A Division of Aggarwal Educare
Aggarwal CLASSES ™

JEE (MAIN)

TEST PAPER

SUBJECT : PHYSICS,CHEMISTRY, MATHEMATICS ***TEST CODE : TSJMT218***

ANSWER PAPER

TIME : 3 HRS ***MARKS : 300***

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark(-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or “un- darken” a darkened bubble.

Part A - PHYSICS

Q.1 In an experiment, the angles are required to the measured using an instrument. 29 divisions of the main scale exactly coincide with 30 divisions of the Vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5°), then the least count of the instrument is

(a) One minute (b) Half minute (c) One degree (d) Half degree

Ans: (d)

Sol: Least count $= \frac{\text{Value of main scale division}}{\text{Number of division on vernier scale}} = \frac{1}{30}\ MSD = \frac{1}{30} \times \frac{1^o}{30} = \frac{1^o}{60} = 1 \text{ min}$

Q.2 The range R of projectile is same when its maximum heights are h_1 and h_2. What is the relation between R and h_1 and h_2?

(a) $R = \sqrt{h_1 h_2}$ (b) $R = \sqrt{2h_1 h_2}$ (c) $R = 2\sqrt{h_1 h_2}$ (d) $R = 4\sqrt{h_1 h_2}$

Ans: (d)

Sol: For equal ranges, the body should be projected with angle θ or $(90^o - \theta)$ from the horizontal and for these angles :

$$h_1 = \frac{u^2 \sin^2\theta}{2g} \quad \text{and} \quad h_2 \frac{u^2 \cos^2\theta}{2g}$$

By multiplication of both height :

$$h_1 h_2 = \frac{u^2 \sin^2\theta \cos^2\theta}{4g^2} = \frac{1}{16}\left(\frac{u^2 \sin 2\theta}{g}\right)^2$$

$$\Rightarrow \quad 16 h_1 h_2 = R^2 \Rightarrow R = 4\sqrt{h_1 h_2}$$

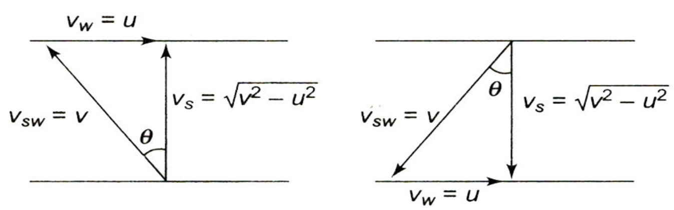
Q.3 Two boys P and Q, are playing on a river bank. P plans to swim across the river directly and comes back. Q plans to swim down stream by a length equal to the width of the river and comes back. The boy succeeding in less time wins. Assuming the swimming rate of both P and Q to be the same, it can be concluded the :

(a) P wins (b) Q wins

(c) A draw take place (d) Nothing certain can be stated.

Ans: (b)

Sol: As shown in figure, let speed of P as well as Q is v and river speed is u. Also let h be the width of river. For P,



Crossing the river

Coming back to starting point

Time to cross the river, $t_1 = \frac{h}{\sqrt{v^2 - u^2}}$

Time to come back to starting point $t_2 = \frac{h}{\sqrt{v^2 - u^2}}$

Hence, total time to cross the river

$$T = t_1 + t_2 = \frac{2h}{\sqrt{v^2 - u^2}} = \frac{2h}{v\sqrt{1 - u^2 / v^2}}$$

Time taken by $Q = \frac{h}{v+u} + \frac{h}{v-u}$

$$T' = \frac{h}{v+u} + \frac{h}{v-u} = \frac{2hv}{v^2 - u^2} = \frac{2h}{v(1 - u^2 / v^2)}$$

Solving, we find that the time taken by Q is less than that of P.

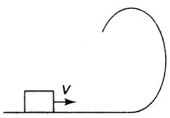
Q.4 A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is

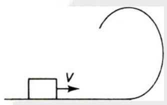
(a) $\frac{PM}{M+m}$ **(b)** $\frac{Pm}{M+m}$ **(c)** $\frac{Pm}{M-n}$ **(d)** P

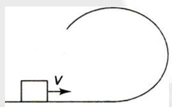
Ans: (a)

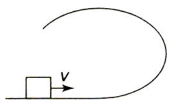
Sol: Required force $F = Ma = MP / (M+m)$

Q.5 A small block is shot into each of the four tracks as shown in the options below. Each of the tracks rises to the same height. The speed with which the block entres the track is same in all cases. At the heighest point of the track, the normal reaction is maximum in

(a) 

(b) 

(c) 

(d) 

Ans: (a)

Sol: Normal reaction at the heighest point of the path $R = \frac{mv^2}{r} - mg$

For maximum R, value of the radius of curvature (r) should be minimum and it is minimum in first condition.

Q.6 The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constant and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{at\ equilibrium}]$, D is

(a) $\frac{b^2}{2a}$ **(b)** $\frac{b^2}{12a}$ **(c)** $\frac{b^2}{4a}$ **(d)** $\frac{b^2}{6a}$

Ans: (c)

Sol: $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$ $U(x = \infty) = 0$

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6} \quad U(x = \infty) = 0$$

At equilibrium, $F = 0$

$$x^6 = \frac{2a}{b} \qquad U_{at\ equilibrium} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = \frac{-b^2}{4a}$$

$$D = \left[U_{(x=\infty)} - U_{at\ equilibrium}\right] = \frac{b^2}{4a}$$

Q.7 **A bag *P* (mass *M*) hangs by a long thread and a bullet (mass *m*) comes horizontally with velocity *v* and gets caught in the bag. Then for the combined (bag + bullet) system,**

(a) Momentum is $\frac{mvM}{M+m}$ **(b) Kinetic energy is $\frac{mV^2}{2}$**

(c) Momentum is $\frac{mv(M+m)}{M}$ **(d) Kinetic energy is $\frac{m^2V^2}{2(M+m)}$.**

Ans: (d)

Sol: Velocity of combined system $V = \frac{mv}{m+M}$

Momentum for combined system, $= (m+M)\,V = (m+M)\,\frac{mv}{m+M}$

Kinetic energy for combined system

$$= \frac{1}{2}(m+M)V^2 \qquad = \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2$$

$$= \frac{1}{2}(m+M)\,\frac{m^2v^2}{(m+M)} = \frac{m^2v^2}{2(m+M)}$$

Q.8 **Two circular disc A and B are of equal masses and thickness but made of metals with densities d_A and d_B ($d_A > d_B$). If their moments of inertia about an axis passing through centres and normal to the circular faces be I_A and I_B. then**

(a) $I_A = I_B$ **(b) $I_A > I_B$** **(c) $I_A < I_B$** **(d) $I_A >=< I_B$**

Ans: (c)

Sol: Moment of inertia of circular disc about an axis passing through centre and normal to the circullar foce

$$I = \frac{1}{2}MR^2 = \frac{1}{2}M\left(\frac{M}{\pi/\rho}\right) \qquad \left[\text{as } M = V\rho = \pi R^2 t\rho \Rightarrow R^2 = \frac{M}{\pi/\rho}\right]$$

$$\Rightarrow \quad I = \frac{M^2}{2\pi/\rho} \quad \text{or} \quad I \propto \frac{1}{\rho} \quad \text{[If mass and thickness are constant]}$$

So, in the equation $\frac{I_A}{I_B} = \frac{d_B}{d_A}$

$\therefore\ I_A < I_B$ [as $d_A > d_B$]

Q.9 **Two bodies of masses m and 4m are placed at a distance *r*. The gravitational potential at a point on the line joining them where the gravitational field is zero is**

(a) zero **(b) $-\frac{4Gm}{r}$** **(c) $-\frac{6Gm}{r}$** **(d) $-\frac{9Gm}{r}$**

Ans: (d)

Sol: Let us find the point where gravitational field is zero.

$$\frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2} \Rightarrow \frac{1}{x} = \frac{2}{r-x} \Rightarrow x = \frac{r}{3}$$

$$V = -\frac{Gm}{r/3} - \frac{G(4m)}{2r/3}$$

$$= -\frac{3Gm}{r} - \frac{6Gm}{r} = -\frac{9Gm}{r}$$

Q.10 A wave is represented by the equation $Y = 7\sin\left(7\pi t - 0.04\pi x\ 9 + \frac{\pi}{3}\right)$ x is in meters and t is in seconds. The speed of the wave is

(a) 175 m/s **(b) 49π m/s** **(c) $\frac{49}{\pi}$ m/s** **(d) 0.28π m/s**

Ans: (a)

Sol: Standard equation $y = A\sin(\omega t - kx + \phi_0)$

In a given equation $\omega = 7\pi$, $k = 0.04\pi$

$$v = \frac{\omega}{k} = \frac{7\pi}{04\pi} = 175 \text{ m/s}$$

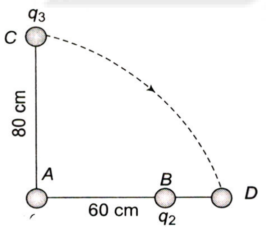
Q.11 Two equal negative charges $-q$ are fixed at points $(0, a)$ and $(0, -a)$ on y-axis. A positive charge Q is released from rest at the point $(2a, 0)$ on the x-axis. The charge Q will
(a) Execute simple harmonic motion about the origin.
(b) Move to the origin and remain at rest.
(c) Move to infinity
(d) Execute oscillatory but not simple harmonic motion.

Ans: (d)

Sol: By symmetry of problem the components of force on Q due to charges at A and B along y-axis will cancel each other while along x-axis will add up and will be along CO. Under the action of this force, charge Q will move towards O. If at any time, charge Q is at a distance x form O, then

$$F \Rightarrow 2F\cos\theta = 2\,\frac{1}{4\pi\varepsilon_0}\frac{-qQ}{(a^2+x^2)} \times \frac{x}{(a^2+x^2)^{1/2}}$$

Q.12 In figure, are shown charges $q_1 = +2 \times 10^{-8}\,C$ and $q_2 = -0.4 \times 10^{-8}\,C$. A charge $q_3 = 0.2 \times 10^{-8}\,C$ in moved along the arc of a circle from C to D. The potential energy of q_3 will



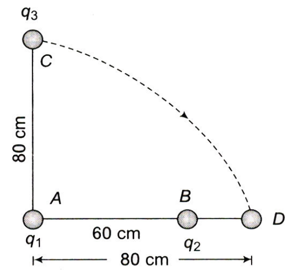
(a) Increase approximately by 76 % **(b) decrease approximately by 76%**
(c) remain same **(d) increase approximately by 12%.**

Ans: (b)

Sol: Initial potential energy q_3. $U_i = \left(\frac{q_1 q_3}{0.8} + \frac{q_2 q_3}{1}\right) \times 9 \times 10^9$

Final potential energy q_3. $U_f = \left(\frac{q_1 q_3}{0.8} + \frac{q_2 q_3}{0.2}\right) \times 9 \times 10^9$

Change in potentia energy $= U_f - U_i$

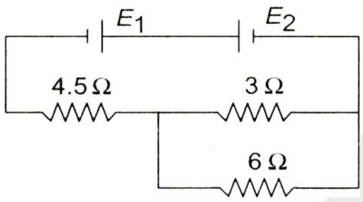


Now percentage change in potential energy

$$= \frac{U_f - U_i}{u_i} \times 100 = \frac{q_2 q_3 \left(\frac{1}{0.2} - 1 \right) \times 100}{q_3 \left(\frac{q_1}{0.8} + \frac{q_2}{1} \right)}$$

On putting the values $= -76\%$

Q.13 In the circuit shown below the cells E_1 and E_2 have emfs 4 V and 8 V and internal resistance in figure. 0.5 Ω and 1 Ω, respectively. Then the potential difference across cell E_1 and E_2 will be



(a) 3.75 V, 7.5V **(b) 4.25 V, 7.5 V**
(c) 3.75 V, 3.5 V **(d) 4.25 V, 4.25 V**

Ans: (b)

Sol: In the given circuit diagram, external resistance $R = \frac{3 \times 6}{3 + 6} + 4.5 = 6.5\ \Omega$, Hence remain current through the circuit

$$i = \frac{E_2 - E_1}{R + r_{eq}} = \frac{8 - 4}{6.5 + 0.5 + 0.5} = \frac{1}{2} A$$

Cell 1 is charging, so from its emf equation $E_1 = V_1 - ir_1$

$$\Rightarrow \quad 4 = V_1 - \frac{1}{2} \times 0.5 \quad \Rightarrow \quad V_1 = 4.25V$$

Cell 2 is discharging, so from its emf equation $E_2 = V_2 + ir_2$

Q.14 A potentiometer has uniform potential gradient across it. Two cells connected in series (i) to support each other and (ii) to oppose each other are balanced over 6 m and 2 m respectively, on the potentiometer wire. The emf's of the cells are in the ratio of
(a) 1 : 2 **(b) 1 : 1** **(c) 3 : 1** **(d) 2 : 1**

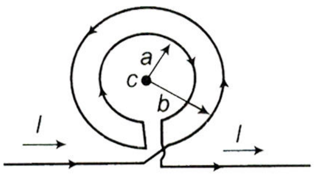
Ans: (d)

Sol: If suppose emf's of the cells are E_1 and E_2, respectively, then

$E_1 + E_2 = x \times 6$ ($x =$ potential gradient) ....(i) and $E_1 - E_2 = x \times 2$ ....(ii)

$$\Rightarrow \frac{E_1 + E_2}{E_1 - E_2} = \frac{3}{1} \Rightarrow \frac{E_1}{E_2} = \frac{2}{1}$$

Q.15 An otherwise infinite, straight wire has two concentric loops of radii *a* and *b* carrying equal currents in opposite directions as shown in figure. The magnetic field at the common centre is zero for



(a) $\frac{a}{b} = \frac{\pi - 1}{\pi}$ **(b)** $\frac{a}{b} = \frac{\pi}{\pi + 1}$ **(c)** $\frac{a}{b} = \frac{\pi - 1}{\pi + 1}$ **(d)** $\frac{a}{b} = \frac{\pi + 1}{\pi - 1}$

Ans: (b)

Sol: $B_{centre} = 0$ $\frac{\mu_0 I}{4\pi b} \Theta + \frac{\mu_0 I}{2b} \Theta + \frac{\mu_0 I}{2b} \Theta + \frac{\mu_0 I}{2a} \otimes = 0$

$$\frac{\mu_0 I}{2\pi b} + \frac{\mu_0 I}{2b} - \frac{\mu_0 I}{2a} = 0$$

$$\frac{1}{2\pi b} + \frac{1}{2b} = \frac{1}{2a} = \frac{a}{b} = \frac{\pi}{\pi = 1}$$

Q.16 A bar magnet suspended by a horse's hair lies in the magnetic meridian where there is no twist in the hair, on turning the upper end of the hair through 150°. the magnet is deflected through 30° from the meridian. Then the angle through which upper end of the hair has to be twisted to deflect the magnet through 90° from the meridian is
(a) 450° **(b) 360°** **(c) 330°** **(d) 150°**

Ans: (c)

Sol: During the twist of hair, two couples come in action.
(1) A deflecting couple due to torsion in the hair. This couple is proportional to the angle of the torsion.
(2) A restoring couple which tends to bring the magnet back into the meridian.
In the first case, the upper end is turned through 150° and magnet deflects through 30°.

$\therefore$ Net twist in hair $= 150^o - 30^o = 120^o$.

Restoring couple $= MB \sin 30^o$

Now, $120^o \propto MB \sin 30^o$ ...(1)

In second case, let required angle be θ.

Then, net twist in hair $= \theta - 90^o$ and $\theta = 90^o \propto MB \sin 90^o$ ...(2)

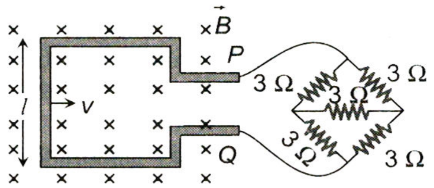
Dividing (ii) by (i),

$$\frac{\theta - 90^o}{120^o} = \frac{MB \sin 90^o}{MB \sin 30^o} = \frac{\sin 90^o}{\sin 30^o} = 2$$

$\therefore$ $\theta = 330^o$

Q.17 A square metalic wire loop of side 0.1 m and resistance of 1 Ω is moved with a constant velocity in a magnetic field 2 wb/m² as shown in figure. The magnetic field is perpendicular to the plane of the loop, which is connected to a network of

resistances. What should be the velocity of loop so as to have a steady current of 1 mA in loop



(a) 1 cm/s **(b) 2 cm/s** **(c) 3 cm/s** **(d) 4 cm/s**

Ans: (b)

Sol: Equivalent resistance of the given wheastone bridge circuit (balanced) is **3 Ω** so, total resistance is circuit is $R = 3 + 1 = 4\ \Omega$. The emf induces in the loop $e = Bvl$.

So, induced current $i = \frac{e}{R} = \frac{Bvl}{R} \Rightarrow 10^{-3} = \frac{2 \times v \times (10 \times 10^{-2})}{4}$

$\Rightarrow \quad v = 2$ cm/s

Q.18 Let frequency $v = 50$ Hz, and capacitance $C = 100\mu F$ in an ac circuit containing a capacitor only. If the peak value of the current in the circuit is 1.57 A. The expression for the instantaneous voltage across the capacitor will be

(a) $E = 50 \sin\left(100\pi t - \frac{\pi}{2}\right)$ **(b)** $E = 100 \sin(50\pi t)$

(c) $E = 50 \sin(100\pi t)$ **(d)** $E = 50 \sin\left(100\pi t + \frac{\pi}{2}\right)$

Ans: (a)

Sol: Peak value of voltage $V_0 = i_0 X_C = \frac{i_0}{2\pi v C}$

$\Rightarrow \quad \frac{1.57}{2 \times 3.14 \times 50 \times 100 \times 10^{-6}} = 50\,V$

Hence, if equation of current $i = i_0 \sin \omega t$ then in capacitive circuit voltage is $V = V_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

$\Rightarrow V = 50\left(\sin 2\pi \times 50t - \frac{\pi}{2}\right) = 50 \sin\left(10\pi t - \frac{\pi}{2}\right)$

Q.19 An electromagnetic wave of $v = 3$ MHz passes from vaccum into dielectric medium with $\in = 4.0 \in_0$ Then,

(a) Wavelength is doubled and frequency becomes half.
(b) Wavelength is doubled and frequency same
(c) Wavelength and frequency both remain unchanged
(d) Wavelength is halved but frequency remains same.

Ans: (d)

Sol: $C = \frac{1}{\sqrt{\mu_0 \in_0}} C' \qquad = \frac{1}{\sqrt{\mu_0 4 \in_0}} = \frac{1}{2} C$

$\therefore \quad C = v\lambda \Rightarrow C \propto \lambda$

So, then $C' = C / 2\lambda' = \lambda / 2$

Q.20 Direction : The question has a paragraph followed by two statements, Statement-1 and Statement-2. Of the given four alternative after the statements, choose the one that describes the statements. A thin air film formed by putting the convex surfaace of a plane-convex lens over a plane glass plate. With monochromoatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement -1 : When light reflects from the air glass plate interface, the reflected wave suffers a phase change of π.

Statements -2 : The centre of the interference pattern is dark.

(a) Statement-1 is true, Statement-2 is false

(b) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1

(c) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1

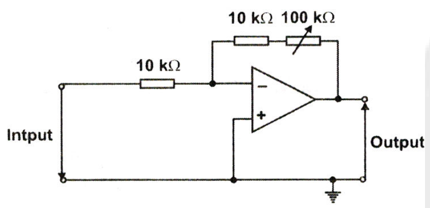
(d) Statement-1 is flase, Statement-2 is true.

Ans: (a)

Sol: Statement-1. When light reflects from denser medium (glass), a phase difference of π is generated.

Statement-2. Centre maxima of minima depend on thickness of the lens.

Q.21 The circuit shown in figure includes an ideal operational amplifier.



Which of the following gives the minimum and maximum values of the voltage gain of the circuit ?

Sol: Gain of the inverting amplifer is given by

$$\text{Gain} = \frac{10\,k\Omega + R}{10k\Omega} \quad \text{Where } R = 0 \text{ to } 100\,k\Omega$$

$$\Rightarrow \quad \text{Minumum gain} = \frac{10\,k\Omega}{10\,k\Omega} = 1 \text{ in magnitude}$$

Q.22 The plates of a capacitor are charged to a potential difference of 320 V and are then connected across a resistor. The potential difference across the capacitor decays exponentially with time. After 1s, the potential difference between the plates of the capacitor is 240 V, then after 2 and 3 s the potential difference between the plates will be ?

(a) 200 and 180 V **(b) 180 and 135 V**

(c)160 and 80 V **(d) 140 and 20 V**

Ans: (b)

Sol: During discharging potential difference across the capacitor falls exponetially as

$V = V_0 e^{-\lambda t}$ $(\lambda = 1/RC)$

Where $V =$ Instantaneous P.D. and $V_0 =$ max P.D. across capacitor

After 1 second $V_1 = 320\ (e^{-\lambda})$

$\Rightarrow\ 240 = 320\,(e^{-\lambda})$ $\qquad \Rightarrow\ e^{-\lambda} = \frac{3}{4}$

After 2 second $V_2 = 320\,(e^{-\lambda})^2$

$\Rightarrow \qquad 320 \times \left(\frac{3}{4}\right)^2 = 180$ volt

After 3 second $V_3 = 320(e^{-\lambda})^3 = 320 \times \left(\frac{3}{4}\right)^3 = 135$ volt

Q.23 A body of mass 10 kg is lying on a rough plane inclined at an angle of 30° to the horizontal and the coefficient of friction is 0.5. The minimum force required to pull the body up the plane is ______?

Sol: $F = mg(\sin\theta + \mu\cos\theta)$

$= 10 \times 9.8(\sin 30 + 0.5 \cos 30) = 91.4$ N

Q.24 A particle travels 10 m in the first 5 s and 10 m in next 3 s. Assuming constant acceleration what is the distance traveled in next 2 s?

Sol: Let initial $(t = 0)$ velocity of particle $= u$ For first five second of motion, $s_5 = 10$ m, so by using

$s = ut + \frac{1}{2}at^2$

$10 = 5u + \frac{1}{2}a(5)^2 \Rightarrow 2u + 5a = 4$ ....(i)

For first eight second of motions, $s_8 = 20$ m

$20 = 8u + \frac{1}{2}a(8)^2 \Rightarrow 2u + 8a = 5$ ...(ii)

By solving (i) and (ii), $u = \frac{7}{6}$ m/s, $a = \frac{1}{3}$ m/s^2

Now distance traveled by particle in total 10 s.

$s_{10} = u \times 10 + \frac{1}{2}a(10)^2$

By substituting the value of u and a, we will get $s_{10} = 28.3$ m

So, the distance in last two seconds $= s_{10} - s_8 = 28.3 - 20 = 8.3$ m

Q.25 Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of -0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in the main scale as 35. The diameter of the wire is _____?

Sol: diameter = MSR + CSR + $\times$LC + ZE

$= 3 + 35 \times (0.5/50) + 0.03 = 3.38\,\text{mm}$

Part - B - CHEMISTRY

Q.26 The fraction of volume occupied by the nucleus with respect to the total volume of an atom is :

(a) 10^{-15} **(b) 10^{-5}** **(c) 10^{-30}** **(d) 10^{-10}**

Ans: (a)

Sol: Volume fraction $= \dfrac{\text{Volume of nucleus}}{\text{Total volume of atom}} = \dfrac{(4/3)\pi(10^{-13})}{(4/3)\pi(10^{-8})} = 10^{-15}$

Q.27 Among the following species, identify the isostructural pairs : NF_3, NO_3^-, BF_3, H_3O^+, HN_3 :

(a) [NF_3, NO_3^-] and [BF_3, $H_3\overset{\oplus}{O}$] **(b) [NF_3, HN_3] and [NO_3^-, BF_3]**

(c) [NF_3, H_3O^+] and [NO_3^-, BF_3] **(d) [NF_3, H_3O^+] and [HN_3, BF_3]**

Ans: (c)

Sol: NF_3 and H_3O^+ are pyramidal, while NO_3^- and BF_3 are planar, Hence option (c) is correct.

Q.28 Helium atom is two times heavier than a hydrogen molecule. At 298 K, the average kinetic energy of a helium atom is :

(a) two times that of hydrogen molecules
(b) same as that of hydrogen molecule
(c) four times that of a hydrogen molecule
(d) half that of a hydrogen molecule

Ans: (b)

Sol: The average kinetic energy of an atom is given as (3/2)kT. Therefore, it does not depend on the mass of the atom.

Q.29 $\mathbf{C_2H_{6(g)} + 3.5\,O_{2(g)} \rightarrow 2CO_{2(g)} + 3H_2O_{(g)}}$

$\Delta S_{vap}(H_2O, 1) = x_1$ **cal/K (boiling point + T_1)**

$\Delta H_f(H_2O, 1) = x_2$

$\Delta H_f(CO_2) = x_3$

$\Delta H_f(C_2H_6) = x_4$

Hence ΔH for the reaction is :

(a) $2x_3 + 3x_2 - x_4$ **(b)** $2x_3 + 3x_2 - x_4 + 3x_1T_1$

(c) $2x_3 + 3x_2 - x_4 - 3x_1T_1$ **(d)** $x_1T_1 + x_2 + x_3 - x_4$

Ans: (b)

Sol: $C_2H_{6(g)} + 3.5\,O_{2(g)} \rightarrow 2CO_{2(g)} + 3H_2O_{(g)}$ $\quad H_2O_{(l)} \rightarrow H_2O_{(g)}$

$\Delta H_f = (X_1T_1)$ ...(i)

$H_{2(g)} + \frac{1}{2}O_{2(g)} \rightarrow H_2O_{(l)}$

$\Delta H = [X_2]$ ...(ii)

$\Delta H_F^o(H_2O, g) = [X_1T_1 + X_2]$

$\Delta H_r = 2X_3 + 3X_1T_1 + 3X_2 - X_4 = 2X_3 + 3X_2 - X_4 + 3X_1T_1$

Q.30 The exothermic formation of ClF_3 is represeneted by the equation :

$Cl_{2(g)} + 3F_{2(g)} \rightleftharpoons 2ClF_{3(g)}$; $\Delta H = -329$ kJ Which of the following will increase the quantity to ClF_3 in an equilibrium mixture of Cl_2, F_2, ClF_3 ?

(a) Increasing the temperature **(b) Removing Cl_2**

(c) Increasing the volume of container **(d) Adding F_2**

Ans: (d)

Sol: Favorable condition for forward reaction according to Le Chatelier principle are :

(i) Decrease in temperature (ii) Increase in concentration of reactant

(iii) Increase in pressure

Q.31 **The solubility product of AgI at 25°C is 1.0×10^{-16} mol^2 / L^2. The solubility if AgI in 10^{-4} N Solubility of KI at 25°C is approximetely (in mol/L) :**

(a) 1.0×10^{-8} **(b) 1.0×10^{-16}** **(c) 1.0×10^{-12}** **(d) 1.0×10^{-10}**

Ans: (c)

Sol: $AgI \rightleftharpoons \underset{(s)}{Ag^+} + \underset{(s)}{I^+}$; $K_{sp} = S^2 = 10^{-4} \times S$

$S = \frac{1.0 \times 10^{-16}}{10^{-4}} = 1 \times 10^{-12}$ mol^2/L^2

Q.32 **A body centered cubic lattice is made up of hollow spheres of B. Spheres of solid A are present in hollow spheres of B. Radius A is half radius of B. What is the ratio of the total volume of spheres of B unoccupied by A in a unit cell and volume of unit cell ?**

(a) $\frac{7\sqrt{3}\pi}{64}$ **(b) $\frac{7\sqrt{3}}{128}$** **(c) $\frac{7\pi}{24}$** **(d) none of these**

Ans: (d)

Sol: Effective number of atoms of B present in a unit cell $= 2$

Total valume of B unoccupied by A in a unit cell

$$= 2 \times \frac{4}{3}(R^3 - r^3) \times \pi = \frac{7\pi R^3}{3} \qquad \left(\because r = \frac{R}{2}\right)$$

Volume of unit cell $= a^3$

$$\Rightarrow \left(\frac{4R}{\sqrt{3}}\right)^3 = \frac{64}{3\sqrt{3}} R^3 \qquad \left(\because \sqrt{3a} = 4R\right)$$

$$\text{Desired ratio} = \frac{3}{(643\sqrt{3})R^3} = \frac{7\pi}{64\sqrt{3}}$$

Q.33 **Resistance of a conductivity cell filled with a solution of an electrolyte of concentration 0.1 M is 100 ohm. The conductivity of this solution is 1.29 S/m. Resistance of the same cell filled with 0.02 M of the same solution if the electrolyte is 520 ohm. The molar conductivity of 0.02 M solution of electrolyte would be :**

(a) 124×10^{-4} S-m²/mol **(b) 1.24×10^{-4} S-m²/mol**

(c) 1240×10^{-4} S-m²/mol **(d) 12.4×10^{-4} S-m²/mol**

Ans: (a)

Sol: $c = 0.1\ M$; $\frac{\ell}{a} kR = 1.29 \times 100$

at $c = 0.02\ M$; $k = \dfrac{\ell}{a \times R} = \dfrac{1.29 \times 100}{520} = 0.248\ \text{S/m}$

Also, $\Lambda = \dfrac{x}{M\,(\text{in mol}/1)} = \dfrac{k}{M \times 10^3\,(\text{in mol}/\text{m}^3)}$

$= \dfrac{0.248}{0.02 \times 10^3} = 124 \times 10^{-4}\ \text{Sm}^2/\text{mol}$

Q.34 In a reaction involving on single reactant, the fraction of the reactant consumed may be defined as $f = [I - (C/C_0)]$, where C_0 and C are the concentration of the reactant at the start and after time *t*. For a first-order reaction.

(a) $\dfrac{df}{dt} = k(1-f)$ (b) $-\dfrac{df}{dt} = kf$ (c) $-\dfrac{df}{dt} = k(1-f)$ (d) $\dfrac{df}{dt} = kf$

Ans: (a)

Sol: Given : $f = \left(1 - \dfrac{c}{c^o}\right)$, Then $\dfrac{c}{c^o} = (1-f)$

$\dfrac{df}{dt} = \dfrac{1}{c^o}\dfrac{dc}{dt}$ for first -order reaction

$-\dfrac{dc}{dt} = K[c]$ $\dfrac{df}{dt} = \dfrac{1}{c^o}$

Then $\dfrac{dc}{dt} = K(1-f)$

Q.35 Equal volume each of two sols of Agl, one obtained by adding $AgNO_3$ to slight excess of KI and another obtained by adding KI to slight excess of $AgNO_3$, are mixed together. Then :

(a) The two soles will stabilize each other

(b) The sole particels will acquire more electric charge

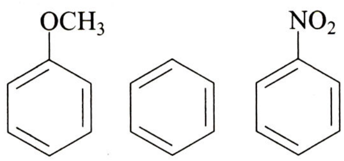
(c) The sole will coagulated each other mutually

(d) A true solution will be obtained.

Ans: (c)

Sol: The sole obtained in the two cases will be oppositiely charged and hence coagulate each other.

Q.36 Among the following compounds (I-III), the correct order of reaction with electrophilic reagent is :

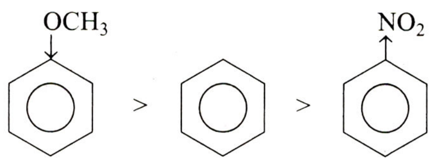


I II III

(a) II > III > I (b) III < I < II (c) I > II > III (d) I = II > III

Ans: (c)

Sol:

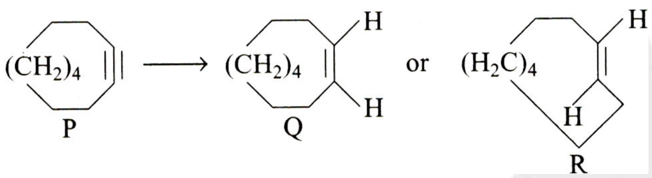


Methoxy group is electron releasing; thus it increases electron density of benzene nucleus. However, $-NO_2$ decreases electron density of benzene.

Q.37 A molecule can be said to have plane of symmetry if :
(a) it can be divided into two equal halves – one half being the mirror image of the other half
(b) it can be divided into two halves – one half is not the mirror image of the other half
(c) it does not have centre of symmetry
(d) it does not have axis of symmetry.

Ans: (a)
Sol: ___

Q.38 Reactant P gives products Q or R.

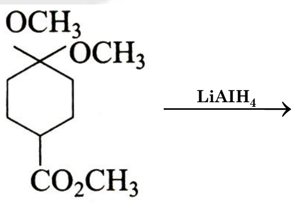


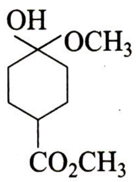
The possible reagents are :
(I) 2Na/liq. NH_3 (II) H_2/Pd/$CaCO_3$ (quinoline) (III) $2H_2$/Pd/C
The correct statement(S) with respect to the above conversion is/are :
(a) Q is obtained on treatment with reagent (I)
(b) R and Q are obtained on treatment with reagent (II)
(c) R is obtained on treatment with reagent (I)
(d) R is obtained on treatement with reagent (II).

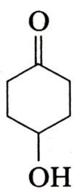
Ans: (c)
Sol: ___

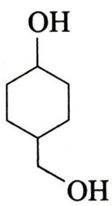
Q.39 Which of the following statements is incorrect ?
(a) An S_N1 reaction proceeds with the inversion of configuration
(b) An S_N2 reaction proceeds with sterochemical inversion
(c) An S_N2 reaction follows second-order kinetics
(d) The reaction of tert-butyl bromide with OH^- follows first-order kinetics

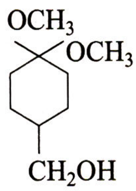
Ans: (a)
Sol: In S_N1, recemic mixture is obtained

Q.40 Product (A) is 

(a) 

(b) 

(c) 

(d) 

Ans: (b)
Sol: ___

Q.41 **An organic compound (A) has the molecular formula C_3H_6O. It undergoes iodoform test. When saturated with dil HCl, it gives (B) of molecular formula $C_9H_{14}O$. A and B, respectively, are :**
(a) propanal and mesitylene
(b) propanone and mesityl oxide
(c) propanone and 2, 6-dimethyl 2,5-heptadien-4-one
(d) propanone and mesitylene oxide

Ans: (c)
Sol: The compound A is propanone which gives the iodoform test and have formula C_3H_6O. 2,6-Dimethyl-2,5-heptadian 4-one is compound B having carbon atoms three times the number of carbon atoms in propanone.

Q.42 **In the given reaction : [X] + Acetic anhydride $\rightarrow$ Aspirin**
[X] will be :
(a) benzoic acid **(b) *o*-methoxybenzoic acid**
(c) *o*-hydrozybenzoic acid **(d) *p*-hydroxybenzoic acid**

Ans: (c)
Sol: --------------------

Q.43 **The order of basic strength among the following amines in benzene solution is :**
(a) $CH_3NH_2 > (CH_3)_3N > (CH_3)_2NH$ **(b) $(CH_3)_2NH > CH_3NH_2 > (CH_3)_2N$**
(c) $CH_3NH_2 > (CH_3)_2NH > (CH_3)_2N$ **(d) $(CH_3)_3N > CH_3NH_2 > (CH_3)_2NH$**

Ans: (b)
Sol: $CH_3NH_2 > (CH_3)_2NH > (CH_3)_2N$
$K_b = 5.4 \times 10^{-4} \quad 4.5 \times 10^{-4} \quad 0.6 \times 10^{-4}$

Q.44 **How many H-atoms are present in 0.046 g of ethanol ?**
(a) 6×10^{20} **(b) 1.2×10^{21}** **(c) 3×10^{21}** **(d) 3.6×10^{21}**

Ans: (d)
Sol: Molecular weight of $C_2H_5OH = 2 \times 12 + 5 + 16 + 1 = 64$

$\because$ 48 g C_2H_5OH has H atoms $= 6 \times N_A$

$\therefore$ 0.046 g C_2H_5OH has H atoms $= \dfrac{6 \times 6.02 \times 10^{23} \times 0.046}{46} = 3.6 \times 10^{21}$

Q.45 **Chargaff's rule states that in an organism :**
(a) amounts of all bases are equal
(b) amount of adenine (A) is equal to that of thymine (T) and the amount of guanine (G) is equal to that of cytosine (C)
(c) Amount of adenine (A) is equal to that guanine (G) and the amount of thymine (T) is equal to that of cytosine (C)
(d) Amount of adenine (A) is equal to that of cytosine (C) and the amount of thymine (T) is equal to guanine (G)

Ans: (b)
Sol: According to Chargaff's rule, the amount of adenine (A) is equal to that of thymine (T) and the amount of guanine (G) is equal to that of cytosine (C).

Q.46 **The boiling point of a solution of 0.1050 g of a substance in 15.84 g of ether was found to be 100°C higher than that of pure ether. What is the molecular weight of the substance [molecular elevation constant of ether per 100 g = 21.6] ?**

Sol: $m = \dfrac{K_b \times w \times 1000}{\Delta T_b \times W} = 143.18$

Q.47 If 0.50 mol of $BaCl_2$ is mixed with 0.20 mol of Na_3PO_4, the maximum number of moles of of $Ba(PO_4)_2$ that can be formed is :

Sol: (i) Write balanced chemical equation for chemical change
(ii) Find limiting reagent.
(iii)Amount of product formed will be determined by amount of limiting reagent.

The balanced equation is $\underset{\substack{0.5 \\ 3\text{mol}}}{3BaCl_2} + \underset{\substack{0.2 \\ 2\text{mol}}}{2Na_3PO_4} \rightarrow \underset{1\text{ mol}}{Ba_3(PO_4)_2} + 6NaCl$

Limiting reagent is Na_3PO_4 (0.2 mol), $BaCl_2$ is in excess

From the above equation, 2.0 mol of Na_3PO_4 yields $Ba_3(PO_4)_2 = 1$ mol.

Therefore, 0.2 mol of Na_3PO_4 will yield $\frac{1}{2} \times 0.2 = 0.1$ mol of $Ba_3(PO_4)_2$

Q.48 In a crystal, at 827°C, one out of 10^{10} lattice site is found to be vacant, while in the same solid, one out of 2×10^9 lattice site is found to be vacant at 927°C. What is the enthalphy of vacancy formation in kJ/mol unit?

Sol: $K(827^\circ C) = 10^{-10}$ and $K(927^\circ C) = 5\times10^{-10}$

$$\Rightarrow \quad \text{In}\left(\frac{5\times10^{-10}}{10^{-10}}\right) = \frac{\Delta H}{R}\left(\frac{100}{1100\times1200}\right)$$

$$\Rightarrow \quad \Delta H = 176.8 \text{ kJ}$$

Q.49 The density of a solution lprepared by dissolving 120 g of urea (mol. mass = 60 u) in 1000 g of water is 1.15 g/mL. The molarity of this solution is _______?

Sol: Mass of solute taken = 120 g
Molecular mass of solute = 60 u
Density of solution = 1.15 g/mL
Total mass of solution = 1000 + 120 = 1120 g

$$\text{Volume of solution} = \frac{\text{Mass}}{\text{Density}} = \frac{1120}{1.15}\text{mL}$$

$$\text{Molarity} = \frac{\dfrac{\text{Mass of solute}}{\text{Molecular mass of solute}}}{\text{Volume of solution}}\times1000$$

$$= \frac{120/60}{1120/1.15}\times1000 = \frac{2\times1000\times1.15}{1120} = 2.05 \text{ M}$$

Q.50 In an atomic solid with FCC arrangements of atom, on an average, a face centre is left unoccupied per unit cell. The packing fraction of this solid would be closest to ?

Sol: No. of atoms per unit cell $= 4 - \frac{1}{2} = \frac{7}{2}$.

Part - C - MATHEMATICS

Q.51 The coefficient of two consecutive terms in the expansion of $(1+x)^n$ will be equal, if

(a) n is any integer **(b) n is an odd integer**
(c) n is an even integer **(d) none of these**

Ans: (b)

Sol: Let consecutive terms are ${}^nC_r$ and ${}^nC_{r+1}$

$\Rightarrow \quad \frac{n!}{(n-r)!r!} = \frac{n!}{(n-r-1)!(r+1)!} \quad \Rightarrow \quad \frac{1}{(n-r)(n-r-1)!r!} = \frac{1}{(n-r-1)!(r+1)r!}$

$\Rightarrow \quad r+1 = n-r \Rightarrow n = 2r+1.$ Hence n is odd

Q.52 Solution set of the inequality $\log_3(x+2)(x+4) + \log_{1/3}(x+2) < \frac{1}{2}\log_{\sqrt{3}} 7$ is

(a) $(-2, -1)$ **(b) $(-2, 3)$** **(c) $(-1, 3)$** **(d) $(3, \infty)$**

Ans: (b)

Sol: $(x+2)(x+4) > 0,\ x+2 > 0$

$\Rightarrow \quad x > -2$ **...(i)**

Now (i) can be written as $\log_3(x+2)(x+4) - \log_3(x+2) < \frac{(\log 7)/2}{(\log 3)/2}$

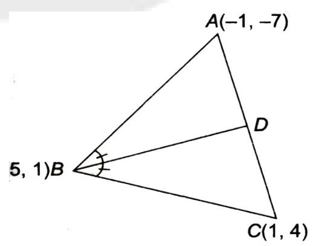
$\Rightarrow \quad \log_3(x+4) < \log_3 7 \quad \Rightarrow \quad x+4 < 7\ x < 3$

Q.53 The vertices of a triangle are $A(-1,-7)$, $B(5,1)$, and $C(1,4)$. The internal angle bisector of the angle $\angle ABC$ meets opposite side in D, the coordinates of which are

(a) $\left(\frac{1}{3}, \frac{1}{3}\right)$ **(b) $(0, -3/2)$** **(c) $(3/11, 0)$** **(d) none of these**

Ans: (a)

Sol: Let BD be the bisector of $\angle ABC$



Then $AD : DC = AB : BC$ and $AB = \sqrt{(5+1)^2 + (1+7)^2} = 10$

$BC = \sqrt{(5-1)^2 + (1-4)^2} = 5$

$\therefore \quad AD : DC = 2 : 1 \quad \therefore$ By section formula $D\left(\frac{1}{3}, \frac{1}{3}\right)$

Q.54 The equation of the lines on which the perpendicular from the origin make 30^o angle with x-axis and which form a triangle of area $50/\sqrt{3}$ with axes are

(a) $x+\sqrt{3}y \pm 10 = 0$ **(b) $\sqrt{3}x + y \pm 10 = 0$**

(c) $x \pm \sqrt{3}y - 10 = 0$ **(d) none of these**

Ans: (b)

Sol: Let p be the length of the perpendicular from the origin on the given line. Then its equation in normal form is $x \cos 30^o + y \sin 30^o = p$ or $\sqrt{3}x + y = 2p$. This meets the coordinate axes at

$A\left(\frac{2p}{\sqrt{3}}, 0\right)$ and $B(0, 2p)$

$\therefore$ Area of $\Delta OAB = \frac{1}{2}\left(\frac{2p}{\sqrt{3}}\right)2p = \frac{2p^2}{\sqrt{3}}$

By hypothesis, $\frac{2p^2}{\sqrt{3}} = \frac{50}{\sqrt{3}} \Rightarrow p = \pm 5.$

Q.55 If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct point P and Q, then the line $5x + by - a = 0$ passes through P and Q for

(a) no value of a **(b) exactly one value of a**

(c) exactly two values of a **(d) infinitely many values of a**

Ans: (a)

Sol: The equation of PQ is $5ax + (c-d)\ y + a + 1 = 0$ ...(i)

Also, the equation of PQ is

$5x + by - a = 0$ ...(ii)

$\therefore \quad \frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a}$

$\Rightarrow \quad a = \frac{a+1}{-a} \quad \Rightarrow a^2 + a + 1 = 0$

$\Rightarrow$ No value of a $[\because D < 0]$

Q.56 If two tangents drawn from the point (α, β) to the parabola $y^2 = 4x$ be such that the slope of one tangent is double of the other then

(a) $\beta = \frac{2}{9}\alpha^2$ **(b) $\alpha = \frac{2}{9}\beta^2$** **(c) $2\alpha = 9\beta^2$** **(d) none of these**

Ans: (b)

Sol: Any tangent to the parabola having slope m is $y^2 = 4ax$ is $y = mx + \frac{1}{m}\cdot$

It passes through (α, β)

$\therefore \quad \beta = m\alpha + \frac{1}{m} \quad$ or $\quad \alpha m^2 - \beta m + 1 = 0$

According to question it has roots $m_1, 2m_1$.

Now,

$$m_1 + 2m_1 = \frac{\beta}{\alpha} \text{ and } m_1 \cdot 2m_1 = \frac{1}{\alpha}$$

$\Rightarrow \quad 2\cdot\left(\frac{\beta}{3\alpha}\right)^2 = \frac{1}{\alpha} \quad \Rightarrow \quad \alpha = \frac{2}{9}\beta^2$

Q.57 If the chords of contact of tangents from two points (x_1, y_1) and (x_2, x_2) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are at right angles, then $\frac{x_1x_2}{y_1y_2}$ is equal to

(a) $\frac{a^2}{b^2}$ (b) $-\frac{b^2}{a^2}$ (c) $-\frac{a^4}{b^4}$ (d) $-\frac{b^4}{a^4}$

Ans: (c)

Sol: The equation of the chords of contact of tangents drawn from (x_1, y_1) and (x_2, y_2) to the ellipes $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1 \quad \text{...(i)}$$

$$\frac{xx_2}{a^2}+\frac{yy_2}{b^2}=1 \quad \text{...(ii)}$$

It is given that (i) and (ii) are at right angles.

$\therefore \quad \frac{-b^2}{a^2}\frac{x_1}{y_1}\times\frac{-b^2}{a^2}\frac{x_2}{y_2}=-1 \quad \Rightarrow \quad \frac{x_1x_2}{y_1y_2}=-\frac{a^4}{b^4}$

Q.58 Let a and b be non-zero real numbers. Then, the equation $(ax^2+by^2+c)\,(x^2-5xy+6y^2)=0$ represents

(a) four straight lines, when $c=0$ and a, b are of the same sign

(b) two straight lines and a circle, when $a=b$, and c is of sign opposite to that of a. (c) two straight lines and hyperbola, when a and b are of the same sign and c of sign opposite to that of a

(d) a circle and an, ellipse, when a and b are of the same sign and c is of sign opposite to that of a.

Ans: (b)

Sol: $(ax^2+by^2+c)\,(x^2-5xy+6y^2)=0$

$\Rightarrow \quad ax^2+by^2+c=0 \quad$ or $\quad x^2-5xy+6y^2=0$

$\Rightarrow \quad x^2+y^2=\left(-\frac{c}{a}\right)$

If $a=b$, $x-2y=0$ and $x-3y=0$

Hence the given equation represents two straight lines and a circle, when $a=b$ and c is a sign opposite to that of a.

Q.59 The value of $\lim\limits_{x\to a}\sqrt{a^2-x^2}\ \cot\frac{x}{2}\sqrt{\frac{a-x}{a+x}}$ is

(a) $\frac{2a}{\pi}$ (b) $-\frac{2a}{\pi}$ (c) $\frac{4a}{\pi}$ (d) $-\frac{4a}{\pi}$

Ans: (c)

Sol: $\lim_{x\to a}\sqrt{a^2-x^2}\ \cot\frac{x}{2}\sqrt{\frac{a-x}{a+x}}$

$$=\lim_{x\to a}\frac{\sqrt{a^2-x^2}}{\tan\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}} \qquad =\frac{\pi}{2}\lim_{x\to a}\frac{\frac{\pi}{2}\sqrt{\frac{a-x}{a-x}}}{\tan\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}}(a+x)$$

$$=\frac{4a}{\pi}$$

Q.60 If $y = x + e^x$ then $\frac{d^2x}{dy^2}$ is

(a) e^x **(b) $-\frac{e^x}{(1+e^x)}$** **(c) $-\frac{e^x}{(1+e^x)^2}$** **(d) $\frac{-1}{(1-e^x)}$**

Ans: (b)

Sol: $y = x + e^x \Rightarrow \frac{dy}{dx} = 1 + e^x \Rightarrow \frac{dx}{dy} = \frac{1}{1+e^x}$

$$\Rightarrow \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{1}{1+e^x}\right) \qquad \Rightarrow \frac{d^2x}{dy^2} = \frac{d}{dx}\left(\frac{1}{1+e^x}\right)\frac{dx}{dy}$$

$$\Rightarrow \frac{d^2x}{dy^2} = \frac{-e^x}{(1-e^x)^2}\ \frac{1}{(1+e^x)} = -\frac{e^x}{(1+e^x)^3}$$

Q.61 If $f(x)\begin{cases}|x+1|; & x \le 0\\ x; & x > 0\end{cases}$ and $g(x)\begin{cases}|x+1|; & x \le 1\\ -|x-2|; & x > 1\end{cases}$. Then $f(x) + g(x)$ is discontinuous at exactly

(a) one point **(b) two points** **(c) three points** **(d) four points**

Ans: (b)

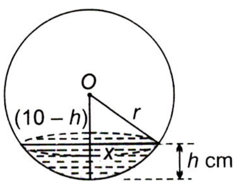
Sol: Since $f(x)$ is discontinuous at $x = 0$ and $g(x)$ is continuous at $x = 0$, then $f(x) + g(x)$ is discontinous at $x = 0$. Since $f(x)$ is continuous at $x = 1$ and $g(x)$ is discontinuous at $x = 1$, then $f(x) + g(x)$ is discontinuous at $x = 1$.

Q.62 Suppose that water is emptied from a spherical tank of radius 10 cm. If the depth of the water in the tank is 4 cm and is decreasing at the rate of 2 cm/sec. then the radius of the top surface of water is decreasing at the rate of

(a) 1 **(b) 2/3** **(c) 3/2** **(d) 2**

Ans: (c)

Sol:



$\frac{dh}{dt} = -2;\ r = 10\text{ cm}\quad \frac{dx}{dt} = ?$ where $h = 4$

Where x is the radius of the top surface.

Now $r^2 = x^2 + (10-h)^2$

$$2x\frac{dx}{dt} = -20(10-h)\frac{dh}{dt} \qquad \frac{dx}{dt} = \frac{(10-h)}{x}(-2)$$

$$\frac{dx}{dt} = \frac{2(10-4)}{x} = \frac{12}{x} \qquad \text{...(i)}$$

When $h = 4$ then $x^2 = 10^2 - 6^2 = 64$ $\quad x = 8$

$$\therefore \frac{dx}{dt} = \frac{12}{8} = \frac{3}{2} \qquad \therefore \frac{dh}{dt} = \frac{77000 \times 4 \times 7}{22 \times 70 \times 70} = 20 \text{ cm/min.}$$

Q.63 **If $y = a\log_c|x| + bx^2 + x$ has its extreme values at $x = -1$ and $x = 2$ then**

(a) $a = 2, b = -1$ **(b)** $a = 2, b = -\frac{1}{2}$ **(c)** $a = -2, b = \frac{1}{2}$ **(d) none of these**

Ans: (b)

Sol: $y = a\log_c|x| + bx^2 + x$ ha its extreme values at $x = -1$ and 2

$\therefore \frac{dy}{dx} = 0$ at $x = -1$ and 2.

$\Rightarrow \frac{a}{x} + 2bx + 1 = 0$ or $2bx^2 + x + a = 0$, which

has -1 and 2 as its roots

$\therefore 2b - 1 + a = 0$ ....(i)

$8b - 2 + a = 0$ ...(ii)

Solving (i) and (ii), we get $a = 2,\ b = -1/2$

Q.64 $\int \frac{\sin x}{\sin(x-a)}dx$ **is equal to**

(a) $(x-a)\cos a + \sin a \log \sin(x-a) + c$ **(b)** $(x-a)\cos x + \log \sin(x-a) + c$

(c) $\sin(x-a) + \sin x + c$ **(d)** $\cos(x-a) + \cos x + c$

Ans: (a)

Sol: $\int \frac{\sin x}{\sin(x-a)}dx$

$$= \int \frac{\sin(x-a+a)}{\sin(x-a)}dx$$

$$= \int \frac{\sin(x-a)\cos a + \cos(x-a)\sin a}{\sin(x-a)}dx$$

$$= \cos a \int dx + \sin a \int \frac{\cos(x-a)}{\sin(x-a)}dx$$

$$= (\cos a)\, x + \sin a \log \sin(x-a) + c$$

$$= (x-a)\cos a + \sin a \log \sin(x-a) + c$$

Q.65 The differential equation whose solution is $Ax^2 + By^2 = 1$, Where A and B are arbitrary constants is of

(a) second order and second degree
(b) first order and second degree
(c) first order and first degree
(d) second order and first degree

Ans: (d)

Sol: $Ax^2 + By^2 = 1,$ ...(i)

$\Rightarrow \quad Ax + By\dfrac{dy}{dx} = 0$ ...(ii)

$\Rightarrow \quad A + By\dfrac{d^2y}{dx^2} + B\left(\dfrac{dy}{dx}\right)^2 = 0$ ...(iii)

From (ii) and (iii)

$$x\left\{-By\frac{d^2y}{dx^2} - B\left(\frac{dy}{dx}\right)^2\right\} + By\frac{dy}{dx} = 0$$

$$\Rightarrow \quad xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

Q.66 $\mathbf{2\tan^{-1}\left[\sqrt{\dfrac{a-b}{a+b}}\tan\dfrac{\theta}{2}\right] =}$

(a) $\cos^{-1}\left(\dfrac{a\cos\theta + b}{a + b\cos\theta}\right)$
(b) $\cos^{-1}\left(\dfrac{a + b\cos\theta}{a\cos\theta + b}\right)$

(c) $\cos^{-1}\left(\dfrac{a\cos\theta}{a + b\cos\theta}\right)$
(d) $\cos^{-1}\left(\dfrac{b\cos\theta}{a\cos\theta + b}\right)$

Ans: (a)

Sol: $2\tan^{-1}\left[\sqrt{\dfrac{a-b}{a+b}}\tan\dfrac{\theta}{2}\right] =$

$$= \cos^{-1}\left[\frac{1-\left(\dfrac{a-b}{a+b}\right)\tan^2\dfrac{\theta}{2}}{1+\left(\dfrac{a-b}{a+b}\right)\tan^2\dfrac{\theta}{2}}\right] \qquad \left(\because 2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}\right)$$

$$= \cos^{-1}\left[\frac{(a+b) - (a-b)\tan^{-2}\dfrac{\theta}{2}}{(a+b) + (a-b)\tan^{-2}\dfrac{\theta}{2}}\right]$$

$$= \cos^{-1}\left[\frac{a\left(1-\tan^{+2}\dfrac{\theta}{2}\right) + b\left(1+\tan^{+2}\dfrac{\theta}{2}\right)}{a\left(1+\tan^{+2}\dfrac{\theta}{2}\right) + b\left(1-\tan^{+2}\dfrac{\theta}{2}\right)}\right]$$

$$= \cos^{-1}\left[\frac{\dfrac{a\left(1-\tan^{+2}\dfrac{\theta}{2}\right)}{1+\tan^{+2}\dfrac{\theta}{2}}+b}{a+b\left(\dfrac{1-\tan^{+2}\dfrac{\theta}{2}}{1-\tan^{+2}\dfrac{\theta}{2}}\right)}\right]$$

$$= \cos^{-1}\left[\frac{a\cos\theta+b}{a+b\cos\theta}\right]$$

Q.67 The unit vector which is orthogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+6\hat{k}$ is

(a) $\frac{2\hat{i}+6\hat{j}+\hat{k}}{\sqrt{41}}$ (b) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$ (c) $\frac{3\hat{i}-\hat{k}}{\sqrt{10}}$ (d) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

Ans: (c)

Sol: Any vector coplanar to $\vec{a}$ and $\vec{b}$ can be written as $\vec{r}=\vec{a}+\lambda\vec{b}$

$$\vec{r}=(1+2\lambda)\hat{i}+(-1+\lambda)\hat{j}+(1+\lambda)\hat{k}$$

Since $\vec{r}$ is orthogonal to $5\hat{j}+2\hat{j}+6\hat{k}$

$\Rightarrow \quad 5(1+2\lambda)+2(-1+\lambda)+6(1+\lambda)=0$

$\Rightarrow \quad 9+18\lambda=0 \Rightarrow \lambda=-\frac{1}{2} \qquad \therefore \quad \vec{r}=3\hat{j}-\hat{k}$

Since $\vec{r}$ is a unit vector $\therefore \quad \vec{r}=\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$

Q.68 The equation of the plane through the intersection of the planes $x+2y+3z-4=0$ and $4x+3y+2z+1=0$ and passing through the origin is

(a) $17x+14y+11z=0$ (b) $7x+4y+z=0$

(c) $x+14y+11z=0$ (d) $17x+y+z=0$

Ans: (a)

Sol: Any plane through the given planes is $x+2y+3z-4+\lambda(4x+3y+2z+1)=0$

It passes through (0, 0, 0)

$\therefore \quad -4+\lambda=0 \qquad \therefore \quad \lambda=4$

$\therefore$ Required plane is $x+2y+3z+4(4x+3y+2z)=0$ or $17x+14y+11z=0$

Q.69 A pie chart is to be drawn for representing the following data :

Items of expenditure	Number of families
Education	150
Food and clothing	400
House rent	40
Electricity	250
Miscellaneous	160

The value of the centre angle for food and cloting would be
(a) 90° **(b) 2.8°** **(c) 150°** **(d) 144°**

Ans: (d)
Sol: Required angle for food and cloting

$$= \frac{400}{1000} \times 360^o = 144^o$$

Q.70 Which of the following is true for any two statements *p* and *q* ?
(a) $\sim[p \vee (\sim q)] \equiv (\sim p) \wedge q$ **(b)** $(p \vee q) \vee (\sim p) \wedge q$
(c) $(p \wedge q) \wedge (\sim q)$ **is a contradiction** **(d)** $\sim p\ [p \wedge (\sim p)]$ **is a tautology**

Ans: (a)

Sol:

p	q	$\sim p$	$\sim q$	$p \vee \sim q$	$\sim(p \vee \sim q)$	$\sim p \wedge q$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	F	F

Q.71 India plays two matches each with West Indies and Australia. In any match the probabilities of India getting point 0, 1, and 2 are 0.45, 0.05, and 0.50 respectively. Assuming that the outcomes are independent, the probability of the India getting at least 7 points is ______?

Sol: P (at least 7 points) = P (7 points) + P(8 points) [∵ At most 8 point can be scored]
Now 7 points can be scored by scoring 2 points in three matches and 1 point in one match, similarly 8 points can be scored by scoring 2 points in each of four matches.
∴ Require property

$$= {}^4C_1 \times [P(2pts)]^3\ P(1\,pt) + [p(2\ pts)]^4$$

$$= 4\ (0.5)^3 \times 0.05 + (0.50)^4$$

$$= 0.0250 + 0.0625 = 0.0875$$

Q.72 The number of numbers that can be formed with the help of the digits 1, 2, 3, 4, 2, 1 so that odd digits always occupy odd places, is ______?

Sol: The four odd digit 1, 3, 3, 1 can be arranged in the four odd place in $\frac{4\,!}{2\,!2\,!} = 6$ ways and three even digits 2, 4, 2 can be arranged in the three even places in $\frac{3\,!}{2!} = 3$ ways. Hence the requird number of ways $= 6 \times 3 = 18$

Q.73 If the algebraic sum of deviations of 20 observations from 30 is 20, then the mean of observations is ______?

Sol: $\sum_{i=1}^{20}(x_i - 30) = 20$

$\Rightarrow \sum_{i=1}^{20} x_i - 20 \times 30 = 20 \Rightarrow \sum_{i=1}^{20} x_i = 620$

Mean $= \dfrac{\sum_{i=1}^{20} x_i}{20} = \dfrac{620}{20} = 31$

Q.74 A dice is thrown 100 times and getting and even number is considered a success. The varience of the number of successes will be?

Sol: Let E = Event of getting an even number from {2, 4, 6}

$\Rightarrow n$ (E) = 3

$\therefore$ Probability of success, $p = \dfrac{3}{6} = \dfrac{1}{2}$

and probability of failure, $q = \dfrac{1}{2}$

Now, variance $= npq = 100 \times \dfrac{1}{2} \times \dfrac{1}{2} = 25 \cdot$

Q.75 $\lim_{x \to 0} \dfrac{x^n - \sin x^n}{x - \sin^n x}$ **in none–zero finite, then n must be equal to ?**

Sol: For $n = 0$, we have $\lim_{x \to 0} \dfrac{1 - \sin 1}{x - 1} = \sin 1 - 1$

For $n = 1$, $\lim_{x \to 0} \dfrac{x - \sin x}{x - \sin x} = 1$

For $n = 2$, $\lim_{x \to 0} \dfrac{x^2 - \sin^2 x}{x - \sin^2 x} = \lim_{x \to 0} \dfrac{1 - \dfrac{\sin^2 x1}{x^2}}{\dfrac{1}{x} - \dfrac{\sin^2 x}{x^2}} = 0$

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ROUGH WORK

ROUGH WORK

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