of oscillation will be

(a) $\left(\frac{k_1+k_2}{m}\right)^{1/2}$ (b) $\left[\frac{k_1k_2}{m(k_1+k_2)}\right]^{1/2}$ (c) $\left[\frac{k_1k_2}{(k_1-k_2)m}\right]^{1/2}$ (d) $\left[\frac{k_1^2+k_2^2}{(k_1+k_2)m}\right]^{1/2}$

Ans: (a)

Sol: Given condition match with parallel combination.

So, $k_{eff} = k_1 + k_2$

$$\therefore \quad \omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{k_1+k_2}{m}}$$

Q.10 An electric dipole is placed at an angle of 30° to a non uniform electric field. The dipole will experience

(a) a translational force only in the direction of the field.

(b) a translational force only in direction normal to the direction of the field

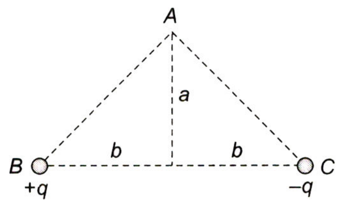
(c) a torque as well as translational force

(d) a torque only.

Ans: (c)

Sol: In non-uniform field, a dipole experience force, $F = q\vec{E}_1 - q\vec{E}_2$ as well as torque.

Q.11 As shown in figure, charges +q and -q are placed at the vertices B and C of aisosceles triangle. The potential at the vertex A is



(a) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{2q}{\sqrt{a^2+b^2}}$ **(b)** $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{a^2+b^2}}$ **(c)** $\frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{\sqrt{a^2+b^2}}$ **(d) Zero**

Ans: (d)

Sol: Potential at A = Potential due to $(+q)$ charge + Potential due to $(-q)$ charge

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{a^2+b^2}} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{\sqrt{a^2+b^2}} = 0$$

Q.12 A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitances of the capacitor

(a) decreases **(b) remains unchanged**
(c) becomes infinite **(d) increases**

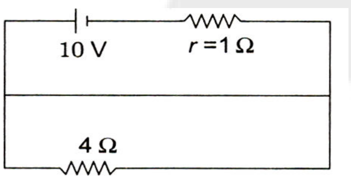
Ans: (b)

Sol: The capacitance of capacitor with air as dielectric is given by $C = \varepsilon_0 A / d,$ where A is the area of the each plate and d is distance between them. The capacitance of capacitor with a conducting shell of thickness t is given by

$$C = \frac{\varepsilon_0 A}{d - t}$$

Now, If $t \to 0$, then $C = \frac{\varepsilon_0 A}{d}$

Q.13 The potential difference across the terminals of the bettery shown in figure, is (r = internal resistance of battery)



(a) 8V **(b) 10 V** **(c) 6 V** **(d) Zero**

Ans: (d)

Sol: Since the battery is short-circuited, so potential difference is zero.

Q.14 An iron core shaped as a toroid with mean radius 250 mm, supports a winding with the total number of turns 1000. The core has a cross-cut of width 1.0 mm. With current I = 0.85 A flowing through the winding, the magnetic induction in the gap is 0.75 T, then the permeability of iron is :

(a) 2×10^3 **(b)** 0.3×10^3 **(c)** 3.7×10^3 **(d)** 10^3

Ans: (c)

Sol: $\oint \vec{B}_0 \cdot \vec{dl} = 2\pi RB$ is the line integral of $\vec{B}$ and line integral of magnetic induction in air is given as $\oint \vec{B}_0 \cdot \vec{dl} = \mu_0 NI - Bb$

Now, $\mu = \dfrac{\int \vec{B}_0 \cdot \vec{dl}}{\oint \vec{B}_0 \cdot \vec{dl}}$ $\mu = \dfrac{2\pi RB}{\mu_0 NI - bB}$

Then, $m = 3.7 \times 10^3$

Q.15 In LCR ciruit $R = 100\ \Omega$. When capacitance C is removed, the current lags behind the voltage by $\pi/3$. When inductance L is removed, the current leads the voltage by $\pi/3$. The impedance of the circuit is

(a) 50Ω (b) 100Ω (c) $200\ \Omega$ (d) 400Ω

Ans: (b)

Sol: When C is removed circuit becomes RL, circuit hence $\tan\dfrac{\pi}{3} = \dfrac{X_L}{R}$ ...(i)

When L is removed circuit becomes RC circuit hence $\tan\dfrac{\pi}{3} = \dfrac{X_C}{R}$ ...(ii)

From (i) and (ii), we obtain $X_L = X_C$. This is the condition of resonance and in resonance $Z = R = 100\ \Omega$

Q.16 In a region of free space the electric at field intensity some instant of time t is $\vec{E} = (80\hat{i} + 32\hat{j} - 64\hat{k})$ and the magnetic field is $\vec{B} = (0.2\hat{i} + 0.08\hat{j} - 0.29\hat{k})\mu$T, The pointing vector for these fields is

(a) $-11.52\hat{i} + 28.8\hat{j}$ (b) $-28.8\hat{i} + 11.52\hat{j}$ (c) $28.8\hat{i} - 11.52\hat{j}$ (d) $11.52\hat{i} - 28.8\hat{j}$

Ans: (d)

Sol: The pointing vector, $\vec{S} = \dfrac{1}{\mu_0}(\vec{E} \times \vec{B})$

$$= \frac{1}{4\pi \times 10^{-7}} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 80 & 32 & -64 \\ 0.2 & 0.08 & 0.29 \end{vmatrix}$$

$$= 11.52\,\hat{i} - 28.8\,\hat{j}$$

Q.17 A man can see the objects upto a distance of one metre from his eyes. For correcting his eye sight so that he can see an object at infinity, he requires a lens whose power is

OR

A man can see upto 100 cm of the distant object. The power of the lens required to see for objects will be

(a) +0.5D (b) +1.0D (c) +2.0D (d) -1.0D

Ans: (d)

Sol: $f = -$(defected far point) $= -100\,cm$, So power of the lens $P = \dfrac{100}{f} = \dfrac{100}{-100} = -1D$

Q.18 A particle of mass M at rest decays into two masses m_1 and m_2 with non-zero velocities. The ratio λ_1 / λ_2 of de-Broglie wavelengths of the particles is

(a) $\frac{m_2}{m_1}$ **(b) $\frac{m_1}{m_2}$** **(c) $\frac{\sqrt{m_1}}{\sqrt{m_2}}$** **(d) 1 : 1**

Ans: (d)

Sol: $\lambda = \frac{h}{mv}$ Here, $0 \times M = M_1 v_1 + m_2 v_2$

Clearly, $m_1 v_1 = -m_2 v_2$

In magnitude, $m =$ constant

$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{1}{1}$

Q.19 An atom makes a transition from a state of energy E_2 to one of lower energy E_1. Which of the following gives the wavelenegth of the radiation emitted in terms of the Planck's constants h and the speed of light c ?

(a) $\frac{E_2 - E_1}{hc}$ **(b) $\frac{hc}{E_2} - \frac{hc}{E_2}$** **(c) $\frac{hc}{E_1} - \frac{hc}{E_2}$** **(d) $\frac{hc}{E_2 - E_1}$**

Ans: (d)

Sol: By quantum theory of radiation, the energy change E between energy levels is proportional to the frequency of electromangnetic radiation f and is given by the equation below

$$E = hf = \frac{hc}{\lambda} \quad \text{...(i)}$$

Where, h is the Plank's constant, c is the speed of light λ is the wavelength of the radiation. Now, energy change by transition from E_2 to E_1 is given by

$$E = E_2 - E_1 \quad \text{...(ii)}$$

From (i) and (ii) we have

$$E_2 - E_1 = \frac{hc}{\lambda} \quad \Rightarrow \quad \lambda = \frac{hc}{(E_2 - E_1)}$$

Q.20 A step index fiber has relative refractive index of 0.88%. What is the critical angle at the corecladding interface ?

(a) 60° **(b) 75°** **(c) 45°** **(d) None of these**

Ans: (d)

Sol: $\frac{\mu_1 - \mu_2}{\mu_1} = \frac{0.88}{100}$

Q.21 Two full turns of the circular scale of a screw gauge cover a distance of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of -0.03 mm. While measuring the diameter of a thin wire, a students notes tha main scale reading of 3 mm and the number of circular scale divisions in the line with the main scale as 35. The diameter of the wire is_____?

Sol: Least count of the screw gauge $= \frac{0.5\,mm}{50} = 0.01\,mm$

Main scale reading $= 3\,mm$

Vernier scale reading $= 35$

$\therefore$ Observed reading $= 3 + 0.35 = 0.35$

Zero error $= -0.03$

$\therefore$ Actual diameter of the wire $= 3.35 - (-0.03) = 3.38\,mm$

Q.22 A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincide with fourth bright fringe of unknown light. From this data, the wavelength of the unknown light is _____?

Sol: $3\lambda_1 = 4\lambda_2$

$\Rightarrow \quad \lambda_2 = \frac{3}{2}\lambda_1 = \frac{3}{4} \times 590 = \frac{1770}{4} = 442.5\,nm$

Q.23 A boat is moving due east in a region where Earth's magnetic field is 5.0×10^{-5} NA/m due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 m/s, the magnetic of the induced emf in the wire of aerial is_____?

Sol: $E_{ind} = Bvl \quad = 5.0 \times 10^{-5} \times 1.50 \times 2 \quad = 10.0 \times 10^{-5} \times 1.5$

$= 15 \times 10^{-5}\,V \quad = 0.15\,mV$

Q.24 A motor cycle starts from rest and accelerates along a straight path at 2 m/s². At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest ? (speed of sound = 330 m/s).

Sol: Motor cycle, $u = 0$, $a = 2\,m/s^2$

$\Rightarrow \quad n' = n\frac{v - v_0}{v + v_s} \qquad \Rightarrow \frac{94}{100}n = n\frac{330 - v_0}{330}$

$\Rightarrow \quad 330 - v_0 = \frac{330 \times 94}{100} \qquad \Rightarrow v_0 = 330 - \frac{94 \times 33}{10} = \frac{33 \times 6}{10}\,m/s$

$s = \frac{v^2 - u^2}{2a} = \frac{9 \times 33 \times 33}{100} = \frac{9 \times 1098}{100} = 98\,m$

Q.25 Two bulbs of 100 W and 200 W, rated at 220 V are connected in series. On supplying 220 V, the consumption of power will be ?

Sol: In series, $P_{\text{Consumed}} = \frac{P_1 P_2}{P_1 + P_2}$

$\Rightarrow \quad P_{\text{Consumed}} = \frac{100 \times 200}{300} = 66\,W$

Part - B - CHEMISTRY

Q.26 In the mixture of ($NaHCO_3$ + Na_2CO_3), the volume of HCl required is *x* mL with phenolphthalein indicator and *y* mL with methyl orange indicator in the same titration. Hence, the volume of HCl for complete reaction of Na_2CO_3 is :

(a) 2*x* **(b) *y*** **(c) *x*/2** **(d) (*y* - *x*)**

Ans: (a)

Sol: Since phenolphthalein indicates only conversion of Na_2CO_3 into $NaHCO_3$. Hence x mL of HCl will be firther required to convert $NaHCO_3$ into H_2CO_3. So, total volume of HCl required to convert Na_2CO_3 into H_2CO_3 is given by $x + x = 2x\ mL$

Q.27 If wavelength is equal to the distance traveled by the electron in one second then :

(a) $\lambda = \frac{h}{p}$ **(b)** $\lambda = \frac{h}{m}$ **(c)** $\lambda = \sqrt{\frac{h}{p}}$ **(d)** $\lambda = \sqrt{\frac{h}{m}}$

Ans: (d)

Sol: $\lambda = v$

Then $\lambda = \frac{h}{mV}$ or $\lambda^2 = \frac{h}{m}$

So, $\lambda = \sqrt{\frac{h}{m}}$

Q.28 N_2 and O_2 are converted into monocation N_2^+ and O_2^+ respectively. Which of the following statements is wrong ?

(a) In N_2, then $N-N$ bond weakness **(b) In O_2, the $O-O$ bond order increase**

(c) In O_2, paramagnetism decrease **(d) N_2^+ becomes diamagnetic**

Ans: (d)

Sol: N_2^+ has one unpaired electron, so it would be paramagnetic.

Q.29 If some moles of O_2 diffuse in 18 s and same moles of other gas diffuse in 45 s, then what is the molecular weight of the unknown gas ?

(a) $\frac{45^2}{18^2} \times 32$ **(b)** $\frac{18^2}{45^2} \times 32$ **(c)** $\frac{18^2}{45^2 \times 32}$ **(d)** $\frac{45^2}{18^2 \times 32}$

Ans: (a)

Sol: $r_{O_2} = \frac{x}{18}\ mol/s \quad \Rightarrow \quad r_s = \frac{x}{45}$

$$M_g = M_{O_2}\left(\frac{r_{O_2}}{r_g}\right)^2 = 32\left(\frac{x}{18} \times \frac{45}{x}\right)^2 = 32 \times \frac{45^2}{18^2}$$

Q.30 In the dissociation of PCl_5 ; $PCl_{5(g)} \rightleftharpoons PCl_{3(g)} + Cl_{2(g)}$, If the degree of dissocitation is α at equilibrium pressure P, then the equilibrium constant for the reaction is :

(a) $K_p = \frac{\alpha^2}{1+\alpha^2 P}$ **(b)** $K_p = \frac{\alpha^2 P^2}{1-\alpha^2}$ **(c)** $K_p = \frac{\alpha P^2}{1-\alpha^2}$ **(d)** $K_p = \frac{\alpha^2 P}{1-\alpha^2}$

Ans: (d)

Sol:

$PCl_{5(g)}$	$\rightleftharpoons$	$PCl_{3(g)}$	+	$Cl_{2(g)}$
1		0		0
$1-\alpha$		α		α

$$K_p = \frac{n_{pcl_3} \times n_{Cl_2}}{n_{pCl_2}} \times \left[\frac{P}{\sum n}\right]^1 = \frac{\alpha^2}{(1-\alpha)}\frac{P}{(1+\alpha)} = \frac{\alpha^2 P}{1-\alpha^2}$$

Q.31 **If pK_b for fluoride ion at 25°C is 10.83, the ionization constant of hydrofluoric acid in water at this temperature is :**

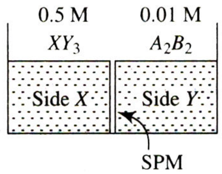
(a) 1.74×10^{-3} **(b) 3.52×10^{-3}** **(c) 6.75×10^{-4}** **(d) 5.38×10^{-2}**

Ans: (c)

Sol: $K_a \times K_b = K_w$

$$\therefore \quad K_a = \frac{K_w}{K_b} = \frac{10^{-14}}{1.48 \times 10^{-11}} = 6.75 \times 10^{-4}$$

Q.32 **X_3Y_2 ($i = 5$) when reacted with A_2B_3 ($i = 5$) in aqueous solution gives brown color. These are separated by a semipermeable membrane AB as shown in the adajcent figure. Due to osmosis, there is :**



(a) brown color formation in side X
(b) brown color formation in side Y
(c) brown color formation in both sof the side X and Y
(d) no brown color formation

Ans: (d)

Sol: Only solvent molecules can passed through semipermeable membrane, so only dilution is possible.

Q.33 **The reaction $[Co(NH_3)_5Br]^{2+} + H_2O \rightarrow [Co(NH_3)_5(H_2O)]^{3+} + Br^-$ is followed by measuring a property of the solution known as the optical density of the solution which may be taken to be linearly related to the concentration of the reactant. The values of optical density are0.80, 0.35 and 0.20 at the end of 20 min, 40 min, and infinite time the start of the reaction which is first order. Calculate the rate constant**

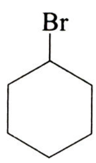
(a) 6.93×10^{-3} min^{-1} **(b) 3.51×10^{-2} min^{-1}**

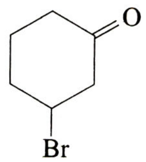
(c) 6.93×10^{-2} min^{-1} **(d) 3.51×10^{-3} min^{-1}**

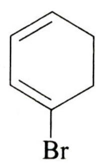
Ans: (c)

Sol: $$k = \frac{2.303}{(40-20)} \log \frac{(.80-.20)}{(.35-.20)} = 6.93 \times 10^{-2}$$

Q.34 **Which of the following compounds has asymmertic centre ?**

(a) 

(b) 

(c) 

(d) Both (b) and (c)

Ans: (b)

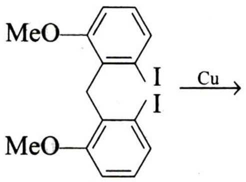
Sol: ----------

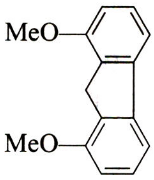
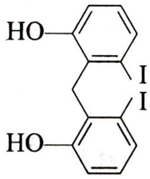
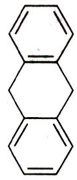
Q.35 **Select the incorrect statement :**
(a) Bromine is more selective and less reactive
(b) Chlorine is less selective and more reactive

(c) Benzyl free radical is more stable than 2° free radical
(d) Vinyl free radical more stable than allyl free radical

Ans: (d)
Sol: Conceptual

Q.36 What would be the major product of the given reaction ?



(a)  **(b)**  **(c)**  **(d)** 

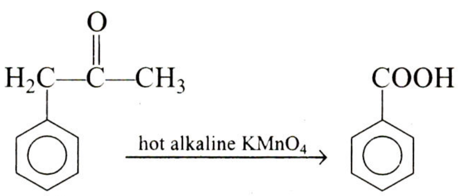
Ans: (a)
Sol: ---------------

Q.37 When a mixture of ethanol and methanol is heated in the presence of concentrated H_2SO_4, the resulting organic product(s) is /are :
(a) $CH_3OC_2H_5$ **(b) CH_3OCH_3 and $C_2H_5OC_2H_5$**
(c) $CH_3OC_2H_5$ and CH_3OCH_3 **(d) $CH_3OC_2H_5$, CH_3OCH_3, and $C_2H_5OC_2H_5$**

Ans: (d)
Sol:
$$CH_3OH + CH_3OH \xrightarrow{H_2SO_{4(conc)}} CH_3OCH_3$$
$$CH_3OH + C_2H_5OH \xrightarrow{H_2SO_4\,(conc)} CH_3OC_2H_5$$
$$C_2H_5OH + C_2H_5OH \xrightarrow{H_2SO_{4(conc)}} C_2H_5OC_2H_5$$

Q.38 Compound (A), $C_9H_{10}O$, is inert to Br_2 in CCl_4. Vigorous oxidation with hot alkanline $KMnO_4$ yields benzoic acid. (A) gives red precipitates with 1, 4-dinitrophenylhydrazine and yellow with NaOl. The possible structure of compound (A) will be :
(a) $PhCOCH_2CH_3$ **(b) $PhCH_2COCH_3$**
(c) $PhCH_2CH_2CHO$ **(d) $PhCH(CH_3)CHO$**

Ans: (b)
Sol: (A) must be a ketone having CH_3 group attached to carbonyl group because only methyl kentons will give yellow precipitate with NaOl.



(A)

Q.39 Zone refining is a technique used primarily for which one of the following process ?
(a) Alloying **(b) Tempering** **(c) Sintering** **(d) Purification**

Ans: (d)

Sol: Zone refining is a method of purification used for semiconductors such as Si, Ge, and Ga.

Q.40 Out of all the known elements, the percentage of transitional elements is approximately :
(a) 30% **(b) 50%** **(c) 60%** **(d) 75%**

Ans: (c)

Sol: $\frac{\text{Transition element} + \text{Inner transition element}}{\text{Total element}} \times 100 \quad \frac{33+28}{105} \times 100 = 58.09 \approx 60\%$

Q.41 Molecular weight of an organic acid is given by :

(a) Equivalent weight $\times$ Basicity **(b) $\frac{\text{Equivalent weight}}{\text{Basicity}}$**

(c) $\frac{\text{Basicity}}{\text{Equivalent weight}}$ **(d) Equivalent weight $\times$ Valency**

Ans: (a)
Sol: Molecular mass of and acid $=$ Equivalent weight $\times$ Basicity.

Q.42 Chargaff's rule states that in an organism :
(a) amount of all bases are equal
(b) amount of adenine (A) is equal to that of thymine (T) and the amount of guanine (G) is equal to that of cytosine (C)
(c) amount of adenine (A) is equal to that of guanine (G) and the amount of thymine (T) is equal to that of cytosine (C)
(d) amount of adenine (A) is equal to that of cytosine (C) and the amount of thymine (T) is equal to guanine (G)

Ans: (b)
Sol: According to Chargaff's rule, the amount of adenine (A) is equal to that of thymine (T) and the amount of guanine (G) is equal to that of cytosine (C).

Q.43 The enthalpy change for a given reaction at 298 K is -*x* J/mol (*x* being positive). If the reaction occurs spontaneously at 298 K, the entropy change at the temperature :
(a) can be negative but numerically larger than *x*/298
(b) can be negative but numerically smaller than *x*/298
(c) can be negative
(d) cannot be positive

Ans: (b)

Sol: It is because of the fact that for spontaneity the value of $\Delta G = (\Delta H - T\Delta S)$ should be less than 0.

If ΔS is negative, the value of $T\Delta S$ shall have to be less than ΔH $\Delta G = \Delta H - T\Delta S$

At equalibrium, $\Delta G = 0$

$\Delta H = T\Delta S \qquad \Delta H = T\Delta S \qquad \Delta S = \frac{-x}{298}$

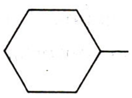
$\Delta G = \Delta H\ (-ve) - T\Delta S\,(-ve)$

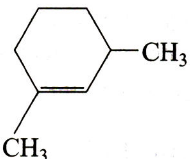
Q.44 The ability of an ion to bring about coagulation of a given colloid depends on :
(a) its size
(b) the magnetic of its charge only
(c) the sign of its charge
(d) both the magnitude and the sign of tis charge

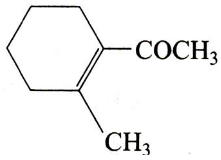
Ans: (d)

Sol: The abilityof an ion to bring about coagulation of a given colloid depends on both the magnitude and sign of its charge.

Q.45 In which of the following molecules all the effects namely inductive, mesomeric, and hyperconjugation operate ?

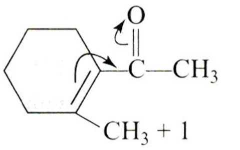
(a) 

(b) 

(c) 

(d)  *

Ans: (c)

Sol: 

Hyperconjugation

Has all effect inductive, measomeric and hyperconjugation.

Q.46 Calculate the electrode potential at 25°**C of Cr^{3+}, $Cr_2O_7^{2-}$ electrode at pOH = 11in a solution of 0.01 M of both Cr^{3+} and $Cr_2O_7^{2-}$ in solution E°, value for the cell.**

Sol: $\underset{0.01}{Cr_2O_7^-} + 14\underset{10^{-3}}{H^+} + 6e \longrightarrow 2Cr^{3+} + 7H_2O;\ E^o = 1.33\,V$

Port =11

pH = 3

$(H^+) = 10^{-3}$

$$E_{cell} = E^o_{cell} - \frac{0.0591}{6}\log\frac{\left[Cr^{3+}\right]^2}{\left[Cr_2O_7^-\right]\left[H^+\right]^{14}} = 0.936\ V$$

Q.47 A 0.500 g sample of magnetite ore (impure Fe_3O_4) is treated so thet he iron is precipitate as Fe-*111* hydroxide. The precipitate is heated and converted to 0.4980 g Fe_2O_3. What is the percentage Fe_3O_4 in the ore ?

Sol: moles of $Fe_2O_3 = \dfrac{0.498}{160}$

$\Rightarrow$ moles of $Fe_3O_4 = \dfrac{2}{3}\times\dfrac{0.498}{160}$

$\Rightarrow$ mass of $Fe_3O_4 = 2.075 \times 10 -3 \times 232 = 0.48$ g

$\%\ Fe_3O_4 = \dfrac{0.48}{0.50}\times 100 = 96\%$

Q.48 Electromagnetic radiations of wavelength 242 nm are just sufficient to ionize sodium atom. Then the ionization energy of sodium in kJ/mol is:

Sol: IE of one sodium atom $= \dfrac{hC}{\lambda}$

and IE of one mole Na atom $= \frac{hC}{\lambda} N_A$

$$= \frac{6.62 \times 10^{34} \times 3 \times 10^{8} \times 6.02 \times 10^{23}}{242 \times 10^{-9}}$$

$= 494.65 \text{kJ} \cdot \text{mol}$

Q.49 NH_3 is produced according to the following reaction :

$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$

In an experiment 0.25 mol of NH_3 is formed when 0.5 mol of N_2 is reacted with 0.5 mol of H_2. What is % yield ?

Sol: H_2 is the limiting reactant and theoretical moles of NH_3 would be

$$n(NH3) = \frac{2}{3} \times 0.5 = \frac{1}{3} \text{ (theoretical)}$$

$$\% \text{ yield} = 0.25 \times 300 = 75 \%$$

Q.50 Naturally occurring thallium consists of two stable isotopes, Tl-203 and Tl-205 (atomic mass = 203.0 and 205.0 respectivaly) and has an average atomic mass of 204.4 what is percentage of Tl -205 ?

Sol: 70.0

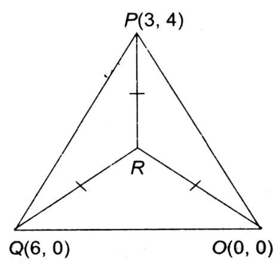
Part - C - MATHEMATICS

Q.51 Let $O(0, 0)$, $P(3, 4)$ and $Q(6, 0)$ be the vertices of the triangle OPQ. The point R insides the triangle OPQ is such that the triangle OPQ, PQR, and OQR are of equal area. The coordinates of R are

(a) $\left(\frac{4}{3}, 3\right)$ **(b)** $\left(3, \frac{2}{3}\right)$ **(c)** $\left(3, \frac{4}{3}\right)$ **(d)** $\left(\frac{4}{3}, \frac{2}{3}\right)$

Ans: (c)

Sol: $\text{Ar}(\Delta OPR) = Ar\ (\Delta PQR) = Ar(\Delta OQR)$



R should be the centroid of ΔPQO

$$\Rightarrow \quad R = \left(\frac{3+6+0}{3}, \frac{4+0+0}{3}\right) = \left(3, \frac{4}{3}\right)$$

Q.52 On the ellips $4x^2 + 9y^2 = 1$, point at which the tangent are parallel to the line $8x = 9y$ are

(a) $\left(\frac{2}{5}, \frac{1}{5}\right)$ (b) $\left(-\frac{2}{5}, \frac{1}{5}\right)$ (c) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$ (d) none of these

Ans: (b)

Sol: Ellipse is $\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{9}} \Rightarrow a^2 = \frac{1}{4}, b^2 = \frac{1}{9}$

The equation of its tangent is $4xx' + 9yy' = 1$

$\therefore \quad m = -\frac{4x'}{9y'} = \frac{8}{9} \Rightarrow x' = -2y'$ and $4x'^2 + 9y'^2 = 1$

$\Rightarrow \quad 4x'^2 + 9\frac{x'^2}{4} = 1 \Rightarrow x' = \pm\frac{2}{5}$

When $x' = \frac{2}{5}, y' = -\frac{1}{5}$ and when $x' = -\frac{2}{5}, y' = \frac{1}{5}$

Hence points are $\left(\frac{2}{5}, -\frac{1}{5}\right)$ and $\left(-\frac{2}{5}, \frac{1}{5}\right)$

Q.53 $y = x + 2$ is any tangent to the parabola $y^2 = 8x$. The point P on this tangent such that the other tangent from it which is perpendicular to it is
(a) (2, 4) (b) (-2, 0) (c) (-1, 1) (d) (2, 0)

Ans: (b)

Sol: Clearly P is the point of intersection of two perpendicular tangents to the parabola $y^2 = 8x$.

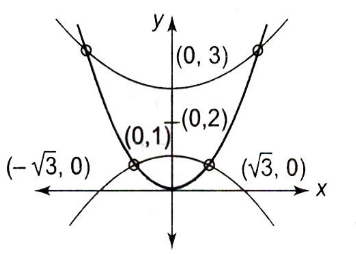
Hence P must lie on the directrix $x + a = 0$ or $x + 2 = 0$

$\therefore \quad x = -2$ Hence the point is $(-2, 0)$

Q.54 Consider the of $y = Ax^2$ and $y^2 + 3 = x^2 + 4y$, where A is a positive constant and $x, y \in R$. Number of points in which tht two graph intersect is
(a) exactly 4
(b) exactly 2
(c) at least 2 but the number of points varies for different positive values of A
(d) zero for at least one positive A

Ans: (a)

Sol:



We have $y = Ax^2, y^2 + 3 = x^2 + 4y; A > 0.$

Now $\qquad y^2 - 4y = x^2 - 3$

$\Rightarrow \quad (y-2)^2 = x^2 + 1 \quad \Rightarrow (y-2)^2 - x^2 = 1$

If $x = 0, y - 2 -1$ or -1 $\quad \Rightarrow y = 3$ or 1

Hence the two graphs of $y = Ax^2\,(A > 0)$ and the hyperbola $(y^2 - 2) - x^2 = 1$ are as shown which can intersect in 4 points.

Q.55 If $f : R \to R$ satisfies $f(x+y) = f(x) + f(y)$ all $x, y \in R$ and $f(1) = 7$ Then $\sum_{r=1}^{n} f(r)$ is

(a) $\frac{7(n+1)}{2}$ (b) $7n(n+1)$ (c) $\frac{7n(n+1)}{2}$ (d) $\frac{7n}{2}$

Ans: (c)

Sol: We have $f(x+y) = f(x) + f(y)$

Putting $x = 1, y = 0 : f(1+0) = f(1) + R(0) \Rightarrow f(0) = 0$

Also $f(1) = 7$

Putting $x = 1,\ y = 1 : f(2) = f(1) + f(1) = 2f(1) = 2(7).$

Similarly $f(3) = 3 \cdot 7$ an so on

$$\Rightarrow \sum_{r=1}^{n} f(r) = 7\ (1 + 2 + ... + n) = \frac{7n(n+1)}{2}$$

Q.56 Which of the following is not true about the function $f(x) = \begin{cases} 5x - 4 & \text{for } 0 < x \le 1 \\ 4x^2 - 3x & \text{for } 1 < x < 2 \\ 3x + 4 & \text{for } x \ge 2 \end{cases}$

(a) continuous at $x = 1$ and $x = 2$

(b) continuous at $x = 1$ but not derivable at $x = 2$

(c) continuous at $x = 2$ bu not derivable at $x = 1$

(d) none of these

Ans: (d)

Sol: $f(1^-) = 1 : f(1^+) = 1 : f(1) = 1$

$f'(1^-) = 5 : f'(1^+) = 5;\ f(2^+) = 10$

$f(2^-) = 10 : f'(2^+) = 3;\ f'(2^-) = 13$

Q.57 If $x = \log\ p$ and $y = \frac{1}{p}$, then

(a) $\frac{d^2y}{dx^2} - 2p = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$

(c) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (d) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

Ans: (c)

Sol: Let $u = f(\tan x) \Rightarrow \frac{dv}{dx} = g'(\sec x) \cdot \sec x \tan x.$

Let $v = g(\sec x)$

$\Rightarrow \quad \frac{dv}{dx} = g'(\sec x) \cdot \sec x \tan x$

$\therefore \quad \frac{du}{dv} = \frac{f'(\tan x)\sec^2 x}{g'(\sec x)\sec x \tan x} = \frac{f'(\tan x)}{g'(\sec x)} \cdot \text{cosec } x$

$\Rightarrow \quad \left.\frac{du}{dv}\right|_{x=\frac{\pi}{4}} = \frac{f'(1)}{g'(\sqrt{2})} \cdot (\sqrt{2}) = \frac{2}{4} \cdot \sqrt{2} = \frac{\sqrt{2}}{2}$

Q.58 If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, where $0 < x \le 1$. then in this interval

(a) both $f(x)$ and $g(x)$ are increasing function

(b) both $f(x)$ and $g(x)$ are decreasing function

(c) $f(x)$ is an increasing function

(d) $g(x)$ is an increasing function

Ans: (c)

Sol: We have $f(x) = \frac{x}{\sin x}$, $0 < x \le 1$

$\Rightarrow \quad f(x) = \frac{\sin x - x\cos x}{\sin^2 x}$

$\Rightarrow \quad f'(x) = \frac{\cos x(\tan x - x)}{\sin^2 x} > 0$ as $\tan x > for\ x \in (0, \pi/2)$

Hence $f(x)$ is increasing function ,

Also, $g(x) = \frac{x}{\tan x}$

$\Rightarrow \quad g'(x) = \frac{x\sec^2 x - \tan x}{\tan^2 x}$ $\qquad g'(x) = \frac{x\sec^2 x - \tan x}{\tan^2 x}$

$= \frac{2x - \sin 2x}{2\sin^2 x} < 0$ as $(2x < \sin 2x \text{ for } x > 0)$

Q.59 The integral $\frac{dx}{(1+\sin x)^{1/2}}$ is

(a) $\sqrt{2}\log\left|\cot\frac{3\pi}{8} - \frac{\pi}{4}\right| + c$

(b) $\sqrt{2}\log\left|\text{cosec}\left(\frac{\pi}{4}+\frac{x}{2}\right) - \cot\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + c$

(c) $\sqrt{2}\log\left|\tan\left(\frac{\pi}{8}+\frac{\pi}{4}\right)\right| + c$

(d) $\sqrt{2}\log\left|\sec\left(\frac{\pi}{4}-\frac{x}{2}\right) + \tan\left(\frac{\pi}{4}-\frac{\pi}{2}\right)\right| + c$

Ans: (c)

Sol: $\int \frac{dz}{(1+\sin x)^{1/2}} = \int \frac{dx}{\cos\frac{x}{2} + \sin\frac{x}{2}}$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)} = \frac{1}{\sqrt{2}} \int \operatorname{cosec}\left(\frac{\pi}{2}+\frac{\pi}{4}\right) dx$$

$$= \frac{1}{\sqrt{2}} \frac{\log\left|\tan\left(\frac{\pi}{4}+\frac{\pi}{8}\right)\right|}{\frac{1}{2}} + c \quad = \sqrt{2}\,\log\left(\tan\left(\frac{x}{4}+\frac{\pi}{x}\right)\right] + c$$

Q.60 **Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}\,dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$. then which one of the following is true ?**

(a) $I > \frac{2}{3}$ **and** $J > 2$

(b) $I < \frac{2}{3}$ **and** $J < 2$

(c) $I < \frac{2}{3}$ **and** $J > 2$

(d) $I > \frac{2}{3}$ **and** $J < 2$

Ans: (b)

Sol: $\int_0^1 \frac{\sin x}{\sqrt{x}}\,dx < \int_0^1 \frac{x}{\sqrt{x}}\,dx = \int_0^1 \sqrt{x}\,dx$

$$= \frac{2}{3} x^{3/2}\Big|_0^1 = \frac{2}{3}$$

$\Rightarrow \quad I = \frac{2}{3}$

$J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx < \int_0^1 \frac{1}{\sqrt{x}}\,dx \quad = 2\sqrt{x}\Big|_0^1 = 2$

$\therefore \quad J \le 2$

Q.61 **The differential equation of the family of curves $y = e^x (A\cos x + B \sin x)$, where A and B are arbitrary costants, is**

(a) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

(b) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 2y = 0$

(c) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + y = 0$

(d) $\frac{d^2y}{dx^2} - 7\,\frac{dy}{dx} + 2y = 0$

Ans: (a)

Sol: $y = e^x (A\cos x + B\sin x)$

$$\frac{dy}{dx} = e^x[-A\ \sin x + B\cos\ x] + e^x\ [A\cos\ x + B\ \sin\ x]$$

$$\frac{dy}{dx} = e^x[-A\ \sin x + B\ \cos\ x] + y \qquad \text{...(i)}$$

Again differentiating w.r.t. "x" we get,

$$\frac{d^2y}{dx^2} = e^x[-A\ \sin x + B\ \cos\ x] + e^+[-A\ \cos x - B\sin x] + \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx} - y\right) - y + \frac{dy}{dx} \qquad \text{[Using (i)]}$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Q.62 **If $\frac{\sin x}{\sin y} = \frac{1}{2}$, $\frac{\cos x}{\cos y} = \frac{3}{2}$ where $x, y \in \left(0, \frac{\pi}{2}\right)$ then the value of $\tan(x+y)$ is equal to**

(a) $\sqrt{13}$ **(b)** $\sqrt{14}$ **(c)** $\sqrt{17}$ **(d)** $\sqrt{15}$

Ans: (d)

Sol: $\frac{\sin x}{\sin y} = \frac{1}{2}, \frac{\cos x}{\cos y} = \frac{3}{2} \quad \Rightarrow \quad \frac{\tan x}{\tan y} = \frac{1}{3}$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y} = \frac{4\tan x}{1 - 3\tan^2 x}$$

Also, $\sin y = 2\sin x$, $\cos y = \frac{2}{3}\cos x$

$\Rightarrow \quad \sin^2 y + \cos^2 y = 4\sin^2 x + \frac{4\cos^2 x}{9} = 1$

$\Rightarrow \quad 36\tan^2 x + 4 = 9\sec^2 x = 9(1 + \tan^2 x)$

$\Rightarrow \quad 27\tan^2 x = 5 \Rightarrow \tan x = \frac{\sqrt{5}}{3\sqrt{3}}$

$\Rightarrow \quad \tan(x+y) = \frac{\frac{4\sqrt{5}}{3\sqrt{3}}}{1 - \frac{15}{27}} = \frac{4\sqrt{5}\cdot 27}{12.3\sqrt{3}} = \sqrt{15}$

Q.63 **In a triangle ABC, *a*, *b*, *c* are the lengths of its sides and *A*, *B*, *C* are in the angles of triangle *ABC*. The correct relation is given by**

(a) $(b-c)\sin\left(\frac{B-C}{2}\right) = a\cos\frac{A}{2}$ **(b)** $(b-c)\cos\left(\frac{A}{2}\right) = a\sin\frac{B-C}{2}$

(c) $(b-c)\cos\left(\frac{A}{2}\right) = a\sin\frac{B-C}{2}$ **(d)** $(b-c)\cos\left(\frac{A}{2}\right) = 2a\sin\frac{B+C}{2}$

Ans: (b)

Sol: Let us consider $\frac{b-c}{a}$, which is involved in three of the option $\frac{b-c}{a}$

$$= \frac{\sin B - \sin C}{2} \qquad = \frac{2\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)}{2\sin A/2\cos A/2}$$

$$= \frac{\sin A/2\sin\left(\frac{B+C}{2}\right)}{\sin A/2\cos A/2} \qquad = \frac{\sin\left(\frac{B+C}{2}\right)}{\cos A/2}$$

$\therefore \quad (b-c)\cos A/2 = a\sin\left(\frac{B-C}{2}\right)$

Q.64 A man from the top of a 100 m high tower sees a car moving towards the tower at an angle of depression of 30°. After some time, the angle of depression becomes 60°. The distance (in meters) travelled by the car daily this time is

(a) $100\sqrt{3}$ (b) $(200\sqrt{3})/3$ (c) $(100\sqrt{3})/3$ (d) $200\sqrt{3}$

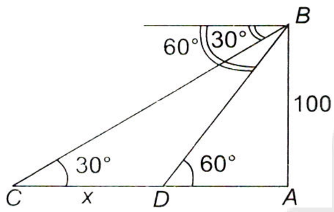
Ans: (b)

Sol: Let A be the foot of the tower AB

Let the initial and final position of the car be C and D respectively

Now, $AB = 100\,m$

From right -angled ΔCAB



$$\frac{AC}{AB} = \cot 30^o = \sqrt{3}$$

$\therefore \quad AC = 100\sqrt{3}$

Again $\frac{AD}{100} = \cot 60^o = \frac{1}{\sqrt{3}} \qquad \therefore \quad AD = \frac{100}{\sqrt{3}}$

$\therefore \quad CD = AC - AD = 100\sqrt{3} - \frac{100}{\sqrt{3}} \quad = \frac{300-100}{\sqrt{3}} = \frac{200}{\sqrt{3}} = \frac{200\sqrt{3}}{3}$

Q.65 $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$, then the vector $\vec{c}$ such that $\vec{a}\cdot\vec{c} = 2$ and $\vec{a}\times\vec{c} = \vec{b}$ is

(a) $\frac{1}{3}(\hat{i} - 2\hat{j} + \hat{k})$ (b) $\frac{1}{3}(-\hat{i} + 2\hat{j} + 5\hat{k})$ (c) $\frac{1}{3}(\hat{i} + 2\hat{j} - 5\hat{k})$ (d) $\frac{1}{3}(-\hat{i} + 2\hat{j} - 5\hat{k})$

Ans: (b)

Sol: $\vec{a}\times\vec{b} = \vec{a}\times(\vec{a}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{a} - (\vec{a}\cdot\vec{a})\vec{c} = 2\vec{a} - 3\vec{c}$

But $\quad \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{k}$

Hence $3\vec{c} = 2\vec{a} - (3\hat{i} - 3\hat{k}) \quad = (2\hat{i} + 2\hat{j} + 2\hat{k}) - (3\hat{i} - 3\hat{k})$

$= -\hat{i} + 2\hat{j} + 5\hat{k}$

$\Rightarrow \quad \vec{c} = \frac{1}{3}(-\hat{i} + 2\hat{j} + 5\hat{k})$

Q.66 Two system of rectangular axes have the same origin. If a plane cuts, then at distance *a*, *b*, *c* and *a'*, *b'*, *c'* from the origin, then

(a) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$ **(b)** $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(c) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$ **(d)** $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$

Ans: (c)

Sol: The plane are $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and $\frac{x}{a'}+\frac{y}{b'}+\frac{z}{c'}=1$

Since the perpendicular distance of origin on the planes is sme

$$\therefore \quad \left|\frac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right|=\left|\frac{-1}{\sqrt{\frac{1}{a'}+\frac{1}{b'}+\frac{1}{c'}}}\right|$$

$$\Rightarrow \quad \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$$

Q.67 At a telephone enquiry system the number of phone calls regarding relevent enquiry follow Poisson distribution with an average of 5 phone cells during 10-minute time intervals. The probability that there is at the most one phone call during a 10-minutes time period is

(a) $\frac{6}{5}$ **(b)** $\frac{5}{6}$ **(c)** $\frac{6}{55}$ **(d)** $\frac{6}{e^5}$

Ans: (d)

Sol: $P(X=r)=\frac{e^{-m}m^r}{r!}$

$$\therefore \quad P(X\le 1)=P(X=0)+P(X=1) \quad = e^{-m}+\frac{e^{-m}\cdot m}{1!}$$

Given, $m = mean = 5$

$$P(X\le 1)=e^{-5}+e^{-5}\cdot 5=\frac{6}{e^5}$$

Q.68 The following data is given the distribution of height of students

Height (in cm)	**160**	**150**	**152**	**161**	**156**	**154**	**155**
Number of studunts	**12**	**8**	**4**	**4**	**3**	**3**	**7**

The median of the distribution is

(a) 154 **(b) 155** **(c) 160** **(d) 161**

Ans: (b)

Sol: Arranging the data in ascending order of magnitude, we obtain

Height	150	152	154	155	156	160	161
Number of students	8	4	3	7	3	12	4
Cumulative frequency	8	12	15	22	25	37	41

Here, total number of items is 41, *i.e.* an odd number

Hence, the medium is $\frac{41+1}{2}$ th *i.e.* 21st item. from cumulative frequency table, we find that medium *i.e.* 21st item is 155. (All items from 16th to 22nd are equal, each 155.)

Q.69 If each of the following statements it true, then $p \Rightarrow -q$; $q \Rightarrow r$; $-r$

(a) *p* is false (b) *p* is true (c) *q* is true (d) none of these

Ans: (a)

Sol: Since -*r* is true, therefore, r is false

Also $q \Rightarrow r$ is true, therefore, q is false. ($\therefore$ A true statement cannot impty a false one) Also, $p \Rightarrow q$ is true, therefore, p must be false.

Q.70 $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is

(a) a tautology

(b) a contraction

(c) tautology and contradiction

(d) neither a tautology nor a contradiction

Ans: (b)

Sol:

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge q$	$(p \wedge \sim q \wedge (\sim p \wedge q))$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

Q.71 A survey shows that 63% of the Indian like cheese whereas 76% like apples. If *x*% of the Indian like both cheese and apples, then *x* can be ?

(a) 40 (b) 65 (c) 39 (d) none of these

Ans: (c)

Sol: Let *A* denote the set of Indians who like cheese and Let *B* denot the set of Indians who like apples.

Let the population of Indians be 100.

Then, $n(A) = 63$, $n(B) = 76$

Now, $m(A \cup B) = n(A) + n(B) - n(A \cap B) = 63 + 76 - n(A \cap B)$

$\therefore$ $n(A \cup B) + n(A \cap B) = 139$ $n(A \cap B) = 139 - n(A \cup B)$

But $n(A \cup B) \leq 100$ $139 - n(A \cup B) \geq 139 - 100 = 39$

$n(A \cap B) \geq 39$, i,e; $39 \leq n(A \cap B)$ ...(i)

Again $A \cap B \subseteq A$, $A \cap B \subseteq B$

$\therefore$ $n(A \cup B) \leq n(A) = 63$ and $n(A \cap B) \leq n(B) = 76$

$\therefore$ $n(A \cap B) \leq 63$ ...(ii)

Then, $39 \leq n(A \cap B) \leq n(A) \Rightarrow 39 \leq x \leq 63$

Q.72 In a 12-storey building three persons enter a lift cabin, It is known that they will leave the lift at different storeys. In how many ways can they do so if the lift does not stop at the second storey ?

Sol: Each of the three persons can leave the lift at one of the ten floors (others that 2nd storey and one at which they enter the lift). Since they leave the lift at diffferent storeys.

Q.73 $f(x) = \dfrac{3x^2 + ax + a + 1}{x^2 + x - 2}$ **and** $\lim\limits_{x \to -2} f(x)$ **exists, then the value of** a **is _____?**

Sol: $f(x) = \dfrac{3x^2 + ax + a + 1}{(x+2)(x-1)}$

as $x \to -2, D^r \to 0 \quad x \to -2, N^r \to 0$

$\therefore\ 12 - 2a + a + 1 = 0 \Rightarrow a = 13$

Q.74 **Let** $f(x)$ **be a continuous function defined for** $1 \le x \le 3$**. If** $f(x)$ **takes rational values for all x and** $f(2) = 10$ **then the value of** $f(1.5)$ **is _____?**

Sol: As $f(x)$ is continuous in [1,3], $f(x)$ will attain all values bewteen $f(1)$ and $f(3)$. As $f(x)$ takes rational values for all x and there are innumerable irrational values between $f(1)$ and $f(3)$, $f(x)$ can take rational values for all x if $f(x)$ has a constant rational value at all points between x = 1 and x = 3. So, $f(2) = f(1.5) = 10$.

Q.75 **If** $\int e^x \sin 2x\ dx$**, then for what value of K,** $KI = e^x(\sin 2x - 2\cos 2x) +$ **constant?**

Sol: $I = \int e^x \sin 2x\, dx = \sin 2x.e^x - 2\int \cos 2x.e^x dx$

$= \sin 2x.e^x - 2\cos 2x.e^x - 4\int e^x \sin 2x\, dx$

$\Rightarrow 5I = e^x(\sin 2x - 2\cos 2x) +$ constant

Equating the given value, we get K = 5.

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