Aggarwal CLASSES™
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JEE (MAIN)

TEST PAPER

SUBJECT : PHYSICS,CHEMISTRY, MATHEMATICS ***TEST CODE : TSJMT217***

ANSWER PAPER

TIME : 3 HRS ***MARKS : 300***

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark(-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

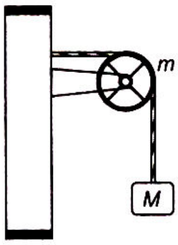
6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or "un- darken" a darkened bubble.

Part A - PHYSICS

Q.1 **A string if negligible mass going over a clamped pulley of mass *m* supports a block of mass *M* as shown in the figure. The force on the pulley by the clamp is given by**

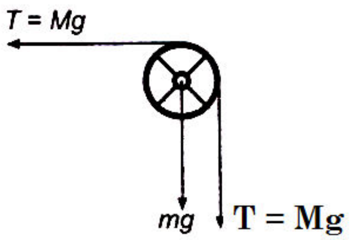


(a) $\sqrt{2}Mg$ **(b)** $\sqrt{2}mg$

(c) $g\sqrt{(M+m)^2+m^2}$ **(d)** $g\sqrt{(M+m)^2+M^2}$

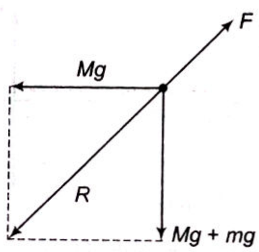
Ans: (d)

Sol: Free body diagram of pulley is shown in figure. Pulley is in equilibrium under four forces. Thee forces as shown in figure and the fourth, which is equal and opposite to the resultant of these three forces, is the force applied by the clamp on the pulley (say F). Resultant R of these three forces is



$$R=\sqrt{(M+m)^2+M^2)}g$$

Therefore, the force F is equal and opposite to R as shown in figure.



$$\therefore \quad F=\sqrt{(M+m)^2+M^2)}g$$

Q.2 **Two masses of 1 g and 4 g are moving with equal kinetic energies. The ratio of the magnitudes of their momenta is**

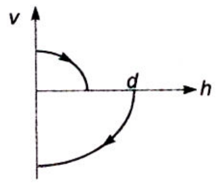
(a) 4 : 1 **(b)** $\sqrt{2}:1$ **(c) 1 : 2** **(d) 1 : 16**

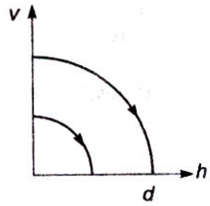
Ans: (c)

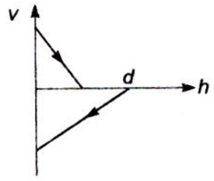
Sol: $p=\sqrt{2Km}$ *or* $p\propto\sqrt{m}$,

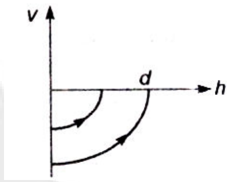
$\frac{m_1}{m_2} = \frac{1}{4} \Rightarrow \quad \therefore \ \frac{p_1}{p_2} = \frac{1}{2}$

Q.3 **A ball is fropped vertically from a height d above he ground. It hits the ground and bounces up vertically to a height *d*/2. Neglecting subsequent motion and air resistance, its velocity *v* varies with height *h* above the ground as**

(a) 

(b) 

(c) 

(d) 

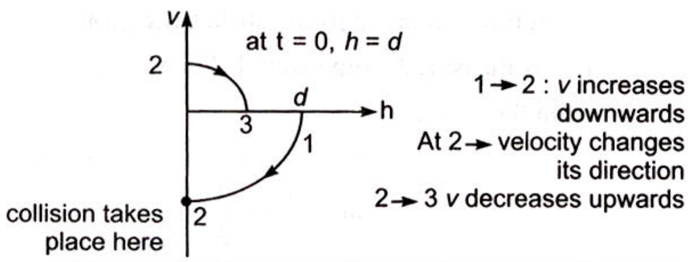
Ans: (a)

Sol: (a) For uniformly accelerated/decelerated motion

$$\text{v}^2 = u^2 \pm 2gh$$

i.e. $\text{v} - h$ graph will b a parabola (because equation is quadractic).

(b) Initially velocity is downwards $(-\text{v}e)$ and then after collision, it reverses its direction with lesser megnitude, i.e. velocity is upwards $(+\text{v}e)$. Graph (a) satisfies both these conditions.



Q.4 **Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is *R* and its height is *h*, then is z_0 equal to**

(a) $\frac{3h}{4}$ **(b)** $\frac{h^2}{4R}$ **(c)** $\frac{5h}{8}$ **(d)** $\frac{3h^2}{8R}$

Ans: (a)

Sol: Centre of mass of uniform solid cone of height h is at a height of $\frac{h}{4}$ from base. Therefore, from vertex it's $\frac{3h}{4}$.

Q.5 **A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is**

(a)ring **(b) solid sphere**

(c) hollow sphere **(d) disc**

Ans: (d)

Sol: $\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = mg\left(\frac{3v^2}{4g}\right)$

$\therefore \quad I = \frac{1}{2}mR^2$

$\therefore$ Body is disc.

Q.6 **A highly rigid cubical block A of small mass Ma and side L is fixed rigidly on to anoher cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the the force is withdrawn, block A executes small oscillations, the time period of which is given by**

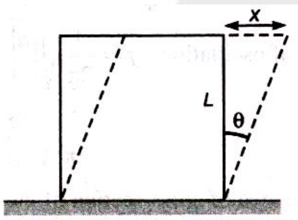
(a) $2\pi\sqrt{M\eta L}$ **(b)** $2\pi\sqrt{\frac{M\eta}{L}}$ **(c)** $2\pi\sqrt{\frac{M\eta}{\eta}}$ **(d)** $2\pi\sqrt{\frac{M}{\eta L}}$

Ans: (d)

Sol: Modulus of rigidity, $\eta = F / A\theta$

Here, $A = L^2$

and $\theta = \frac{x}{L}$



Therefore, restoring force is

$F = \eta - A\theta = -\eta L x$

or acceleration, $a = \frac{F}{M} = -\frac{\eta L}{M}x$

Since, $a \propto -x$, oscillations are simple harmonic in nature, time period of which is given by

$$T = 2\pi\sqrt{\left|\frac{\text{displacement}}{\text{acceleratrion}}\right|}$$

$$= 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{m}{\eta L}}$$

Q.7 A cube has a side of length $1.2\times10^{-2}m$ Calculate its volume.

(a) $1.7\times10^{-6}m^3$ (b) $1.73\times10^{-6}m^3$ (c) $1.70\times10^{-6}m^3$ (d) $1.732\times10^{-6}m^2$

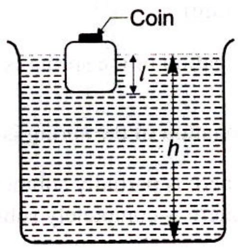
Ans: (a)

Sol: $V = l^3 = (1.2\times10^{-2}m)^3 = 1.728\times10^{-6}m^3$

$\therefore$ Length (l) has two significant figures, the volume (V) will also have two significant figures. Therefore, the correct answer is

$$V = 1.7\times10^{-6}m^3$$

Q.8 A wooden block, with a coin placed on its top, floats in water as shown in figure. The distance *l* and *h* are shown there. After sometime the coin falls into the water. Then



(a) *l* decreases and *h* increases
(b) *l* increases and *h* decreases
(c) Both *l* and *h* increase
(d) Both *l* and *h* decrease

Ans: (d)

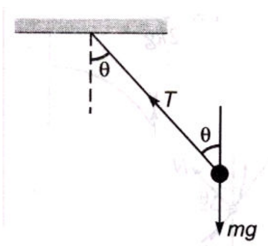
Sol: l will decrease, because the block oves up and h will decrease, because the oin will displace the volume of water equal to its own volume when it is in the water (V_1) whereas when it is on the block it will displace the volume of water (V_2) whose weight is equal to weight of coin and since density of coin is greater than the densicty of water $V_1 < V_2$.

Q.9 A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb bob is suspended from the roof of the car by a light rigid rod. The angle made by the rod with the vertical is (Take g = 10 m/s²)

(a)zero (b) 30° (c) 45° (d) 60°

Ans: (c)

Sol: FBD of bob is $T\sin\theta = \dfrac{mv^2}{R}$



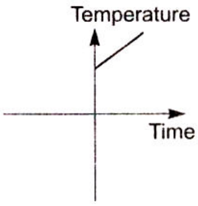
and $T\cos\theta = mg$

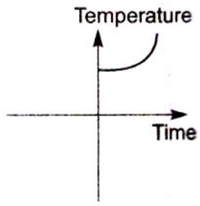
$$\therefore \tan\theta = \frac{v^2}{Rg} = \frac{(10)^2}{(10)(10)}$$

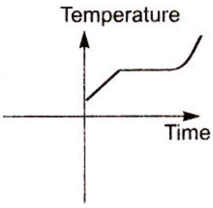
$\tan\theta = 1$

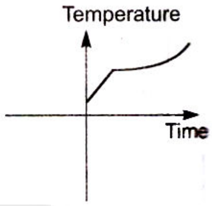
or $\theta = 45^{o}$

Q.10 Liquid oxygen at 50 K is heated to 300 K at constant pressure of 1 atm. The rate of heating is constant. Which of the following graphs represent the variation of temperature with time?

(a) 

(b) 

(c) 

(d) 

Ans: (c)

Sol: Temperature of liquid oxygen will first increase in the same phase. Then, phase change (liquid to gas) will take place. During which temperature will remain constant. After that temperature of oxygen in gaseous state will further increase.

Q.11 Consider an expanding sphere of instataneous radius R whose total mass remains constant. The expansion is such that the instataneous density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{p}\frac{d\rho}{dt}\right)$ is constat. The velocity v of any point of the surface of the expanding sphere is proportional to

(a) R **(b)** $\frac{1}{R}$ **(c)** R^{3} **(d)** $R^{\frac{2}{3}}$

Ans: (a)

Sol: $m = \frac{4\pi R^3}{3}\times\rho$

On taking log both sides, we have

$$\ell n(m) = \ell n\left(\frac{4\pi}{3}\right) + \ell n(\rho) + 3\,\ell n(R)$$

On differenting with respect to time,

$$0 = 0 + \frac{1}{\rho}\frac{d\rho}{dt} + \frac{3}{R}\frac{dR}{dt}$$

$$\Rightarrow \quad \left(\frac{dR}{dt}\right) = -R\times\frac{1}{\rho}\left(\frac{d\rho}{dt}\right)$$

$$\because \quad \frac{dR}{dt} = v$$

$\therefore \quad v = -R \propto \frac{1}{\rho}\left(\frac{d\rho}{dt}\right)$

$\therefore \quad v \propto R$

Q.12 From a tower of height *H*, a particle is thrown vertically upwards with a speed *u*. The time taken by the particle to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between *H*, *u* and *n* is

(a) $\frac{2n-1}{2n}$ **(b)** $\frac{2n+1}{2n-1}$ **(c)** $\frac{2n-1}{2n+1}$ **(d)** $\frac{2n}{2n+1}$

Ans: (c)

Sol: Time taken to reach the maximum height, $t_1 = \frac{u}{g}$

If t_2 is the time taken to hit the ground, then

i.e. $-H = ut_2 - \frac{1}{2}gt_2^2$

But $t_2 = nt_1$ [Given]

So, $-H = u\frac{nu}{g} - \frac{1}{2}g\frac{n^2u^2}{g^2}$

$$-H = \frac{nu^2}{g} - \frac{1}{2}\frac{n^2u^2}{g} \Rightarrow 2gH = nu^2(n-2)$$

Q.13 A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mV and the speed of light is 3×10^8 ms^{-1}. The final momentum of the object is

(a) 0.3×10^{-17} **kg – ms^{-1}** **(b)** 1.0×10^{-17} **kg – ms^{-1}**

(c) 3.0×10^{-17} **kg – ms^{-1}** **(d)** 9.0×10^{-17} **kg – ms^{-1}**

Ans: (b)

Sol: Final momentum of object $= \frac{\text{Power} \times \text{time}}{\text{Speed of light}}$

$$= \frac{30 \times 10^{-3} \times 100 \times 10^{-9}}{3 \times 10^8}$$

$$= 1.0 \times 10^{-17} \text{ kg-m/s}$$

Q.14 From a solid sphere of mass *M* and radius *R*, a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular bisector is *I*. What is the ratio *I*/*R* such that the moment of inertia is minimum?

(a) $\frac{MR^2}{32\sqrt{2}\pi}$ **(b)** $\frac{4MR^2}{9\sqrt{3}\pi}$ **(c)** $\frac{MR^2}{16\sqrt{2}\pi}$ **(d)** $\frac{4MR^2}{3\sqrt{3}\pi}$

Ans: (d)

Sol: Maximum possible volume of cube will occur when

$$\sqrt{3}a = 2R \qquad (a = \text{side of cube})$$

$$\therefore \qquad a = \frac{2}{\sqrt{3}}R$$

Now, density of sphere, $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$

Mass of cube, $m = (\text{volume of cube})(\rho) = (a^3)(\rho)$

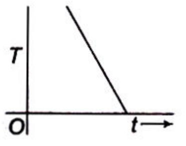
$$= \left[\frac{2}{\sqrt{3}}R\right]^3 \left[\frac{m}{\frac{4}{3}\pi R^3}\right] = \left(\frac{2}{\sqrt{3}\pi}\right)M$$

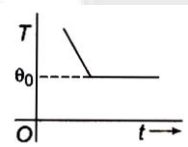
Now, moment of inertia of the cube about the said axis is

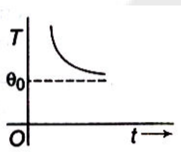
$$I = \frac{ma^2}{6} = \frac{\left(\frac{2}{\sqrt{3}\pi}\right)M\left(\frac{2}{\sqrt{3}}R\right)^2}{\sigma}$$

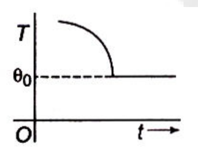
$$= \frac{4MR^2}{9\sqrt{3}\pi}$$

Q.15 If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0. The graph between the temperature T of the metal and time t will be closed to

(a) 

(b) 

(c) 

(d) 

Ans: (c)
Sol: According to Newton's cooling law, option (c) is correct answer.

Q.16 One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C 400 K, 800 K and 600 K, respectively. Choose the correct statement.
(a) The change in internal energy in whole cyclic process is 250 R
(b) The change in internal energy in the process CA is 700 R.
(c) The change in internal energy in the process AB is -350 R.
(d) The change in internal energy in the process BC is -500 R.

Ans: (d)
Sol: According to first law of thermodynamics, we get

(i) Change in internal energy from A to B i.e. ΔU_{AB}

$$\Delta U_{AB} = nC_V(T_B - T_A) = 1\times\frac{5R}{2}(800-400) = 1000\,R$$

(ii) Change in internal energy from B to C

$$\Delta U_{BC} = nC_V(T_C - T_B) = 1\times\frac{5R}{2}(600-800)$$

$$= -500R$$

(iii) $\Delta U_{isthermal=0}$

(iv) Change in internal energy from C to A i.e. ΔU_{CA}

$$\Delta U_{CA} = nC_V(T_A - T_C)$$

$$= 1\times\frac{5R}{2}(400-600) = -500R$$

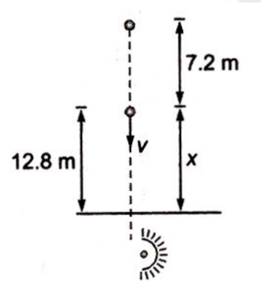
Q.17 A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is $\frac{4}{3}$. A fish inside the lake. in the line of fall of the ball, is looking at the ball At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as

(a) $9\,ms^{-1}$ **(b) $12\,ms^{-1}$** **(c) $16\,ms^{-1}$** **(d) $21.33\,ms^{-1}$**

Ans: (c)

Sol: $v = \sqrt{2gh} = \sqrt{2\times10\times7}$

$= 12\,\text{ms}^{-1}$



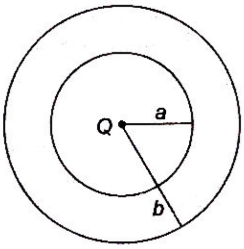
In this case when eye is inside water,

$$x_{app.} = \mu x$$

$$\therefore \quad \frac{dx_{app.}}{dt} = \mu.\frac{dx}{dt}$$

or $\quad v_{app.} = \mu v = \frac{4}{3}\times12 = 16\,\text{ms}^{-1}$

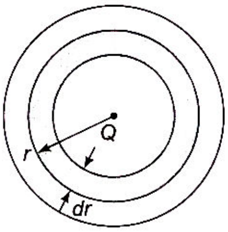
Q.18 The region between teo concentric spheres of radii a and b, respectively (see the figure), has volume charge density $\rho = \frac{A}{r}$, where, A is a constat and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A, such that the electric field in the region between the spheres will be constant, is



(a) $\frac{Q}{2\pi a^2}$ (b) $\frac{Q}{2\pi(b^2-a^2)}$ (c) $\frac{2Q}{\pi(a^2-b^2)}$ (d) $\frac{2Q}{\pi a^2}$

Ans: (a)

Sol: As E is constant,



Hence, $E_a = E_b$

As per Guass theorem, only Q_{in} contributes in electric field.

$$\therefore \quad \frac{kQ}{a^2} = \frac{k\left[Q + \int_a^b 4\pi r^2 dr.\frac{A}{r}\right]}{b^2}$$

Here, $k = \frac{1}{4\pi\varepsilon_0}$

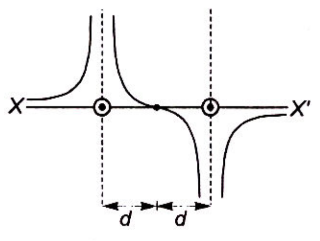
$$\Rightarrow \quad Q\frac{b^2}{a^2} = Q + 4\pi A\left[\frac{r^2}{2}\Bigg|_a^b\right] = Q + 4\pi A.\left(\frac{b^2-a^2}{2}\right)$$

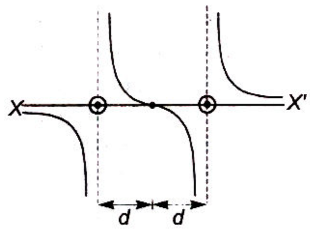
$$\Rightarrow \quad Q\left(\frac{b^2}{a^2}\right) = Q2\pi A(b^2-a^2)$$

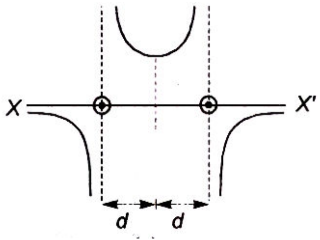
$$\Rightarrow \quad Q\left(\frac{b^2-a^2}{a^2}\right) = 2\pi A(b^2-a^2)$$

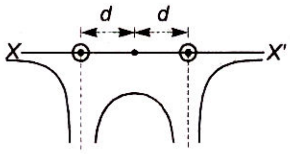
$$\Rightarrow \quad A = \frac{Q}{2\pi a^2}$$

Q.19 Two long parallel wires are at a distance 2d apart. They carry steady equal currents following out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by

(a) 

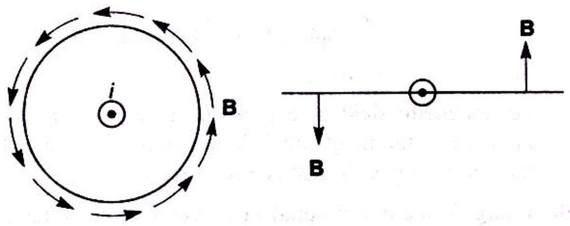
(b) 

(c) 

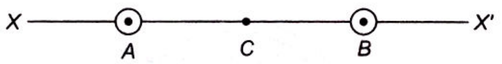
(d) 

Ans: (b)

Sol: If the current flows out of the paper, the magnetic field at points to the right of the wire will be upwards and to the left will be downwards as shown in figure.



Now, let us come to the problem.
Magnetic field at C = 0
Magentic field in region BX' will be upwards (+ve) because all points lying in this region are to the right of both the wires.



Magnetic field in region AC will be upwards (+ve), because points are closer to A, compared to B. Similarly, magnetic field in region BC will be downwards $(-\text{ve})$.
Graph (b) satisfies all these conditions. Therefore, correct answer is (b).

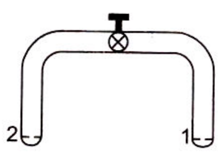
Q.20 A glass tube of uniform internal radius (r) has a value separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherial soap bubble of radius *r*. End 2 has sub-hemispherical soap bubbles as shown in figure. Just after opening the value.

(a) air from end 1 flows towards end 2. No change in the volume of the soap bubbles.

(b) air form end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases.

(c) no change occurs

(d) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 decreases



Ans: (b)

Sol: $\Delta p_1 = \dfrac{4T}{r_1}$ and $\Delta p_2 = \dfrac{4T}{r_2}$

$r_1 < r_2$

$\therefore \quad \Delta p_1 > \Delta p_2$

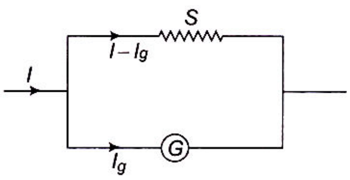
$\therefore$ Air will flow from 1 to 2 and volume of bubble at end 1 will decrease.
Therefore, correct option is (b).

Q.21 A source of sound of frequency 600 Hz is placed inside water. The speed of sound in water is 1500m/s and in air it is 300 m/s. The frequency of sound recorded by an observer who is standing in air is_____?

Sol: The frequency is a characteristic of source. It is independent of the medoium. Hence, the correct option is (d).

Q.22 A galvanometer having a coil resistance of 100Ω gives a full scale deflection when a current of 1 mA is passed through it. The value of the resistance which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is_____?

Sol: In parallel, current distributes in inverse ratio of resistance.
Hence,



$$\frac{I - I_g}{I_g} = \frac{G}{S}$$

$$\Rightarrow \quad S = \frac{GI_g}{I - I_g}$$

As I_g is very small, hence

$$S = \frac{GI_g}{I}$$

$$b = \frac{(100)(1\times 10^{-3})}{10} = 0.01\Omega$$

Q.23 A whistle giving out 450 Hz approches a stationary observer at a speed of 33 m/s. The frequency heard by the observer (in Hz) is (Speed of sound = 330 m/s) ?

Sol: Source is moving towards the observer

$$f' = f\left(\frac{v}{v - v_s}\right) = 450\left(\frac{330}{330 - 33}\right)$$

$$f' = 500 \text{ Hz}$$

Q.24 A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2/T_1 is _____?

Sol: $T \propto \frac{1}{\sqrt{g}}$

$i.e. \frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}}$

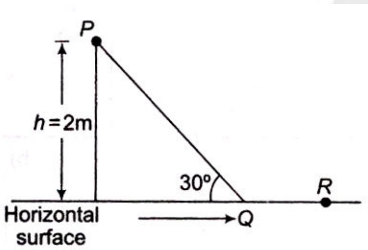
where, $g_1 =$ acceleration due to gravity on earth's surface $= g$

$g_2 =$ acceleartion sue to gravity at a height h = R from earth's surface $= g/4$

$$\left[\text{Using } g\,(h) = \frac{g}{\left(1+\frac{h}{R}\right)}\right]$$

$\Rightarrow \quad \frac{T_2}{T_1} = \sqrt{\frac{g}{g/4}} = 2$

Q.25 A point particle of mass m, moves along the uniformly through track PQR as shown in the figure. The coefficient of frition, between the particle and the rough track equals μ. The particle is released from rest, from the point P and it comes to rest at a point R. The energies lost by the ball, over the parts *PQ* and *QR* of the track, are equal to each other, and no energy is lost when particle changes direction from *PQ* to *QR*. The values of the coefficient of friction μ and the distance $x(=QR)$, are respectively close to ?



Sol: As energy loss is same, thus

$\mu\, mg \cos\theta.\ (PQ) = \mu mg.(QR)$

$\therefore \quad QR = (PQ)\cos\theta$

$\Rightarrow \quad QR = 4 \times \frac{\sqrt{3}}{2}$

$= 2\sqrt{3} = 3.5\ m$

Further, decrease in potential energy = loss due to friction

$\therefore \quad mgh = (\mu\, mg\cos\theta)d_1 + (\mu mg)d_2$

$$m \times 10 \times 2 = \mu \times m \times 10 \times \frac{\sqrt{3}}{2} \times 4 + \mu \times m \times 10 \times 2\sqrt{3}$$

$\Rightarrow \quad 4\sqrt{3}\ \mu = 2$

$\Rightarrow \quad \mu = \frac{1}{2\sqrt{3}} = 0.288 = 0.29$

Part - B - CHEMISTRY

Q.26 Which of the following compound is covalent?

(a) H_2 **(b) CaO** **(c) KCl** **(d) Na_2S**

Ans: (a)

Sol: H_2 is a covalent, diatomic molecule with a sigma covalent bond between two hydrogen atoms.

Q.27 A liquid is in equilibrium with its vapour at it's boiling point. On the average, the molecules in the two phases have equal

(a) inter-molecular forces **(b) potential energy**
(c) kinetic energy **(d) total energy**

Ans: (c)

Sol: At liquid-vapor equilibrium at boling point, molecules in two phase posses the same kinetic energy.

Q.28 The radius of an atomic nucleus is of the order of

(a) 10^{-1}cm **(b) 10^{-13}cm** **(c) 10^{-15}cm** **(d) 10^{-8}cm**

Ans: (b)

Sol: Radius of a nucleus is in the order of 10^{-13}cm a fact.

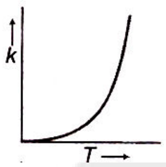
Q.29 The species which by defination has zero standard molar enthalpy of formation at 298 K is

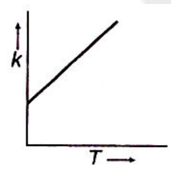
(a) $Br_2(g)$ **(b) $Cl_2(g)$** **(c) $H_2O(g)$** **(d) $CH_4(g)$**

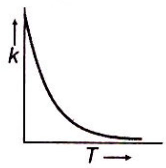
Ans: (b)

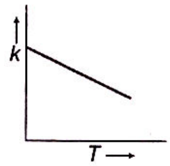
Sol: Elements in its standard state have zero enthalpy of formation. Cl_2 is gas at room temperature, therefore ΔH_f^o of Cl_2 is zero.

Q.30 Plots showing the variation of the rate constat (k) with temperature (T) are given below. The plot that follows Arrhenius equation is

(a)


(b)


(c)


(d)


Ans: (a)

Sol: According to Arrehnius equation, rate constat increases expoentially with temperature :

$$k = Ae^{-E_a/RT}$$

Q.31 Calcium is obtained by

(a) electrolysis of molten $CaCl_2$
(b) electrolysis of soultion of $CaCl_2$ in water
(c) reduction of $CaCl_2$ with carbon
(d) roasting of limestone

Ans: (a)

Sol: Electrolysis of molten $CaCl_2$ gives calcium at cathode

$$Ca^{2+} + 2e^- \rightarrow Ca \text{ (at cathode)}$$

In case of electrolysis in aquesous medium, less electropositive H^+ is reduced at cathode rather than Ca^{2+}.

Q.32 Which one of the following ores is best concentrated by froth floatation method?
(a)Siderite (b) Galena (c)Malachite (d) MAgnetie

Ans: (b)

Sol: Sulphide ores are concentrated by froth flotation method e.g. Galena (PbS).

Q.33 An azeotropic solution of two liquids has boiling point lower than either of them when it
(a) shows negative deviation from Raoult's law
(b) shows no deviation from Raoult's law
(c) shows positive deviation from Raoult's law
(d) is saturated

Ans: (c)

Sol: In case of positive deviation from Raoult's law, the observed vapour pressure is greater than the ideal vapour pressure and boling point of azeotrope becomes lower than either of pure liquid.

Q.34 The statement that is not correct for the periodic classification of elements, is
(a) the properties of elemets are the periodic functions of their atomic numbers.
(b) non-metallic elements are lesser in number than metallic elemets
(c) the first ioniosation energies of elements along a period do not vary in a regular manner with increase in atomic number
(d) for transition elements the d-subshell are filled with electrons monotonically with increase in atomic number

Ans: (d)

Sol: (a) Correct statement According to Moseley's law, the properties of elements are the periodic function of their atomic numbers.
(b) Correct statement The whole s-block, d-b;ock, f-block and heavier p-block elements are metal.
(c) Correct statement Trend is not regular, Be has higher first ionisation energy than B, nitrogen has higher first ionisation energy than oxygen.
(d) Incorrect statement d-subshells are not filled monotonically, regularity break at chromimum and copper.

Q.35 The species present in solution whn CO_2 is dissolved in water are
(a) $CO_2, H_2CO_3, HCO_3^-, CO_3^{2-}$ (b) H_2CO_3, CO_3^{2-}
(c) HCO_3^-, CO_3^{2-} (d) CO_2, H_2CO_3

Ans: (a)

Sol: When CO_2 is dissolved in water, following equilibria are established:

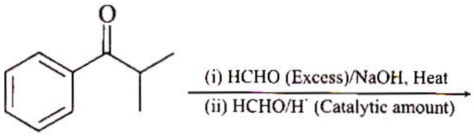
$$H_2 + CO_2 \rightleftharpoons H_2CO_3$$

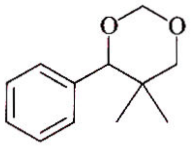
$$H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$$

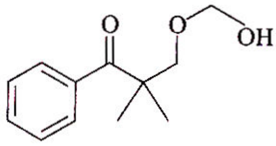
$$HCO_3^- \rightleftharpoons H^+ + CO_3^{2-}$$

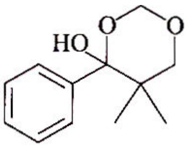
Therefore, in solution, all of the above mentioned species exist.

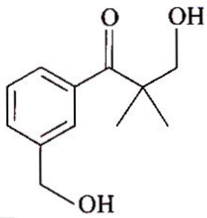
Q.36 The major product of the following reaction sequence is



(a) 

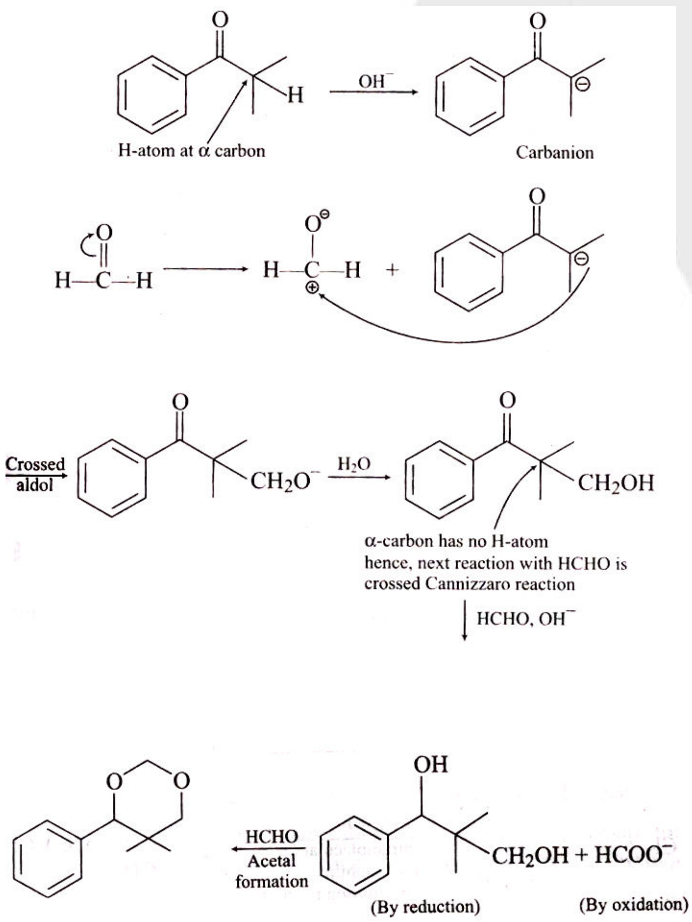
(b) 

(c) 

(d) 

Ans: (a)

Sol:



Q.37 Benzamide on tratement with $POCl_3$ gives

(a) aniline **(b) benzonitrile**

(c) chlorobenzene **(d) benzyl amine**

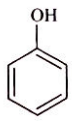
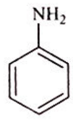
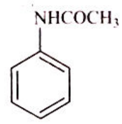
Ans: (b)

Sol:

$$C_6H_5-\overset{\overset{O}{||}}{C}-NH_2 \xrightarrow{POCl_3} C_6H_5-C\equiv N + H_2O$$

$POCl_3$ Brings about dehydration of primary amide.

Q.38 Which of the following compounds will give significant amount of *meta*-product during mononitration reaction?

(a)  **(b)**  **(c)**  **(d)** 

Ans: (c)

Sol: Aniline in presence of nitrating mixture (conc. HNO_3 + conc. H_2SO_4) gives significant amount ($\approx 47\%$) of meta-product because in presence of H_2SO_4 its protonation takes place and anilinium ion is formed.
Here, anilinium ion is strongly deactivating group and *meta*-directing in nature. So, it gives meta-nitration product.

Q.39 Which of the following 0.1 M aqueous solution will have the lowest freezing point?
(a) Potassium sulphate **(b) Sodium chlordie**
(c) Urea **(d) Glucose**

Ans: (a)

Sol:
K_2SO_4: $i=3$
NaCl: $i=2$
Urea: $i=1$
Glucose: $i=1$

Greater the value of i, greater the lowering in freezing point, lower will be the freezing temperature, if molarity in all cases are same. Therefore K_2SO_4 solution has the lowest freezing point.

Q.40 Among the electrolytes $Na_2SO_4, CaCl_2, Al_2(SO_4)_3$ and NH_4Cl, the most effective coagulating agent for Sb_2S_3 sol is
(a) Na_2SO_4 **(b) $CaCl_2$** **(c) $Al2(SO_4)_3$** **(d) NH_4Cl**

Ans: (c)

Sol: Sb_2S_3 is a negative (anionic) sol. According to Hardy Schulze rule, greater the valency of cationic coagulating agent, higher its coagulating power. Therefore, $Al2(SO_4)_3$ will be the most effective coagulating agent in the present case.

Q.41 Which of the following compounds is metallic and ferromagnetic?
(a) CrO_2 **(b) VO_2** **(c) MnO_2** **(d) TiO_2**

Ans: (a)

Sol: Only three elements iron (Fe), cobalt (Co) and nickel (Ni) show ferromagnetism at room temperature. CrO2 is also a metallic and ferromagnetic compound which is used to make magnetic tapes for cassette recorders.

Q.42 Among the following, the compound that can be most readily sulphonated is
(a) benzene **(b) nitrobenzene**
(c) toluene **(d) chlorobenzene**

Ans: (c)

Sol: Toluene is most readily sulphonated among these because methyl group is electron donating (+ I effect), activate benzene ring for electrophilic aromatic substitution.

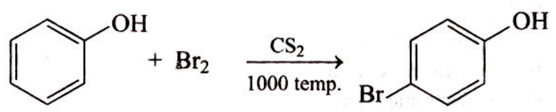
Q.43 Phenol reacts with bromine in carbon disulphide at low temperature to give

(a) *m*-bromophenol **(b) *o*-and *p*-bromophenol**

(c) *p*-bromophenol **(d) 2,4,6-tribromophenol**

Ans: (c)

Sol:



In carbon disulphide, no phenoxide ion exist, therefore only monobromination takes place.

Q.44 A positron is emitted from $^{23}_{11}$Na. The ratio of the atomic mass and atomic number of the resulting nuclide is

(a) 22/10 **(b) 22/11** **(c) 23/10** **(d) 23/12**

Ans: (c)

Sol: The required nuclear reaction is

$$_{11}\text{Na}^{23} \rightarrow _{+1}e^{o} + _{10}\text{Na}^{23}$$

Q.45 Concentrated nitric acid upon long standing, turns yellow-brown due to the formation of

(a) NO **(b) NO_2** **(c) N_2O** **(d) N_2O_4**

Ans: (b)

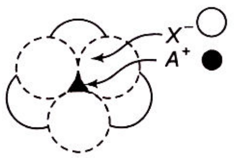
Sol: NO_2 is a brown coloured gas and imparts this colour to concentrated HNO3 during long standing.

$$4HNO_3 \rightarrow 2H_2O + 2NO_2 + 3O_2$$

Q.46 The volume strength of 1.5 N H_2O_2 is_____?

Sol: Volume strength of H_2O_2 = Normality $\times 5.6 = 1.5 \times 5.6 = 8.4\,\text{V}$

Q.47 The arrangement of X^- ions around A^+ ion in solid AX is given in the figure (not drawn to scale). If the radius of X^- is 250 pm, the radius of A^+ is_____?



Sol: $\dfrac{r_+(\text{cation})}{r_-(\text{cation})} = 0.414$

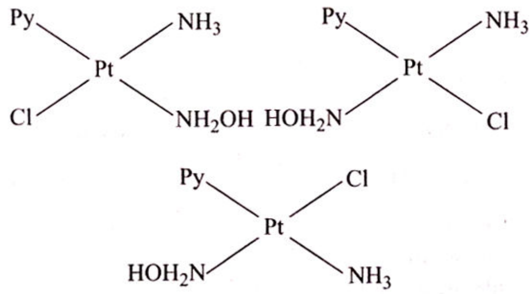
$$\frac{r(A^+)}{r(X^-)} = 0.414$$

$$r(A^+) = 0.414 \times r(X^-) = 0.414 \times 250\text{pm}$$

$$= 103.5\,\text{pm} \approx 104\,\text{pm}$$

Q.48 The number of geometric isomers that can exist for square planar $[Pt(Cl)(py)(NH_3)(NH_2OH)]^+$ is (py = pyridine).

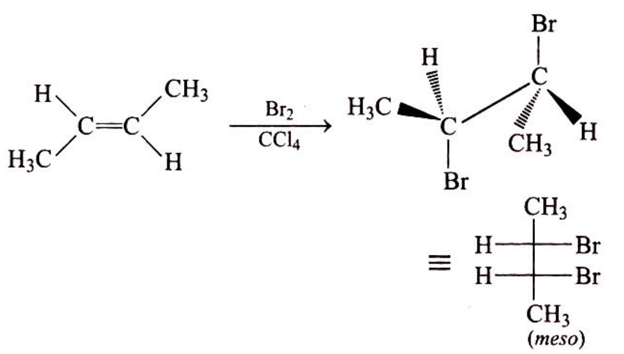
Sol: $[Pt(Cl)(py)(NH_3)(NH_2OH)]^+$ is square planar complex. The structures are formed by fixing a group and then arranging all the groups.



Hence, this complex shows three geometrical isomers.

Q.49 The number of stereoisomers obtained by bromination of *trans*-2-butene is_____?

Sol:



Q.50 If one litre of air is passed repeatedly over heated copper and magnesium till no further reduction in volume takes place, the volume finally obtained would be approximately?

Sol: Noble gases constitute about **1** % of the total volume of air. The ramaining 99% volume is mainly due to oxygen and nitrogen. When 1 L (1000 mL) of air is passed over heated copper, oxygen is consumed and on further passing over magnesium, nitrogen is consumed according to the equations

$$2Cu + O_2 \longrightarrow 2CuO$$

$$3Mg + N_2 \longrightarrow Mg_3N_2$$

In this way about 990 mL of air is consumed. The ramaining volume of about 10 mL is due to mainly Ar and other noble gases.

Part - C - MATHEMATICS

Q.51 The different letters of an alphabet are given. WOrds with five letters are formed from these given letters. The, the number of words which have at least one letter repeated, is

(a) 69760 **(b) 30240** **(c) 99748** **(d) None**

Ans: (a)

Sol: Total number of five letters words formed from ten different letters $=10\times10\times10\times10\times10=10^5$

Number of five letters words having no repetition $=10\times9\times8\times7\times6=30240$

$\therefore$ Number of words which have at least one letter repeated $=10^5-30240=69760$

Q.52 If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3, is

(a) $\frac{4}{55}$ **(b)** $\frac{4}{35}$ **(c)** $\frac{4}{33}$ **(d)** $\frac{4}{1155}$

Ans: (d)

Sol: Since, three distinct numbers are to be selected from first 100 natural numbers.

$\Rightarrow \quad n(A) = {}^{100}C_3$

$E_{(\text{favourable evenets})}$ = All three of them are divisible by both 2 and 3.

$\Rightarrow$ Divisible by 6 i.e. {6,12,18...,96}

$\therefore \quad n(E) = {}^{16}C_3$

$\therefore$ Required probability $= \frac{{}^{16}C_3}{{}^{100}C_3} = \frac{4}{1155}$

Q.53 Let α and β be the roots of equation $x^2-6x-2=0$. If $a_n=\alpha^n-\beta^n$, $n\geq1$, for then the value of $\frac{a_{10}-2a_8}{2a_9}$ is

(a) 6 **(b) −6** **(c) 3** **(d) −3**

Ans: (c)

Sol: Given, α and β are the roots of the equation $x^2-6x-2=0$.

$\therefore \quad a_n=\alpha^n-\beta^n$, for $n\geq1$,

$a_{10}=\alpha^{10}-\beta^{10}$

$a_8=\alpha^8-\beta^8$

$a_9=\alpha^9-\beta^9$

Now consider,

$$\frac{a_{10}-2a_8}{2a_9}=\frac{\alpha^{10}-\beta^{10}-2(\alpha^8-\beta^8)}{2(\alpha^9-\alpha^9)}$$

$$=\frac{\alpha^8(\alpha^2-2)-\beta^8(\beta^2-2)}{2(\alpha^9-\beta^9)}$$

$$=\frac{\alpha^8.6\alpha-\beta^8 6\beta}{2(\alpha^9-\beta^9)}=\frac{6\alpha^9-6\beta^9}{2(\alpha^9-6\beta^9)}=\frac{6}{2}=3$$

$$\left[\begin{array}{l} \because \alpha \text{ and } \beta \text{ are the roots of} \\ \qquad x^2-6x-2=0 \text{ or } x^2=6x+2 \\ \Rightarrow \quad \alpha^2=6\alpha+2 \quad \Rightarrow \alpha^2-2=6\alpha \\ \text{and } \beta^2=6\beta+2 \quad \Rightarrow \beta^2-2=6\beta \end{array}\right]$$

Alternate Solution

Since α and β are the roots of the equation

$$x^2-6x-2=0.$$

or $\quad x^2=6x+2$

$\therefore \quad \alpha^2=6\alpha+2$

$\Rightarrow \quad \alpha^{10}=6\alpha^9+2\alpha^8 \quad$ ...(i)

Similarly, $\quad \beta^{10}=6\beta^9+2\beta^8 \quad$ ...(ii)

On subtracting Eq. (ii) from Eq. (i). we get

$$\alpha^{10}-\beta^{10}=6\alpha^9-6\beta^9+2(\alpha^8-\beta^8) \qquad (\because a_n=\alpha^n-\beta^n)$$

$\Rightarrow \quad a_{10}=6a_9+2a_8$

$\Rightarrow \quad a_{10}-2a_8=6a_9 \Rightarrow \dfrac{a_{10}-2a_8}{2a_9}=3$

Q.54 The system of linear equations $x+\lambda y-z=0;\ \lambda x-y-z=0;\ x+y-\lambda z=0$ has a non-trivial solution for

(a) infinitely many values of λ

(b) exactly one value of λ

(c) exactly two values of λ

(d) exactly three values of λ

Ans: (d)

Sol: Given, System of linear equation is

$x+\lambda y-z=0;\ \lambda x-y-z=0;\ x+y-\lambda z=0$

Note that, given system will have a non-trivial solution only of determinant of coefficient matrix is zero,

i.e. $$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix}$$

$\Rightarrow \quad 1(\lambda+1)-\lambda(-\lambda^2+1)-1(\lambda+1)=0$

$\Rightarrow \quad \lambda+1+\lambda^3-\lambda-\lambda-1=0$

$\Rightarrow \quad \lambda^3-\lambda=0 \Rightarrow \lambda(\lambda^2-1)=0$

$\Rightarrow \quad \lambda=0 \text{ or } \lambda=\pm 1$

Hence, given system of linear equation has a non-trivial solution for exactly three values of λ.

Q.55 The normal to the curve at $x^2+2xy-3y^2=0$ at (1,1)

(a) does not meet the curve again
(b) meets in the curve again the second quadrant
(c) meets the curve again in the third quadrant
(d) meets the curve agin in the fourth quadrant

Ans: (d)

Sol: Given equation of curve is

$$x^2+2xy-3y^2=0 \quad \text{... (i)}$$

On differenting w.r.t x, we get

$$2x+2xy+2y-6yy=0 \Rightarrow y=\frac{x+y}{3y-x}$$

At $\quad x=1, y=1, y=1$

i.e. $\quad \left(\frac{dy}{dx}\right)_{(1,1)}=1$

Equation of normal at (1,1) is

$$y-1=-\frac{1}{1}(x-1) \Rightarrow y-1=-(x-1)$$

$$\Rightarrow \quad x+y=2 \quad \text{... (ii)}$$

On solving Eqs. (i) and (ii) simultaneously, we get

$$\Rightarrow \quad x^2+2x(2-x)-33(2-x^2)=0$$

$$\Rightarrow \quad x^2+4x-2x^2-3(4+x^2-4x)=0$$

$$\Rightarrow \quad -x^2+4x-12-3x^2+12x=0$$

$$\Rightarrow \quad -4x^2+16x-12=0$$

$$\Rightarrow \quad 4x^2-16x+12=0$$

$$\Rightarrow \quad x^2-4x+3=0$$

$$\Rightarrow \quad (1-1)(x-3)=0$$

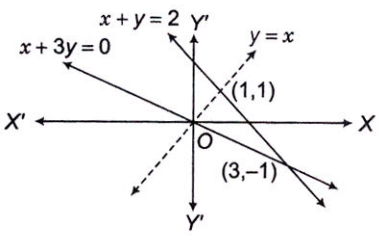
$$\therefore \quad y=1,3$$

Now, when $x=1$, then $y=1$

and when $x=3$, then $y=-1$

$\therefore \quad P=(1,1)$ and $Q=(3,-1)$

Hence, normal meets the curve agin at $(3,-1)$ in fourth quadrant.



Alternate Solution

Given, $\quad x^2+2xy-3y^2=0$

$\Rightarrow \quad (x-y)(x+3y)=0$

$\Rightarrow \quad x-y=0$ or $x+3y=0$

Equation of normal at (1,1) is

$y-1=-1(x-1) \Rightarrow x+y-2=0$

It intersects $x+3y=0$ at $(3,-1)$ and hence normal meeets the curve in fourth quadrant.

Q.56 The value of the integral $\int_{-\pi/2}^{\pi/2}\left(x^2+\log\frac{\pi-x}{\pi+x}\right)\cos x\,dx$ is

(a) 0 **(b) $\frac{\pi^2}{2}-4$** **(c) $\frac{\pi^2}{2}+4$** **(d) $\frac{\pi^2}{2}$**

Ans: (b)

Sol: $I=\int_{-\pi/2}^{\pi/2}\left(x^2+\log\frac{\pi-x}{\pi+x}\right)\cos x\,dx$

As, $\int_{-a}^{a} f(x)dx=0,$ when $f(-x)=-f(x)$

$\therefore \quad I=\int_{-\pi/2}^{\pi/2} x^2\cos x\,dx+0=2\int_0^{\pi/2}(x^2\cos x)\,dx$

$=2\{(x^2\sin x)_0^{\pi/2}-\int_0^{\pi/2}2x.\sin x\,dx\}$

$=2\left[\frac{\pi^2}{4}-2\{(-x.\cos x)_0^{\pi/2}-\int_0^{\pi/2}1.(-\cos x)\,dx\}\right]$

$=2\left[\frac{\pi^2}{4}-2(\sin x)_0^{\pi/2}\right]=2\left[\frac{\pi^2}{4}-2\right]=\left(\frac{\pi^2}{2}-4\right)$

Q.57 The lines $2x-3y=5$ and $3x-4y=7$ are diameters of a circle of area 154 sq units. Then, the equation of this circle is

(a) $x^2+y^2+2x-2y=62$ **(b) $x^2+y^2+2x-2y=47$**

(c) $x^2+y^2-2x+2y=47$ **(d) $x^2+y^2-2x+2y=62$**

Ans: (c)

Sol: Since, $2x-3y=5$ and $3x-4y=7$ are diameters of a circle.

Their point of intersection is centre $(1,-1)$

Also given, $\pi r^2=154$

$\Rightarrow \quad r^2=154\times\frac{7}{22}\Rightarrow r=7$

$\therefore$ Required equation of circle is

$(x-1)^2+(y+1)^2=7^2$

$\Rightarrow \quad x^2+y^2-2x+2y=47$

Q.58 The differential equation determines a family of circles with

(a) variable radii and a fixed centre at (0,1)

(b) variable radii a fixed centre at $(0,-1)$

(c) fixed radius 1 and variable centres along the X-axis
(d) fixed radius 1 and variable centres along the Y-axis

Ans: (c)

Sol: Given, $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$

$\Rightarrow \quad \int \frac{y}{\sqrt{1-y^2}} dy = \int dx$

$\Rightarrow \quad -\sqrt{1-y^2} = x + c \Rightarrow (x+c)^2 + y^2 = 1$

Here, centre$(-c, 0)$ and radius $= 1$

Q.59 The equation $\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1, |r| < 1$ represents

(a) an ellipse **(b) a hyperbola** **(c) a circle** **(d) None of the above**

Ans: (b)

Sol: Given equation is $\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1$, where $|r| < 1$

$\Rightarrow$ $1-r$ is $(+\text{ve})$ and $1+r$ is $(+\text{ve})$

$\therefore$ Given equation is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Hence, it represents a hyperbola when $|r| < 1$.

Q.60 Consider an infinite geometric series with first term *a* and common ratio *r*. If its sum is 4 and the second term is 3/4, then

(a) $a = 4/7,\ r = 3/7$ **(b)** $a = 2,\ r = 3/8$
(c) $a = 3/2,\ r = 1/2$ **(d)** $a = 3,\ r = 1/4$

Ans: (d)

Sol: Since, sum = 4 and second term $= \frac{3}{4}$.

It is given term *a* and common ratio *r*.

$\Rightarrow \quad \frac{a}{1-r} = 4,\ ar = \frac{3}{4}$

$\Rightarrow \quad r = \frac{3}{4a}$

$\Rightarrow \quad \frac{a}{1-\frac{3}{4a}} = 4$

$\Rightarrow \quad \frac{4a^2}{4a-3} = 4$

$\Rightarrow \quad (a-1)(a-3) = 0$

$\Rightarrow \quad a = 1 \ or \ 3$

When $a = 1, r = 3/4$

and when $a = 3, r = 1/4$

Q.61 In the binomial expansion of $(a-b)^n, n \geq 5$ the sum of the 5th and 6th terms is zero. Then a/b equals

(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$ (c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$

Ans: (b)

Sol: Given, $T_5 + T_6 = 0$

$\Rightarrow {}^nC_4 a^{n-4} b^4 - {}^nC_5 a^{n-5} b^5 = 0$

$\Rightarrow {}^nC_4 a^{n-4} b^4 - {}^nC_5 a^{n-5} b^5 \Rightarrow \frac{a}{b} = \frac{{}^nC_5}{{}^nC_4} = \frac{n-4}{5}$

Q.62 Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan)^{\tan\theta}, t_2 = (\tan\theta)^{\cot\theta}$,

$t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\tan\theta}$, then

(a) $t_1 > t_2 > t_3 > t_4$ (b) $t_4 > t_3 > t_1 > t_2$

(c) $t_3 > t_1 > t_2 > t_4$ (d) $t_2 > t_3 > t_1 > t_4$

Ans: (b)

Sol: As when $\theta \in \left(0, \frac{\pi}{4}\right)$, $\tan\theta < \cot\theta$

Since, $\tan\theta < 1$ and $\cot\theta > 1$

$\therefore \quad (\tan\theta)^{\cot\theta} < 1$ and $(\cot\theta)^{\tan\theta} > 1$

$\therefore$ $t_4 > t_1$ which only holds in (b).

Q.63 If $\vec{a} = (\hat{i} + \hat{j} + \hat{k}), \vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, *then* $\vec{b}$ is equal to

(a) $\hat{i} - \hat{j} + \hat{k}$ (b) $2\hat{j} - \hat{k}$

(c) $\hat{i}$ (d) $2\hat{i}$

Ans: (c)

Sol: We know that, $\vec{a} \times (\vec{a} \times \vec{b}) = (\vec{a}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b}$

$\therefore \quad (\hat{i} + \hat{j} + \hat{k}) \times (\hat{j} - \hat{k}) = (\hat{i} + \hat{j} + \hat{k}) - (\sqrt{3})^2 \vec{b}$

$\Rightarrow \quad -2\hat{i} + \hat{j} + \hat{k} = \hat{i} + \hat{j} + \hat{k} - 3\vec{b} \Rightarrow 3\vec{b} = +3\hat{i}$

$\therefore \quad \vec{b} = \hat{i}$

Q.64 The $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx$ value of

(a) $\sin x - 6\tan^{-1}(\sin x) + c$

(b) $\sin x - 2(\sin x)^{-1} + c$

(c) $\sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + c$

(d) $\sin x - 2(\sin x)^{-1} + 5\tan^{-1}(\sin x) + c$

Ans: (c)

Sol: Let $I = \int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx$

$$= \int \frac{\left(\cos^2 x + \cos^4\right).\cos x\ dx}{\left(\sin^2 x + \sin^4 x\right)}$$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$

$$\therefore \quad I = \int \frac{\left[\left(1-t^2\right) + \left(1-t^2\right)^2\right]}{t^2 + t^4} dt$$

$$\Rightarrow \quad I = \int \frac{1 - t^2 + 1 - 2t^2 + t^4}{t^2 + t^4} dt$$

$$\Rightarrow \quad I = \int \frac{2 - 3t^2 + t^4}{t^2(t^2+1)} dt \qquad \text{...(i)}$$

Using partial fraction for

$$\frac{y^2 - 3y + 2}{y(y+1)} = 1 + \frac{A}{y} + \frac{B}{y+1} \qquad [\text{where, } y = t^2]$$

$$\Rightarrow \quad A = 2,\ B = -6$$

$$\therefore \quad \frac{y^2 - 3y + 2}{y(y+1)} = 1 + \frac{2}{y} - \frac{6}{y+1}$$

Now, eq. (i) reduces to, $I = \int\left(1 + \frac{2}{t^2} - \frac{6}{1+t^2}\right)dt$

$$= t - \frac{2}{t} - 6\tan^{-1}(t) + c$$

$$= \sin x - \frac{2}{\sin x} - 6\tan^{-1}(\sin x) + c$$

Q.65 Area of the region is $\{(x, y)\} \in R^2 : y \geq \sqrt{|x+3|}, 5y \leq (x+9) \leq 15\}$ equal to

(a) $\frac{1}{6}$

(b) $\frac{4}{3}$

(c) $\frac{3}{2}$

(d) $\frac{5}{3}$

Ans: (c)

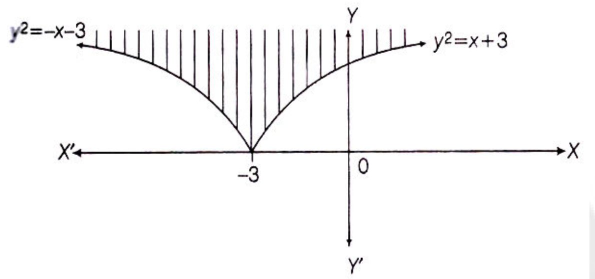
Sol: Here, $\{(x, y)\} \in R^2 : y \geq \sqrt{|x+3|}, 5y \leq (x+9) \leq 15\}$

$$\therefore \quad y \geq \sqrt{x+3}$$

$\Rightarrow \quad y \ge \begin{cases} \sqrt{x+3}, \text{ when } x \ge -3 \\ \sqrt{-x-3}, \text{ when } x \le -3 \end{cases}$

or $\quad y^2 \ge \begin{cases} x+3, \text{ when } x \ge -3 \\ -3-x, \text{ when } x \le -3 \end{cases}$

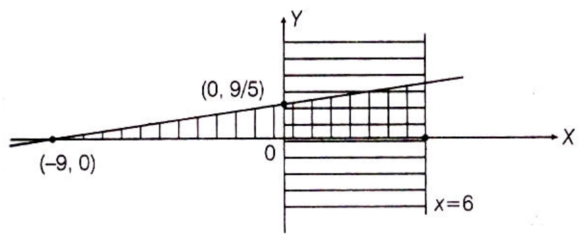
Shown as



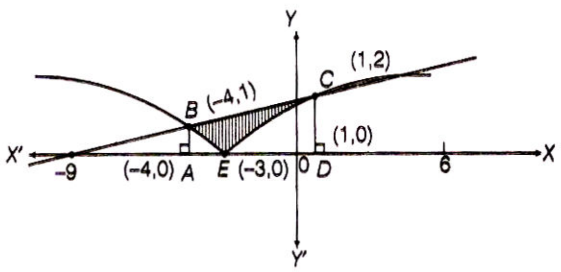
Also, $\quad 5y \le (x+9) \le 15$

$\Rightarrow \quad (x+9) \ge 5y$ and $x \le 6$

Shown as



$\therefore \ \{(x,y)\} \in R^2 : y \ge \sqrt{|x+3|}, 5y \le (x+9) \le 15\}$



$\therefore$ Required area = Area of trapezium ABCD – Area of ABE under parabola
– Area of CDE under parabola

$$= \frac{1}{2}(1+2)(5) - \int_{-4}^{-3} \sqrt{-(x+3)}dx - \int_{-3}^{1} \sqrt{x+3}\ dx$$

$$= \frac{15}{2} - \left[\frac{(-3-x^{3/2})}{-\frac{3}{2}} \right]_{-4}^{-3} - \left[\frac{(x+3)^{3/2}}{\frac{3}{2}} \right]_{-3}^{1}$$

$$=\frac{15}{2}+\frac{2}{3}[0-1]-\frac{2}{3}[8-0]$$

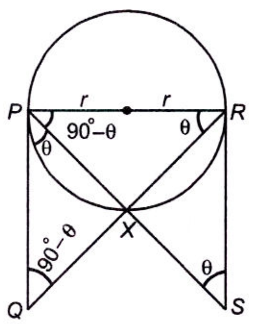
$$=\frac{15}{2}-\frac{2}{3}-\frac{16}{3}=\frac{15}{2}-\frac{18}{3}=\frac{3}{2}$$

Q.66 Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then $2r$ equals

(a) $\sqrt{PQ.RS}$ (b) $\frac{PQ+RS}{2}$ (c) $\frac{2PQ.RS}{PQ+RS}$ (d) $\sqrt{\frac{PQ^2+RS^2}{2}}$

Ans: (a)

Sol: From figure, it is clear that ΔPRQ and ΔRSP are similar.



$$\therefore \quad \frac{PR}{RS}=\frac{PQ}{RP}$$

$$\Rightarrow \quad PR^2=PQ.RS$$

$$\Rightarrow \quad PR=\sqrt{PQ.RS}$$

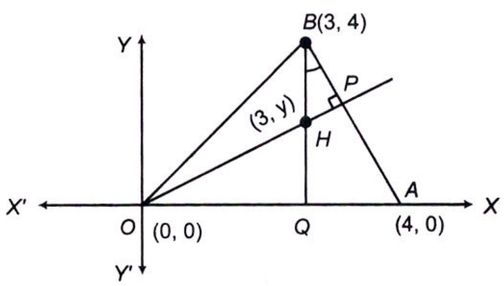
$$\Rightarrow \quad 2r=\sqrt{PQ.RS}$$

Q.67 Orthocentre of triangle with vertics (0,0), (3,4) and (4,0) is

(a) $\left(3,\frac{5}{4}\right)$ (b) (3,12) (c) $\left(3,\frac{3}{4}\right)$ (d) (3,9)

Ans: (c)

Sol: To find orthocentre of the triangle formed by (0,0) (3,4) and (4,0).



Let H be the orthocentre of ΔOAB

$\therefore$ (slope of OP i.e. OH).(slope of BA) $=-1$

$\Rightarrow \quad \left(\frac{y-0}{3-0}\right).\left(\frac{4-0}{3-4}\right)=-1$

$\Rightarrow \quad -\frac{4}{9}y=-1$

$\Rightarrow \quad y=\frac{3}{4}$

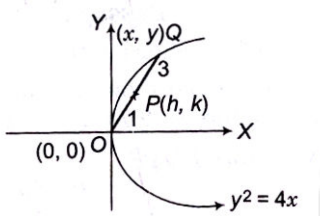
$\therefore$ Required orthocentre $=(3,y)=\left(3,\frac{3}{4}\right)$

Q.68 Let (*x,y*) be any point on the parabola Let P be the point that divides the line segment from (0,0) to (*x,y*) in the ratio 1 : 3. Then, the locus of P is

(a) $x^2 = y$ **(b)** $y^2 = 2x$ **(c)** $y^2 = x$ **(d)** $x^2 = 2y$

Ans: (c)

Sol: By section formula,



$$h=\frac{x+0}{4},k=\frac{y+0}{4}$$

$\therefore \quad x=4\,h,\ y=4k$

Subsituting in $y^2=4x$,

$(4k)^2=4(4h)\Rightarrow k^2=h$

or $y^2=x$ is required locus.

Q.69 The curve described paarametrically by $x=t^2+t+1,\ \ y=t^2-t+1$ represents

(a) a pair of straight lines **(b) an ellipse**
(c) a parabola **(d) a hyperbola**

Ans: (c)

Sol: Given curves are $x=t^2+t+1$ ...(i)

and $y=t^2-t+1$ ...(ii)

On subtracting Eq. (ii) from Eq. (i),

$$x-y=2t$$

Thus, $x=t^2+t+1$

$\Rightarrow \quad x=\left(\frac{x-y}{2}\right)^2+\left(\frac{x-y}{2}\right)+1$

$\Rightarrow \quad 4x=(x-y)^2+2x-2y+4$

$\Rightarrow \quad (x-y)^2 = 2(x+y-2)$

$\Rightarrow \quad x^2+y^2-2xy-2x-2y+4=0$

Now, $\Delta = 1.1.4+2.(-1)(-1)(-1)-1\times(-1)^2-1\times(-1)^2-4(-1)^2$

$= 4-2-1-1-4=-4$

$\therefore \quad \Delta \neq 0$

and $ab-h^2 = 1.1-(-1)^2 = 1-1=0$

Hence, it represents a equation of parabola.

Q.70 Distnace between two parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is

(a) $\frac{3}{2}$ **(b)** $\frac{5}{2}$ **(c)** $\frac{7}{2}$ **(d)** $\frac{9}{2}$

Ans: (c)

Sol: (i) Equation of plane through intersection of two planes,

i.e. $(a_1x+b_1y+c_1z+d_1)+\lambda$

$$(a_2x+b_2y+c_2z+d_2)=0$$

(ii) Distance of a point (x_1, y_1, z_1) from

$ax+by+cz+d=0$

$$= \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

Equation of plane through intesection of two planes

$x+2y+3z=2$ and $x-y+z=3$ *is*

$(x+1y+3z-2)+\lambda(x-y+z-3)=0$

$\Rightarrow (1+\lambda)x+(2-\lambda)y+(3+\lambda)x-(2+3\lambda)=0$

Whose distance from $(3,1-1)$ *is* $\frac{2}{\sqrt{3}}$

$$\Rightarrow \quad \frac{|3(1+\lambda)+1.(2-\lambda)y+(3+\lambda)x-(2+3\lambda)|}{\sqrt{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{|-2\lambda|}{\sqrt{3\lambda^2+4\lambda+14}} = \frac{2}{\sqrt{3}}$$

$\Rightarrow \quad 3\lambda^2 = 3\lambda^2+4\lambda+14$

$\Rightarrow \quad \lambda = -\frac{7}{2}$

$$\therefore \quad \left(1-\frac{7}{2}\right)x+\left(2+\frac{7}{2}\right)y+\left(3-\frac{7}{2}\right)z-\left(2-\frac{21}{2}\right)=0$$

$$\Rightarrow \quad -\frac{5x}{2}+\frac{11}{2}y-\frac{1}{2}z+\frac{17}{2}=0$$

or $5x-1y+z-17=0$

Q.71 The number of integral values of k **for which the equation** $7\cos x+5\sin x=2k+1$ **has solution, is_____?**

Sol: We know that,

$$-\sqrt{a^2+b^3}\le a\sin x+b\cos x\le\sqrt{a^2+b^2}$$

$\therefore$ $-\sqrt{74}\le 7\cos x+5\sin x\le\sqrt{74}$

Since, k isinteger, $-9<2k+1<9$

i.e. $-\sqrt{74}\le 7\cos x+5\sin x\le\sqrt{74}$

$\Rightarrow$ $-10<2k<8\Rightarrow -5<k<4$

$\Rightarrow$ Number of possible interger values of $k=8$.

Q.72 The mean of the data set comprising of 16 observations is 16. If one of the observations valued 16 is deleted and three new observations valued 3,4 and 5 added to the data, then the mean of the resultant data is_____?

Sol: Given, $\dfrac{x_1+x_2+x_3+\ldots+x_{16}}{16}=16$

$\Rightarrow$ $\sum_{i=1}^{16}x_i=16\times16$

Sum of new observations

$=\sum_{i=1}^{18}y_1=(16\times16-16)+(3+4+5)=252$

Number of observations $=18$

$\therefore$ New mean $=\dfrac{\sum_{i=1}^{18}y_1}{18}=\dfrac{252}{18}=14$

Q.73 What is the standard deviation of the following series

Measurments	**0 - 10**	**10 - 20**	**20 - 30**	**30 - 40**
Frequency	**1**	**3**	**4**	**2**

Sol:

Class	f_i	x_i	$d_i=x_i-A$ $A=25$	f_id_i	$f_id_i^2$
0 – 10	1	5	–20	–20	400
10 – 20	3	15	–10	–30	300
20 – 30	4	25	0	0	0
30 – 40	2	35	10	20	200
Total	10			–30	900

$$\sigma^2=\frac{\Sigma f_1d_1^2}{\Sigma f_1}-\left(\frac{\Sigma f_1d_1}{\Sigma f_1}\right)^2$$

$= \frac{900}{10} - \left(\frac{-30}{10}\right)^2$

$= 90 - 9 = 81$

$\Rightarrow \sigma = 9$

Q.74 India plays two matches each with West Indies and Australia. In any match the prob ability of india getting point 0, 1 and 2 are 0.45, 0.05 and 0.05 respectively. Assuming that the outcomes are independents, the probability of india getting at least 7 points is _____?

Sol: Matches played by india are four. Maximum points in any match are 2

$\therefore$ Maximum points in four matches can be 8 only.

Therefore probability $(P) = p(7) + p(8)$

$p(7) = {}^4C_1(0.05)(0.5)^3 = 0.0250$

$p(8) = (0.5)^4 = 0.0625$

$P = 0.0875$

Q.75 Let $x_1, x_2, ..., x_n$ be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$ Then a possible value of n among the following is _____?

Sol: Since root mean square $\geq$ arithmetic mean

$$\therefore \quad \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}} \geq \frac{\sum_{i=1}^{n} x_i}{n} = \sqrt{\frac{400}{n}} \geq \frac{80}{n} \Rightarrow n \geq 16$$

Hence, possible value of n = 18.

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