JEE (MAIN)

TEST PAPER

SUBJECT : PHYSICS,CHEMISTRY, MATHEMATICS ***TEST CODE : TSJMT216***

ANSWER PAPER

TIME : 3 HRS ***MARKS : 300***

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark(-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or “un- darken” a darkened bubble.

Part A - PHYSICS

Q.1 "rad" is the correct unit used to report the measurement of
(a) Biological effect of radiation
(b) Rate of decay of a radioactive source
(c) Ability of a beam of gamma ray photons to produce ions in a target
(d) Energy delivered by radiation to a target.

Ans: (d)

Sol: The unit rad (radiation - absorbed dose) in an old unit. It is a measure of the radiation dose (as energy per unit mass) actually absorbed by a specific object such as a patient's hand or chest.

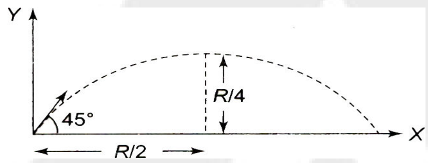
Q.2 A projectile is thrown into space so as to have maximum horizontal range R. Taking the point of projection as origin, the co-ordinates of the point where the speed of the particle is minimum are

(a) (R, R) **(b)** $\left(R, \frac{R}{2}\right)$ **(c)** $\left(\frac{R}{2}, \frac{R}{4}\right)$ **(d)** $\left(R, \frac{R}{4}\right)$

Ans: (c)

Sol: For maximum horizontal range , $\theta = 45^o$

From $R = 4H \cot\theta = 4H$ [as $as\ \theta = 45^o$, for maximum range].

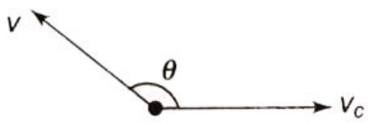


Speed of the particle will be minimum at the heighest point of parabola.
So, the co-ordinates of the heighest point will be (R/2, R/4).

Q.3 A car is moving along a road with a speed of 45 km/h. In what direction must body be projected from it with a velocity of 25 m/s, so that its resultant motion is at right angles to the direction of car ?
(a) At an angle of 120° with the direction of motion of car
(b) At an angle of 60° with the direction of motion of car
(c) At an angle of 90° with direction of motion of car
(d) At an angle of 135° with the direction of motion of car.

Ans: (a)

Sol: $v_c = 45 km/h = \frac{25}{2} m/s$



For the resultant motion to be upwards.

$$v\cos\theta + v_c = 0$$

$$\cos\theta = -\frac{v_c}{v} = -\frac{25/2}{25} = -\frac{1}{2} \Rightarrow \theta = 120^o$$

Q.4 The ratio of the weight of a man in a stationary lift and when it is moving downward with uniform acceleration *a* is 3 : 2. The value of a is (g : acceleration due to gravity on the earth)

(a) $\frac{3}{2}g$ (b) $\frac{g}{3}$ (c) $\frac{2}{3}g$ (d) g

Ans: (b)

Sol: $\frac{\text{Weight of an man in stationary lift}}{\text{Weight of a man in downward moving in}} = \frac{mg}{m(g-a)} = \frac{3}{2}$

$\therefore \quad \frac{g}{g-a} = \frac{3}{2} \Rightarrow 2g = 3g - 3a \quad \text{or} \quad a = \frac{g}{3}$

Q.5 The fraction of the floating object of volume V_0 and density d_0 above the surface of a liquid of density *d* will be

(a) $\frac{d_0}{d}$ (b) $\frac{dd_0}{d+d_0}$ (c) $\frac{d-d_0}{d}$ (d) $\frac{dd_0}{d-d_0}$

Ans: (c)

Sol: For the floatation

$$V_0 d_0 g = V_{10} dg \Rightarrow V_{in} = V_0 \frac{d_0}{d}$$

$$\therefore \quad V_{out} = V_0 - V_{in} = V_0 - V_0 \frac{d_0}{d} = V_0 \left[\frac{d-d_0}{d}\right]$$

$$\Rightarrow \quad \frac{V_{out}}{V_0} = \frac{d-d_0}{d}$$

Q.6 A monoatomic ideal gas, initially at temperature T_1 is enclosed in a cylinder fitted with a frictionless piston. The gas a allowed to expand adiabatically to temperature T_2 by relasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion, respectively. Then T_1/T_2 is given by

(a) $\left(\frac{L_1}{L_2}\right)^{2/3}$ (b) $\frac{L_1}{L_2}$ (c) $\frac{L_2}{L_1}$ (d) $\left(\frac{L_2}{L_1}\right)^{2/3}$

Ans: (d)

Sol: For an adiabatic process, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$\Rightarrow \quad \frac{T_1}{T_2} = \left[\frac{V_2}{V_1}\right]^{\gamma-1} = \left[\frac{L_2 A}{L_1 A}\right]^{(5/3)-1} = \left[\frac{L_2}{L_1}\right]^{2/3}.$$

Q.7 Two thermally insulated vessels 1 and 2 are filled with air at temperature T_1, T_2; volumes V_1, V_2 and pressure P_1, P_2, respectively. If the value joining the two vessels is opened, the temperature inside the vessel at equilibrium will be

(a) $T_1 = T_2$ (b) $(T_1 + T_2)/2$ (c) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$ (d) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_1 + P_2V_2T_2}$

Ans: (c)

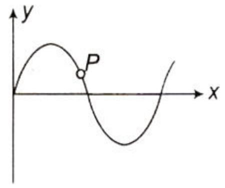
Sol: The guiding principle in this problem is that the total number of moles of the system remain the same.

$$\frac{P_1V_1}{RT_1}+\frac{P_2V_2}{RT_2}=\frac{P(V_1+V_2)}{RT} \quad \text{or} \quad T=\frac{P(V_1+V_2)T_1T_2}{P_1V_1T_2+P_2V_2T_2}$$

By, Boyle's law $P(V_1+V_2)=P_1V_1+P_2V_2$

$$\therefore\ P=\frac{P_1V_1+P_2V_2}{V_1+V_2} \qquad \therefore\ T=\frac{(P_1V_1+P_2V_2)T_1T_2}{P_1V_1T_2+P_2V_2T_1}$$

Q.8 A transverse sinusoidal wave moves along a string in the position *x*-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular amplitude is snap-shot of the wave is shown in the figure. The velocity of point *p* when its displacement is 5 cm, is



(a) $\frac{\sqrt{3}\pi}{50}\hat{j}\,\text{m/s}$ (b) $-\frac{\sqrt{3}\pi}{50}\hat{j}\,\text{m/s}$ (c) $\frac{\sqrt{3}\pi}{50}\hat{i}\,\text{m/s}$ (d) $-\frac{\sqrt{3}\pi}{50}\hat{i}\,\text{m/s}$

Ans: (a)

Sol: From $V=-v\times\frac{\partial y}{\partial x}$

At location of $P, \partial y/\partial x$ is -ve is v is along +ve *x*-axis. So, *vp* is along +ve *y*-axis. So, option (a) is correct.

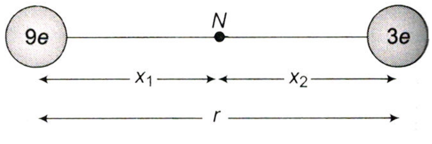
Q.9 Two charges 9*e* and 3*e* are placed at a distance *r*. The distance of the point where the electric fields intensity will be zero is

(a) $\frac{r}{(\sqrt{3}+1)}$ **from 9*e* charge** (b) $\frac{r}{1+\sqrt{1/3)}}$ **from 9*e* charge**

(c) $\frac{r}{(1-\sqrt{3})}$ **from 3*e* charge** (d) $\frac{r}{1+\sqrt{1/3}}$ **from 3*e* charge**

Ans: (b)

Sol: Suppose neutral point is obtained at a distance x_1 from charge 9*e* and x_2 from charge 3*e*.



Q.10 This question Statement-1 and Statement -2. Of the four choices given after the statement, choose the one that best describes the two statements.

Statement -1 : For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement -2 : The net work done by a conservative force on an object moving along a closed loop is zero.

(a) Statement-1 is true, Statement-2 is false.

(b) Statement-1 is true, Statement-2 is true; Statement-2 is correct explanation of Statement-1.

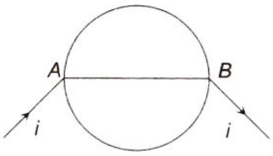
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
(d) Statement-1 is false, Statement-2 is true.

Ans: (b)
Sol: Work done by conservative force done not depend on the path. Electrostatic force is a conservative force.

Q.11 An air capacitor of capacity $C = 10\mu F$ is connected to a constant voltage bettery of 12 V. Now the space between the plates is filled with a liquid of dielectric constant 5. The charge that flows now from battery to the capacitor is
(a) $120\mu F$ (b) $600\mu F$ (c) $480\mu F$ (d) $24\mu F$

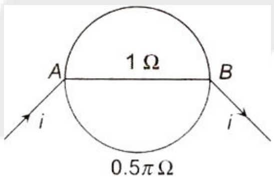
Ans: (c)
Sol: Initially, charge on the capacitor $Q_1 = 10 \times 12 = 120\ \mu C$
When dielectric medium is filled, so capacitance becomes K times, i.e. new capacitance
$C' = 5 \times 10 = 50\ \mu C$
Final charge on the capacitor $Q_f = 50 \times 12 = 60\ \mu C$
Hence, additional charge supplied by the battery $= Q_f - Q_i = 480\ \mu C$

Q.12 A wire of resistance $0.5\ \Omega/m$ is bent into a circle of radius 1 m. The same wire is connected across a diameter AB as shown in figure. The equivalent resistance is



(a) π ohm (b) $\pi(\pi+2)$ ohm (c) $\pi(\pi+4)$ ohm (d) $(\pi+1)$ ohm

Ans: (c)
Sol: Resistance of upper semicircle
$=$ Resistance of lower semicircle
$= 0.5 \times (\pi R) = 0.5\pi\Omega$



Resistance of wire $AB = 0.5 \times 2 = 1\Omega$

Hence, equivalent resistance between A and B, $\frac{1}{R_{AB}} = \frac{1}{0.5\pi} + \frac{1}{1} + \frac{1}{0.5\pi}$

$\Rightarrow \quad R_{AB} = \frac{\pi}{(\pi+4)}\Omega$

Q.13 If an electron and a proton having a same moment enter perpendicular to a magnetic field, then
(a) The curved path of the electron and proton will be same (ignoring the sense of revolution)
(b) They will more undeflected
(c) The curved path of electron will be more curved than that of proton.
(d) The path of proton will be more curved.

Ans: (a)

Sol: $Buq = \frac{mv^2}{r}$ or $r = \frac{mv}{Bq}$ or $r = \frac{p}{Bq}$

All quantities on right hand side remain unchanged so, r remain unchanged.

Q.14 A direct current flows in a long straight conductor whose cross-section has form of a thin half ring of radius R. The same current flows in opposite direction along a thin conductor placed on the axis on first conductor, then the magnetic interaction force between the conductors to a unit of their length is

(a) $\frac{\mu_0 I^2}{\pi^2 R}$ **(b)** $\frac{\mu_0 I}{2\pi R}$ **(c)** $\frac{\mu_0 I}{\pi^2 R}$ **(d)** $\frac{2\mu_0 I}{\pi R}$

Ans: (a)

Sol: The magnetic field at the centre of a long straight conductor whos crossection is in the form of thin half ring is

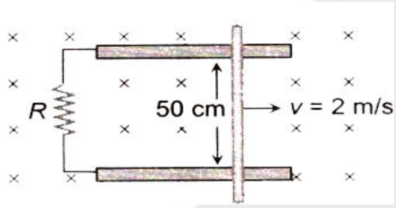
$$B = \frac{\mu_0 I}{\pi^2 R} \quad ....(1)$$

So the force per unit length of the wire carrying current I and placed at the centre of above are shaped conductor is

$$F = BI \times l$$

$$= \frac{\mu_0 I^2}{\pi^2 R} \quad \text{(For unit length)}$$

Q.15 As shown figure, a metal rod makes contact and complete the circuit. The circuit is perpendicular to the magnetic field with $B = 0.15$ Tesla. If the resistance is 3Ω, force needed to move the rod as indicated with a constant spedd of 2 m/s is



(a) 3.75×10^{-3} **N** **(b)** 3.75×10^{-2}**N** **(c)** 3.75×10^{2} **N** **(d)** 3.75×10^{-4}**N**

Ans: (a)

Sol: Force needed to move the rod is $F = \frac{B^2 v l^2}{R} = \frac{(0.15)^2 \times 2 \times (0.5)^2}{3}$

$$= 3.75 \times 10^{-3} N$$

Q.16 The magnetic field in the plane electromagnetic wave is given by
$B_z = 2 \times 10^{-7} \sin(0.5 \times 10^9 x + 1.5 \times 10^{11} t)$. The expression for electric field will be

(a) $E_z = 30\sqrt{2} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ **V / m**

(b) $E_z = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ **V / m**

(c) $E_y = 30\sqrt{2} \sin(0.5 \times 10^{11} x + 0.5 \times 10^3 t)$ **V / m**

(d) $E_y = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ **V / m**

Ans: (d)

Sol: Wave is progagating along positive X-axis; magnetic field is directed along Z-axis to electric field must be directed along $(\hat{k} \times \hat{i}) = \hat{j}$, along Y-axis,

amplitude of electric filed $E_0 = B_0\, c = 2 \times 10^{-7} \times 3 \times 10^{8} = 60 V/m$

Q.17 The diameter of the eye-ball of a normal eye is about 2.5 cm. The power of the eye lens varies from

(a) 2D to 10D **(b) 40D to 32D** **(c) 9 D to 8 D** **(d) 44 D to 40D**

Ans: (d)

Sol: An eye sees distant object with full relaxation

So, $\frac{1}{2.5 \times 10^{-2}} - \frac{1}{-\infty} = \frac{1}{f}$ or $P = \frac{1}{f} = \frac{1}{25 \times 10^{-2}} = 40 D$

An eye sees an object at 25 cm with strain so

$$\frac{1}{2.5 \times 10^{-2}} - \frac{1}{-25 \times 10^{-2}} = \frac{1}{f}$$

Q.18 The eye can detect 5×10^4 photons / m²s of green light $(\lambda = 5000\,\mathring{A})$, while car can detect 10^{-13} W/m². As a power detector, which is more sensitive and by what factor ?

(a) Eye is more sensitive and by a factor of 5.00
(b) Ear is more sensitive by a factor of 5.00
(c) Both are equally sensitive.
(d) Eye is more sensitive by a factor of 10.

Ans: (a)

Sol: Energy received by the eye, $E = \frac{nhc}{\lambda} = \frac{5 \times 10^4 \times 6.6 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}}$

$$= 0.2 \times 10^{-13} \text{ W/m}^2$$

So, eye is more sensitive by a factor $\frac{1}{0.200} = 5.00$.

Q.19 An electron revolving in an orbit of radius $0.5\,\mathring{A}$ in a hydrogen atom executes 10 revolution per second. The magnetic moment of electron due to its orbit motion will be

(a) 1.256×10^{-32} Am² **(b) 653×10^{-26} Am²**

(c) 10^{-3} Am² **(d) 256×10^{-26} Am²**

Ans: (a)

Sol: $M = IA = ef\pi r^2 = 1.6 \times 10^{-9} \times 10^{16} \times 3.14 \times (0.5 \times 10^{-10})^2 \text{ A m}^2$

$$= 1.256 \times 10^{-23} \text{ Am}^2$$

Q.20 The percentage of quantity of a radioactive material that remain after five half lives will be

(a) 31% **(b) 3.125%** **(c) 0.3%** **(d) 1%**

Ans: (b)

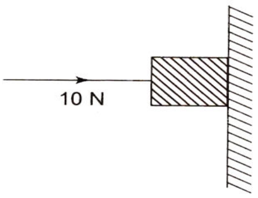
Sol: $\frac{N}{N_0} = \frac{1}{2^{5T/T}} = \frac{1}{2^5} = 100 = \frac{100}{32} = 3.125$

Q.21 A body sliding on a smooth inclined plane requires 4 s to reach the bottom starting from rest at the top. How much time does it take to cover one-fourth distance starting from rest at the top.

Sol: $S = \frac{1}{2}at^2 \Rightarrow t \propto \sqrt{s}$ (As a = constant)

$$\frac{t_2}{t_1} = \sqrt{\frac{s_2}{s_1}} = \sqrt{\frac{s/4}{s}} = \frac{1}{2} \quad \Rightarrow \quad t_2 = \frac{t_1}{2} = \frac{4}{2} = 2s$$

Q.22 A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is ________?



Sol: Here frictional force is equal to weight of the body

$\therefore$ $f = W = mg$ or $\mu R = W$ or $W = 0.2 \times 10 = 2$ N

Q.23 A heating coil is labelled 100 W, 220 V. The coil is cut in half and the two pieces are joined in parallel to the same source. The energy now liberated per second is____?

Sol: Let the resistance of the heating coil be R, when coil cut in two equal parts, resistance of each part will be R/2. When these two parts are corrected in parallel, $R_{eq} = R/4$. So according to $P \propto 1/R$, power becomes 4 times, *i.e.* $P' = 4P = 400 J/s$.

Q.24 In a series LCR circuit R = 200 Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is ________?

Sol: The given circuit is under resonance as $X_L = X_C$

Hence, power dissipated in the circuit is $P = \frac{V^2}{R} = 242\ W$

Q.25 If the potential function is given by V = 4x + 3y, then the magnitude of electric field intensity at the point (2, 1) will be ?

Sol: By using $E = \sqrt{E_x^2 + E_y^2}$

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}(4x+3y) = -4$$

and $$E_y = -\frac{dV}{dy} = -\frac{d}{dy}(4x+3y) = -3$$

$\therefore$ $$E = \sqrt{(-4)^2 + (-3)^2} = 5N/C$$

Part - B - CHEMISTRY

Q.26 The wave number of the first line in the Balmer series of hydrogen is 15200 cm^{-1}, What would be the wave number of the first line in the Lyman series of the Be^{3+} ion?

(a) $2.4 \times 10^5\ cm^{-1}$ (b) $24.3 \times 10^5\ cm^{-1}$ (c) $6.08 \times 10^5\ cm^{-1}$ (d) $1.313 \times 10^6\ cm^{-1}$

Ans: (d)

Sol: Given $15200 = R(1)^2\left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right]$ ....(i)

Then $\bar{v} = R(4)^2\left[\frac{1}{(1)^2} - \frac{1}{(2)^2}\right]$ ....(ii)

From equ (i) and (ii), we have $\bar{v} = 1.313 \times 10^6\ cm^{-1}$.

Q.27 A quantity of heat is confined in a chamber of constant volume. When the chamber is immersed in a bath of melting ice, the pressure of the gas is 1000 torr. Find the temperature when the pressure manometer indicates in absolute pressure of 400 torr.

(a) 109 K (b) 273 K (c) 373 K (d) 0 K

Ans: (a)

Sol: Melting point of ice $= 273\,K$

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ $\frac{1000}{273} = \frac{400}{T_2}$ (at constant V)

$T_2 = 109.2$

Q.28 Anhydrous $AlCl_3$ is a covalent compound from the data given. Predict whether it will remain covalent or become ionic in an aqueous solution. (Lattice energy of $AlCl_3 = 5137$ kJ / mol), ΔH hydration for $Al^{3+} = -4665$ kJ /mol, and ΔH hydration for $Cl^- = -381$ kj/mol.

(a) Ionic (b) Covalent (c) Partially ionic (d) Partially covalent

Ans: (a)

Sol: (i) $Al^{3+}_{(g)} + 3Cl^-_{(g)} \rightarrow AlCl_{3(s)}$ $\Delta H = -5317$ kJ/mol ...(i)

(ii) $Al^{3+}_{(g)}$ + aq. $\rightarrow Cl^-_{(aq)}$ $\Delta H = -4665$ kJ/mol ...(ii)

(iii) $Cl^-_{(g)}$ + aq. $\rightarrow Cl^-_{aq}$ $\Delta H = -381$ kJ/mol ....(iii)

If it is ionic, then following reaction takes place in aqueous solution :

$AlCl_{3(g)}$ +aq. $\rightarrow Al^{3+}_{(aq)} + Cl^-_{(aq)}$ $\Delta H_4 = ?$

$\Delta H = 4$ Eq(ii) + 3 Eq (iii) – Eq(i)

$\Delta H = -4665 + 3(-381) - (-5137)$

$= -4665 - 1143 + 5137$

$= -5808 + 5137 = -671$

$\Delta H = -ve$; so compound is ionic.

Q.29 For $NH_4HS_{(s)} \rightleftharpoons NH_{3(g)} + H_2S_{(g)}$: observed pressure for reaction mixture at equilibrium is 1.12 atm at 106°C. The value of K_p for the reaction is :

(a) 3.316 atm² (b) 0.316 atm² (c) 31.36 atm² (d) 6.98 atm²

Ans: (b)

Sol: $NH_4HS_{(s)} \rightleftharpoons \underset{P}{NH_{3(g)}} + \underset{P}{H_2S_{(g)}}$

Pressure at equilibrium

$\therefore$ Total pressure at equilibrium $= 2P = 1.12$

$\therefore$ $P = 1.121/2$ atm

$\because$ $K = p'_{NH_3} \times p'_{H_2S}$

$\therefore$ $K_p = \frac{1.12}{2} \times \frac{1.12}{2} = 0.3136$ atm^2

Q.30 The degree of hydrolysis in hydrolytic equilibrium $A^- + H_2O \rightleftharpoons HA + OH^-$ at salt concentration of 0.001 M is $(K_a = 1 \times 10^{-5})$:

(a) 1×10^{-3} **(b) 1×10^{-4}** **(c) 5×10^{-4}** **(d) 1×10^{-4}**

Ans: (a)

Sol: $K_h = \frac{K_w}{K_a} = \frac{10^{-14}}{1 \times 10^{-5}} = 10^{-9}$

$K_h = \alpha^2 C;\ \alpha = \sqrt{\frac{K_h}{C}} = \sqrt{\frac{1 \times 10^{-9}}{0.001}} = 1 \times 10^{-3}$

Q.31 A solution is obtained by dissolving 12 g of urea (molecular weight 60) in a litre of water. Another solution in obtained by dissolving 68.4 g of cane sugar (molecular weight 342) in a litre of water at the same temperature. The lowering of vapour pressure in the first solution is :
(a) same as that of the second solution
(b) nearly one-fifth of the second solution
(c) double that of the second solution
(d) nearly five times that of the second solution

Ans: (a)

Sol: We know that in the first urea ,

Number of the moles of urea $= \frac{\text{Mass of urea}}{\text{Molecular weight of urea}} \times \frac{1}{V} = \frac{12}{60} \times \frac{1}{1} = 0.2$

In second solution,

Number of moles of cane sugar $= \frac{\text{Mass of cane sugar}}{\text{Molecular weight of cane sugar}} = \frac{68.4}{242} \times \frac{1}{1} = 0.2$

Q.32 We have taken a saturated solution of AgBr. K_{sp} of AgBr is 12×10^{-14}. If 10^{-7} mol of $AgNO_3$ is added to 1 L of this solution, then the conductivity of this solution in terms of 10^{-7} S/m units will be :

[Given $\lambda^o_{(Ag^+)} = 4 \times 10^{-3}$ Sm^2/mol; $\lambda^o_{(Br)} = 6 \times 10^{-3}$ Sm^2/mol; $\lambda^o_{(NO_3^-)} = 5 \times 10^{-3}$ Sm^2/mol]

(a) 39 **(b) 55** **(c) 15** **(d) 41**

Ans: (a)

Sol: The solubility of AgBr in the presence of 10^{-7} M $AgNO_3$ is $3 \times 10^{-7} M$

Therfore, $[Br^-] = 3 \times 10^{-4}\ m^3, [Ag^+] = 4 \times 10^{-4}\ m^3,$ and $[NO_3^-] = 10^{-4}\ m^3$

Therefore, $K_{total} = K_{Br^-} + K_{Ag^+} + K_{NO_3^-} = 39\ S/m$

Q.33 **$A_{(aq)} \rightarrow B_{(aq)} + C_{(aq)}$ is a first-order reaction,**

Time	**t**	∞
Moles of reagent	n_1	n_2

Reaction progress is measured with the help of titration with reagent R. If all A, B and C reacted with the reagent and have *n* factors [*n* factor; eq. wt. = (mol. wt./*n*)] in the ratio of 1 : 2 : 3 with the reagent, the *k* in terms of *t*, n_1, and n_2 is :

(a) $k = \frac{1}{t}\ln\left(\frac{n_2}{n_2 - n_1}\right)$ **(b)** $k = \frac{1}{t}\ln\left(\frac{2n_2}{n_2 - n_1}\right)$

(c) $k = \frac{1}{t}\ln\left(\frac{4n_2}{n_2 - n_1}\right)$ **(d)** $k = \frac{1}{t}\ln\left(\frac{4n_2}{5(n_2 - n_1)}\right)$

Ans: (d)

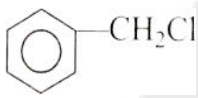
Sol: Let n be the moles of reagent R when R is reacted with A at time $t = 0$.

	A	$\rightarrow$	B	$+$	C
$t = 0$	n		0		0
at t	$n - x$		$2x$		$2x$
at $t \rightarrow \infty$	0		$2n$		$3n$.

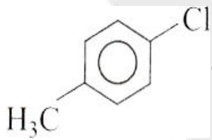
$\therefore\ 5n = n_2 \Rightarrow n = \frac{n_2}{5}$ $\qquad n + 4x = n_1 \Rightarrow x = \frac{n_1 - n}{4}$

$k = \frac{2.303}{t}\log\left(\frac{n}{n-x}\right)$ $\qquad k = \frac{1}{t}\ln\left(\frac{4n_2}{5(n_2 - n_1)}\right)$

Q.34 **Which of the following will be the least reactive towards nucleophilic substitution ?**

(a) 

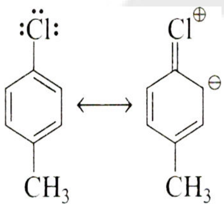
(b) 

(c) 

(d) C_2H_5Cl

Ans: (c)

Sol: Due to partial double bond character, it is difficult to break the double bond.



Q.35 **A molecule can be said to have plane of symmetry if :**

(a) It can be divided into two equal halves – one half being the mirror image of the other half

(b) It can be divided into two halves – one half is not the mirror image of the other half

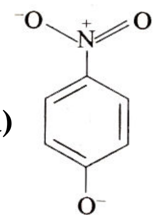
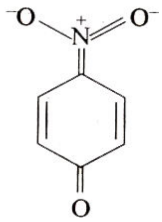
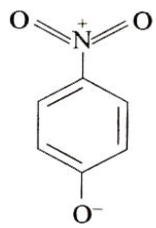
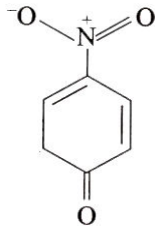
(c) It does not have centre of symmetry

(d) It does not have axis of symmetry

Ans: (a)

Sol: ---------------------

Q.36 The most unlikely representation of resonance structures of *p*-nitrophenoxide ion is

(a) 
(b) 
(c) 
(d) 

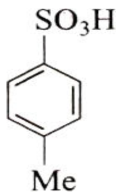
Ans: (c)
Sol: -----------

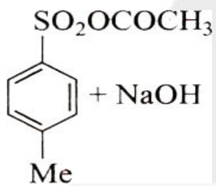
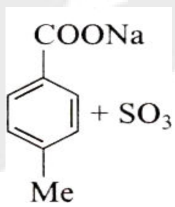
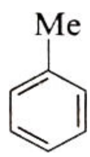
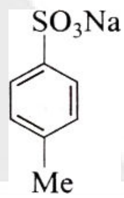
Q.37 Which of the following is an example of associated colloid ?
(a) Protein + water **(b) Soap + water**
(c) Rubber + benzene **(d) As_2O_3 + $Fe(OH)_3$**

Ans: (b)
Sol: Soap + water

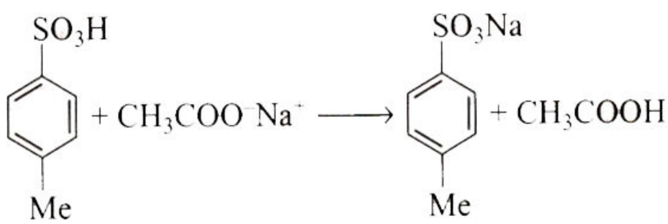
Q.38 Order of the bond strength of $C-H$ bonds involvings *sp*, sp^2, and sp^3 hybridized carbon atoms is :
(a) $sp > sp^2 > sp^3$ **(b) $sp^3 > sp^2 > sp$** **(c) $sp^2 > sp^3 > sp$** **(d) $sp^2 > sp > sp^3$**

Ans: (a)
Sol: ------------------------

Q.39 When  **reacts with $CH_3COO^-Na^+$ (excess), the product formed is :**

(a) 
(b) 
(c) 
(d) 

Ans: (d)
Sol: The reaction is simply an acid -base reaction :



Q.40 Which of the following will be obtained by keeping ether in contact with air for a long time ?
(a) $C_2H_5-O-CH(CH_3)-O-OH$ **(b) $C_2H_5-OCH_2-OH$**
(c) $C_2H_5-O-C_2H_5OH$ **(d) $CH_3-O\ CH\ (CH_3)-O-OH$**

Ans: (a)

Sol: $R-O-R' \xrightarrow{O_2/light} C_2H_5-O-CH(CH_3)-O-OH$

Q.41 Which is not true about acetophenone ?
(a) Reacts to form 2, 4-dinitorphenylhydrazine
(b) Reacts with Tollen's reagent to form silver mirror
(c) Reacts with I_2 / NaOH to form iodoform
(d) On oxidation with alkaline $KMnO_4$ followed by hydrolysis gives benzoic acid

Ans: (b)
Sol: Acetophenone is ketone and does not react with Tollen's reagent to give silver mirror.

Q.42 A primary amine forms an amide by the treatement of bromine and alkali. The primary amine has :
(a) 1 carbon atom less than amide
(b) 1 carbon atom more than amide
(c) 1 hydrogen atom less than amide
(d) 1 hydrogen atom more than amide

Ans: (a)

Sol: $\underset{\text{(Acetamide)}}{CH_3CONH_2} + Br_2 + 4KOH \xrightarrow[-2H_2O]{} \underset{\text{(Methyl amine)}}{CH_3NH_2} + 2KBr + 2K_2CO_3$

Q.43 Empirical formula of a compound is C_2H_5O and its molecular weight is 90. Molecular formula of the compound is:
(a) C_2H_5O **(b) $C_3H_6O_3$** **(c) $C_4H_{10}O_2$** **(d) $C_5H_{14}O$**

Ans: (c)

Sol: Empirical formula mass $= C_2H_5O = 24+5+16 = 45.$

$$n = \frac{\text{Molecular mass}}{\text{Empirical mass}} = \frac{90}{45} = 2$$

Molecular formula $= (C_2H_5O) = C_4H_{10}O_2.$

Q.44 Rayon is :
(a) natural silk **(b) artificial silk**
(c) natural plastic or rubber **(d) synthetic plastic**

Ans: (b)
Sol: Rayon is a man-made fiber which consists of purified cellulose in the form of long threads. Rayon resembles silk in appearance and henced is called as artificial silk.

$$\underset{\text{(from wood pulp)}}{\text{Cellulose}} \xrightarrow[CS_2]{NaOH} \underset{\text{(Syrup-like-liquid)}}{\text{Viscose}} \xrightarrow[\text{Spinneret into dil. } H_2SO_4]{\text{Pass through}} \underset{\text{(Fine silken thread)}}{\text{Rayon}}$$

Q.45 There is mixture of Cu (II) chloride and Fe(II) sulphate. The best way to separate the metal ions from this mixture in qualitative analysis is :
(a) hydrogen sulphide in acidic medium, where only Cu(II) sulphide will be precipitated
(b) ammonium hydroxide buffer, where only Fe(II) hyroxide will be precipitated
(c) hydrogen sulphide in acidic medium, where only Fe(II) sulphide will be precipitated
(d) ammonium hydroxide buffer, where only Cu(II) hydroxide will be precipiated

Ans: (a)
Sol: Cu^{2+}, a second group radical, gets precipitated first due to having lower solubility product $[CuS : K_{sp} = 1\times10^{-44}]$

Q.46 The molarity of Cl^- in an aqueous solution which was (*w/V*) 2% NaCl, 4% $CaCl_2$, and 6% NH_4Cl will be :

Sol: Moles of Cl^- in 100 ml of solution $= \frac{2}{58.5} + \frac{4}{111} \times 2 + \frac{6}{53.5} = 0.2184$

Molarity of Cl^- $= \frac{0.2184}{100} \times 1000 = 2.184$

Q.47 The number of electrons that are paired in oxygen molecule is :

Sol: Configuration of O_2 molecule is

$[\sigma(1s)^2\, \sigma*(1s)^2\, \sigma(2s)^2\, \sigma*(2s)^2\, \pi(2p_x)^2, \pi(2p_y)^2\, \sigma(2p_z)^2\, \pi*(2p_x)^1\, \pi*(2_y)^1]$

Number of pairs are 7. So the total number of paired electrons is 14.

Q.48 A_2B molecules (molar mass = 259.8 g/mol) crystallizes in a hyxagonal lattice as shown in the figure. The lattice constant were $a = 5\ \mathring{A}$ and $b = 8\mathring{A}$. If the density of crystal is 5 g/cm³, the how many molecules are contained in the given unit cell ? (Use $N_A = 6 \times 10^{23}$)

Sol: Volume of unit cell $= a^2 \sin 60^o \times b = 173.2 \times 10^{-24}\, cm^3 \quad (\because a = 5\ \mathring{A})$

Mass of unit cell $= 173.2 \times 10^{-24} \times 5 \times 6 \times 10^{23} = 51936\ g$

Number of molecules present in given unit cell $= \frac{519.6}{259.8} = 2$

Q.49 Oxidation states of vanadium in

$V \rightarrow V^{2+} + 2e$,

$V^{2+} \rightarrow V^{3+} + e$,

are 2 and 3 respectively. The oxidation states of vanadium in this following reaction

$V^{3+} + H_2O \rightarrow VO^{2+} + 2H^+ + e$?

Sol: VO^{2+}

x + 1(-2) = +2

x - 2 = 2

x = + 4.

Q.50 Antimony reacts with sulphur according to the equation

$2Sb(s) + 3S(s) \rightarrow Sb_2S_3(s)$

The molar mass of Sb_2S_3 is 340 g mol $^{-1}$

What is the percentage yield for a reaction in which 1.40 g of Sb_2S_3 is btained from 1.73 g of antimony and a slight excess of sulphur ?

Sol: Theoretically 244 g gives 340 g Sb_2S_3

$\Rightarrow$ 1.73 g of sb should give $\frac{340}{244} \times 1.73 = 2.41$ g Sb_2S_3

$$\%\ \text{yield} = \frac{1.40 \times 100}{2.41} = 58\ \%$$

Part - C - MATHEMATICS

Q.51 If the ratio of the sums to n terms of two AP's is $(5n + 3) : (3n + 4)$, then the ratio of their 17^{th} terms is

(a) 172 : 99 **(b) 168 : 103** **(c) 175 : 99** **(d) 171 : 103**

Ans: (b)

Sol: According to the given condition,

$$\frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]} = \frac{5n+3}{3n+4} \quad \Rightarrow \quad \frac{a+\frac{n-1}{2}d}{A+\frac{n-1}{2}D} = \frac{5n+3}{3n+4}$$

Now, put $\frac{n-1}{2} = 16$ or $n = 33$.

Q.52 If roots of the quadratic equation $ax^2 + bx + c = 0$ with real coefficients are complex, then imaginary part of the roots is

(a) $\frac{\pm\sqrt{b^2+4ac}}{2a}$ **(b) $\frac{\pm\sqrt{b^2-4ac}}{a}$** **(c) $\frac{\pm\sqrt{4ac-b^2}}{2a}$** **(d) none of these**

Ans: (c)

Sol: Roots are imaginary $\Rightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$= \frac{-b\pm\sqrt{4ac-b^2}\sqrt{-1}}{2a} = \frac{-b\pm\sqrt{4ac-b^2}\,i}{2a}$$

Hence the imaginary part is $\frac{\pm\sqrt{4ac-b^2}}{2a}$.

Q.53 If distance of z_1 from the origin is 4 and distance of z_1z_2 from the origin is 2 then distance of z_2 from the origion is

(a) 2 **(b) 6** **(c) 8** **(d) none of these**

Ans: (d)

Sol: Given $|z_1| = 4, |z_1z_2| = 2 \Rightarrow |z_1||z_2| = 2$

$\Rightarrow \quad |z_2| = 1/2$

Q.54 Let A and B be two matrices such that they commute then for any positive integer n.

(i) $AB^n = B^nA$ **(ii) $(AB)^n = A^nB^n$**

(a) Only (a) is correct **(b) both (a) and (b) are correct**

(c) only (b) is correct **(d) none of (a) and (b) are correct**

Ans: (b)

Sol: $AB^n = ABBBBB\ldots B$

$= (AB)BBB\ldots.B$

$= B(AB)BBB\ldots.B$

$= BB(AB)BB\ldots.B$

....

....

$= B^n A$

$(AB)^n = (AB)(AB)(AB) .... (AB)$

$= A(BA)(BA)(BA)...(BA)B$

$= A(AB)(AB)(AB)....(AB)B$

$= A^2(BA)(BA)(BA)....(BA)B^2$

$= A^2(AB)(AB)(AB)... (AB)B^2$

$= A^2(BA)(BA)(BA)...(BA)B^3$

$=$ ....

$=$ ....

$= A^n B^n$

Q.55 Let a, b, and c be the real numbers. Then the following system of equations in x. y, and z, $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$, $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, $-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ has

(a) no solution **(b) unique solution**

(c) infinitely many solution **(d) finitely many solutions.**

Ans: (b)

Sol: Let $\frac{x^2}{a^2}=X$, $\frac{y^2}{b^2}=Y$, $\frac{z^2}{c^2}=Z$

Then the given system of equation is

$X+Y-Z=1$ $X-Y+Z=1$ $-X+Y+Z=1$

Coefficient determinant

$$A=\begin{vmatrix}1 & 1 & -1\\ 1 & -1 & 1\\ -1 & 1 & 1\end{vmatrix} = 1(-1-1)-1(1+1)-1(1-1) = -4 \neq 0$$

Therefore, the given system of equation has unique solution.

Q.56 If the pair of lines $ax^2+2(a+b)xy+by^2=0$ lies along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sectors, then

(a) $3a^2-2ab+3b^2=0$ **(b) $3a^2-10ab+3b^2=0$**

(c) $3a^2+2ab+3b^2=0$ **(d) $3a^2+10ab+3b^2=0$**

Ans: (c)

Sol: Here, angle between the given lines $=\tan^{-1}\frac{2\sqrt{(a+b)^2-ab}}{a+b}$

According to the question,

$$\tan^{-1}\left|\frac{2\sqrt{(a+b)^2-ab}}{a+b}\right| = \frac{\pi}{8}$$

$\Rightarrow$ $3a^2+2ab+3b^2=0$

Q.57 Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), then which of the following is not the equation of L ?

(a) $y - x + 3 = 0$ (b) $y + 3x - 33 = 0$ (c) $y + x - 15 = 0$ (d) $y - 2x + 12 = 0$

Ans: (d)

Sol: $y^2 = 4x$

Equation of normal is $y = mx - 2m - m^3$ · It passes through (9, 6)

$\Rightarrow \quad m^3 - 7m + 6 = 0$

$\Rightarrow \quad m = 1, 2, -3$

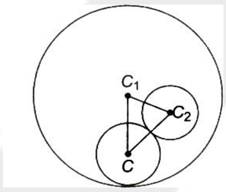
$\Rightarrow \quad y - x + 3 = 0,\ y + 3x - 33 = 0,\ y - 2x + 12 = 0$

Q.58 Two circles are given such that one is completely lying inside other without touching. Then locus of centre of variable circle which touches smaller circle from outside and bigger circle from inside is

(a) ellipse (b) hyperbola (c) parabola (d) circle

Ans: (a)

Sol: In the figure, circle with hard line are given circle with centres C_1 and C_2 and radius r_1 and r_2.



Let the circle with dotted line is variable circle which touches the given two circles as explained in the questions which has centre C and radius r.

Now, $CC_2 = r + r_2$ and $CC_1 = r_1 - r$.

Hence, $CC_1 + CC_2 = r_1 + r_2$ (= constant)

Then locus of C is ellipse whose foci are C_1 and C_2.

Q.59 The locus of the foot of the perpendicular from the centre of the hyperbola $xy = 1$ on a variable tangent is

(a) $(x^2 - y^2)^2 = 4xy$ (b) $(x^2 + y^2)^2 = 2xy$ (c) $(x^2 + y^2) = 4xy$ (d) $(x^2 + y^2)^2 = 4xy$

Ans: (d)

Sol: Slope of $OP = k/h$. Then equation of tangent to hyperbola at point P is

$$y - k = -\frac{h}{k}(x - h) \quad \text{or} \quad hx + ky = h^2 + k^2$$

Solving it with $xy = 1$, we have

$$hx + \frac{k}{x} = h^2 + k^2 \quad \text{or} \quad hx^2 - (h^2 + k^2)x + k = 0$$

This equation must have real and equal roots

$\Rightarrow \quad D = 0 \quad \Rightarrow \quad (h^2 + k^2)^2 - 4hk = 0 \quad \Rightarrow \quad (x^2 + y^2)^2 = 4xy$

Q.60 If R and R' are symmetric relations (not disjoint) on a set A, then the relation $R \cap R'$ is

(a) reflexive (b) symmetric (c) transitive (d) none of these

Ans: (b)

Sol: As $R \cap R'$ are not disjoint , there is a least one ordered pair, say (a, b) in $R \cap R'$

But $(a, b) \in R \cap R'$ $\Rightarrow$ $(a, b) \in R$ and $(a, b) \in R'$

As R and R' are symmetric relations, we get $(b, a) \in R$ and $(b, a) \in R'$ and consequently $(b, a) \in R \cap R'$

Q.61 Which of the following functions is an even functions ?

(a) $f(x) = \frac{a^x + a^{-x}}{a^x - a^{-x}}$ **(b)** $f(x) = \frac{a^x + 1}{a^x - 1}$

(c) $f(x) = x\frac{a^x - 1}{a^x + 1}$ **(d)** $f(x) = \log_2 (x + \sqrt{x^2 + 1})$

Ans: (c)

Sol: For $f(-x) = -x\,\frac{a^{-x} - 1}{a^{-x} + 1} = -x\,\frac{1 - a^x}{1 + a^x} = x \cdot \frac{a^x - 1}{a^x + 1} = f(x)$

Q.62 If $f(x) = \lim_{n \to \infty} n(x^{1/n} - 1)$. The for $x > 0$, $y > 0$, $f(x, y)$ is equal to

(a) $f(x)\, f(y)$ **(b)** $f(x) + f(y)$ **(c)** $f(x) - f(y)$ **(d) none of these**

Ans: (b)

Sol: $f(x) = \lim_{n \to \infty} n(x^{1/n} - 1) = \lim_{n \to \infty} \frac{x^{1/n} - 1}{1/n}$

$= \lim_{m \to 0} \frac{x^m - 1}{m}$ (Where $\frac{1}{n}$ replaced by m)

$= \log_e = x$

$\Rightarrow$ $f(x\,y) = \log\,(x\,y) = \log_e x + \log_e y = f(x) + f(y)$

Q.63 If $y = \tan^{-1}\left(\frac{3a^2x - x^3}{a(a^2 - 3x^2)}\right)$, then $\frac{dy}{dx} =$

(a) $\frac{3a^2}{a^2 + x^2}$ **(b)** $\frac{3a}{a^2 + x^2}$ **(c)** $\frac{a}{a^2 + x^2}$ **(d)** $\frac{3}{a^2 + x^2}$

Ans: (b)

Sol: $y = \tan^{-1}\left(\frac{3a^2x - x^3}{a(a^2 - 3x^2)}\right) = \tan^{-1}\left(\frac{3\frac{x}{a} - \left(\frac{x}{2a}\right)^3}{1 - 3\left(\frac{x}{a}\right)^2}\right) = 3\tan^{-1}\left(\frac{x}{a}\right)$

$\Rightarrow$ $\frac{dy}{dx} = 3 \cdot \frac{1}{1 + \frac{x^2}{a^2}} \cdot \frac{1}{a} = \frac{3a}{a^2 + x^2}$

Q.64 **Given $f(x)$ is a function such that $f(x)=\begin{bmatrix} x^{a}\sin\frac{1}{x} & \text{if } x>0 \\ 0 & \text{if } x=0\end{bmatrix}$ where α is a constant.**

$f(x)$ is a derivable $\forall\, x \geq 0$. then the least integral value of α is

(a) 1 **(b) 0** **(c) 2** **(d) none of these**

Ans: (c)

Sol: $f'(0^{+}) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0}\frac{h^{\alpha}\sin(1/h)}{h}$ $= \lim_{h\to 0} h^{\alpha-1}\cdot\sin\frac{1}{h}$

For this unit to exist $\alpha - 1 > 0$ $\Rightarrow \alpha > 1$

$\Rightarrow$ $\alpha = 2$

Q.65 **Given $g(x) = \frac{x+2}{x-1}$ and the line $3x + y - 10 = 0$, then the line is**

(a) tangent to $g(x)$ **(b) normal to $g(x)$**

(c) chord of $g(x)$ **(d) none of these**

Ans: (a)

Sol: $g(x) = \frac{x+2}{x-1}$ $\Rightarrow$ $g'(x) = \frac{-3}{(x-1)^2}$

Slope of the given line $=-3$ $\Rightarrow$ $\frac{-3}{(x-1)^2} = -3$ $\Rightarrow$ $x = 2$,

also $g(2) = 4$. The (2, 4) lies on given line alos. Hence given line is tangent to the curve.

Q.66 **$\int \frac{\sin x + \cos x}{\sin(x-\alpha)}dx$ is equal to**

(a) $(\cos\alpha - \sin\alpha)(x-\alpha) + (\cos\alpha + \sin\alpha)\log|\sin(x-\alpha)| + c$

(b) $(\cos\alpha + \sin\alpha)(x-\alpha) - (\cos\alpha - \sin\alpha)\log|\sin(x-\alpha)| + c$

(c) $(\cos\alpha + \sin\alpha)(x+\alpha) + (\cos\alpha - \sin\alpha)\log|\sin(x+\alpha)| + c$

(d) none of these

Ans: (a)

Sol: Put $x - \alpha = z \;\therefore\; x = z + \alpha$

$\therefore$ $dx = dz$ $\Rightarrow$ $dx = dz$

$= \int \frac{\sin(z+\alpha) + \cos(z+\alpha)}{\sin z}dz$ $\sin z\cos\alpha + \cos z\sin\alpha$

$= \int \frac{+\cos z\cos\alpha + \cos z\sin\alpha}{\sin z}dz$

$= (\cos\alpha - \sin\alpha)\int dz + (\sin\alpha + \cos\alpha)\int\frac{\cos z}{\sin z}dz$

$= (\cos\alpha - \sin\alpha)\,z + (\sin\alpha + \cos\alpha)\log|\sin z| + c$

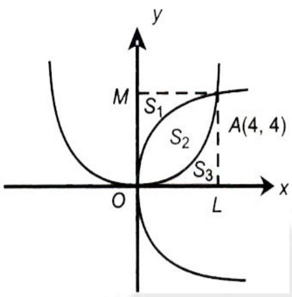
$= (\cos\alpha - \sin\alpha)(x-\alpha) + (\sin\alpha + \cos\alpha)\log|\sin(x-\alpha)| + c$

Q.67 The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4,\ y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the then $S_1 : S_2 : S_2$ is

(a) 1 : 2 : 3 **(b) 1 : 2 : 1** **(c) 1 : 1 : 1** **(d) 2 : 1 : 2**

Ans: (c)

Sol: $y^2 = 4x$ and $x^2 = 4y$ meet at $O(0, 0)$ and $\alpha(4, 4)$



Now $S_3 = \int_0^4 \frac{x^2}{4}\,dx = \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4 = \frac{1}{12}\left[64-0\right] = \frac{16}{3}$

$S_2 = \int_0^4 2\sqrt{x}\,dx - S_3$

$= 2\left[\frac{x^{3/2}}{3/2}\right]_0^4 - \frac{16}{3} = \frac{4}{3}\left[8-0\right] - \frac{16}{3} = \frac{16}{3}$ and $S_1 = 4\times 4 - (S_2 + S_3)$

$= 16 - \left(\frac{16}{3} + \frac{16}{3}\right) = \frac{16}{3}$

Hence, $S_1 : S_2 : S_3 = 1 : 1 : 1$.

Q.68 The minimum value of the expression $\sin\alpha + \sin\beta + \sin\gamma \cdot$ where α, β, γ are real number satisfying $\alpha + \beta + \gamma = \pi$ is

(a) positive **(b) zero** **(c) negative** **(d) -3**

Ans: (c)

Sol: For $\theta = -\pi/2, \beta = -\pi/2$ and $\gamma = 2\pi$

$\sin\alpha + \sin\beta + \sin\gamma = -2 \Rightarrow$ minimum value of the expression is negative.

Q.69 The range of values of p for which the equation $\sin\cos^{-1}(\cos(\tan^{-1} x)) = p$ has a solution is

(a) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ **(b)** $[0, 1)$ **(c)** $\left[\frac{1}{\sqrt{2}}, 1\right)$ **(d)** $(-1, 1)$

Ans: (b)

Sol: $\sin\cos^{-1}(\cos(\tan^{-1} x)) = p$

For $x \in R$ $\tan^{-1} x \in (-\pi/2, \pi/2)$

$\cos(\tan^{-1} x) \in (0, 1]$

$\cos^{-1}\cos(\tan^{-1}x) \in [0, \pi/2)$

$\sin(\cos^{-1}(\cos(\tan^{-1}x))) \in [0, 1)$

Q.70 For a regular polygon, Let r and R be the radii of the inscribed and the circumscribed circles. A false statements among the following is

(a) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

(b) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

(c) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(d) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

Ans: (d)

Sol: $r = \frac{a}{2}\cot\frac{\pi}{n}$ "a" is the side of polygon.

$R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n}$

$\frac{r}{R} = \frac{\cot\frac{\pi}{n}}{\operatorname{cosec}\frac{\pi}{n}} = \frac{\pi}{n}$ $\cos\frac{\pi}{n} \neq \frac{2}{3}$ for any $n \in N$

Q.71 Consider the frequency distribution of the given numbers

Value	**1**	**2**	**3**	**4**
Frequency	**5**	**4**	**6**	***f***

Sol: Since mean $= 3$

$\therefore \frac{1.5 + 2.4 + 3.6 + 4.f}{5 + 4 + 6 + f} = 3$

$5 + 8 + 18 + 4f = 3(15 + f)$ $31 + 4f = 45 + 3f \Rightarrow f = 14$

Q.72 Number of ordered pair(s), (a, b) for each of which the equality, a $(\cos x - 1) + b^2 = \cos(ax + b^2) - 1$ hold true for all $x \in R$ are________

Sol: putting $x = 0 \Rightarrow b^2 = \cos b^2 - 1 \Rightarrow \cos b^2 = 1 + b^2 \Rightarrow b = 0$

For $b = 0$, we have $a(\cos x - 1) = \cos ax - 1$

$\Rightarrow 2a\sin^2\frac{x}{2} = 2\sin^2\frac{ax}{2}$

$\Rightarrow a = 0$ or $a = 1$

Hence the ordered pairs are $(a, b) \equiv (0, 0)$ or $(1, 0)$.

Q.73 A closed vessel tapers to a point both at its top E and its bottom F and is fixed with EF vertical when the depth of the liquid in it is x cm, the volume of the liquid in it

$x^2(15-x)$ cu. cm. The length *EF* is ________

Sol: $\frac{dy}{dx} = 3x(10-x) = 0$

$\Rightarrow \quad x = 0;\ x = 10;\ \left.\frac{d^2v}{dx^2}\right]_{x=10} < 0 \qquad \Rightarrow v$ is maximum at $x = 10$

$\Rightarrow \quad EF = 10\,cm,$

Q.74 If the sum of 99 terms of AP is 198, then the value of the 50th term is _______

Sol: Given sum of 99 terms is 198

$\Rightarrow \frac{99}{2}\left[T_1 + T_{99}\right] = 198$

$\Rightarrow \frac{T1 + T99}{2} = 2$

Now T_1, T_{50}, T_{99} are in AP

$\Rightarrow \quad T_{50} = 2$

Q.75 Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how may ways can we place the balls so that no box reamins empty ?

Sol: Let the boxes be marked as A, B, C. We have to ensure that no box remains empty and in all five balls have to put in. There will be two possibilities.

(i) Any two containing one 3rd containing 3.

A (1) B (1) C (3)

${}^5C_1.{}^4C_1.{}^3C_3 = 5.4.1. = 20.$

Since the box containing 3 balls could be any of the three boxes A, B, C.

Hence the required number is $= 20 \times 3 = 60.$

(ii) Any two containing 2 each and 3rd containing 1.

A (2) B (2) C (1)

${}^5C_2.{}^3C_2.{}^1C_1 = 10 \times 3 \times 1 = 30$

Since the box containing 1 ball could be any of the three boxes A, B, C.

Hence the required number is $= 30 \times 3 = 90.$

Hence total number of ways are $= 60 + 90 = 150.$

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ROUGH WORK

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