A Division of Aggarwal Educare
Aggarwal CLASSES ™

JEE (MAIN)

TEST PAPER

SUBJECT : PHYSICS,CHEMISTRY, MATHEMATICS — ***TEST CODE : TSJMT215***

ANSWER PAPER

TIME : 3 HRS — ***MARKS : 300***

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark(-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or “un- darken” a darkened bubble.

Part A - PHYSICS

Q.1 A particle of mass m is moving in a circuular path of constant radius r such that is centripetal acceleration a_c is varying with time r as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the forces acting on it is

(a) $2\pi nk^2r^2t$ (b) mk^2t^2t (c) $\frac{mk^4r^2t^5}{3}$ (d) zero

Ans: (b)

Sol: $a_c = k^2rt^2 \Rightarrow \frac{v^2}{r} = k^2rt^2 \Rightarrow v^2 = k^2r^2t^2 \Rightarrow v = krt$

Tangential acceleration, $a_t = \frac{dv}{dt} = kr$

As centripetal foce does not work in circular motion, so power delivered by tangential force, $P = F_t v = ma_1 v = m(kr)krt = mk^2r^2t$

Q.2 A body of mass 3 kg moving with a speed of 4 m/s, collides head on with a stationary body of mass 2 kg. Their relative velocity of separation after the collision, is 2 m/s . Then
(a) The coefficient of restitution is 0.5
(b) The impulse of the collision is 7.2 N^{-s}
(c) The loss of kinetic energy due to collision is 3.6 J
(d) All of the above

Ans: (d)

Sol: $m_1 = 3\ kg,\ m_2 = 2kg,\ u_1 = 4, m/s,\ u_2 = 0$

Relative velocity of approach, $u_1 - u_2 = 4\,m/s$

Relative velocity of separation $v_2 - v_1 = 2\,m/s$

Give,

Coefficient of restitution $e = \frac{\text{Relative velocity of sepeartion}}{\text{Relative velocity of approach}} = \frac{2}{4} = \frac{1}{2} = 0.5$

Loss in kinetic energy $= \frac{1}{2}\frac{m_1m_2}{m_1+m_2}(1-e)^2(u_1-u_2)^2$

$$= \frac{1}{2}\frac{3\times 2}{3+2}\left[1-\left(\frac{1}{2}\right)^2\right](4)^2 = 7.2\ \text{J}$$

Final velocity of m1 mass.

$$v_1 = \left(\frac{m_1 - em_2}{m_1+m_2}\right)u_1 + \left[\frac{(1+e)m_2}{m_1+m_2}\right]u_2 = \frac{(3-0.5\times 2)}{3+2}\times 4 + 0 = \frac{8}{5}m/s$$

Implulse of collision = Change in momentum of mass m_1 (or m_2)

$$= mv_1 - m_1u_1 = 3\times\frac{8}{5} - 3\times 4 = \frac{24}{5} - 12 = 4.8 - 12 = -7.2\ \text{N-s}$$

Q.3 If g_E and g_M are the acceleration due to gravity on the Earth and Moon, respectively, and if Millikan oil drop experiment could be performed on the two surfaces, one

will find the ratio $\left(\frac{\text{electronic charge on moon}}{\text{electronic charge on earth}}\right)$ to be

(a) 0 **(b) g_E/g_M** **(c) g_M/g_E** **(d) 1**

Ans: (d)

Sol: Electronic charge $= 1.6 \times 10^{-19}$C is the fundamental property of the electron. Its value remains same on the Moon as well as on the Earth.

$$\therefore \quad \frac{\text{electronic charge on moon}}{\text{electronic charge on earth}} = 1$$

Q.4 In which one of the following cases will the liquid flow in a pipe be most streamlined ?

(a) Liquid of high viscosity and high density flowing through a pipe of small radius
(b) Liquid of high viscosity and low density flowing through a pipe of small radius.
(c) Liquid of low viscosity and low density flowing through a pipe of large radius.
(d) Liquid of low viscosity and high density flowing through a pipe of large radius.

Ans: (b)

Sol: For streamline flow, Reynold's number, $N_R \propto \frac{r\rho}{\eta}$ should be less.

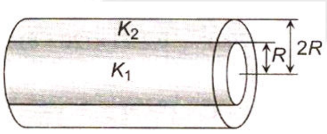
For less value of N_R. radius and density should be small and viscosity should be high.

Q.5 A cylinder of radius R made of material of thermal conductivityf K_1 is surrounded by a cylinderical shell of inner radius R and outer radius 2R made of material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindirical surface and the system is a steady state. The effective thermal conductivity of the system is

(a) $K_1 + K_2$ **(b) $\frac{K_1K_2}{K_1+K_2}$** **(c) $\frac{K_1+3K_2}{4}$** **(d) $\frac{3K_1+K_2}{4}$**

Ans: (c)

Sol: We can consider this arrangement as a parallel combination of two material having different thermal conductivites K_1 and K_2.



For parallel combination $K = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$

$A_1 =$ Area of cross section of intetal cylinder $= \pi R^2$

$Al_2 =$ Area of cross section of outer cylinder $= \pi(2R)^2 - \pi(R)^2 = 3\pi R^2$

$$\therefore \quad K = \frac{K_1 \cdot \pi R^2 + K_2 \cdot 3\pi R^2}{\pi R^2 + 3\pi R^2} = \frac{K_1 + 3K_2}{4}$$

Q.6 A Carnot engine takes 103 kcal of heat from a reservoir at 627°C and exhausts it to a sink a 27°C. The efficiency of the engine will be

(a) 22.2 % **(b) 33.3 %** **(c) 44.4 %** **(d) 66.6 %**

Ans: (d)

Sol: Efficiency of Carnot engine

$$= \frac{T_1 - T_2}{T_1} = \frac{90 - 300}{900} = \frac{6}{9} \quad \text{or} \quad 66.6\%$$

Q.7 Read the given statements and decide which is/are correct on the basis of kinetic energy theory of gases.
(I) Energy of one molecular at absolute temperature is zero.
(II) rms spreeds of different gases are same at same temeperature
(III) For 1 g of all ideal gases kinetic energy is same at same temperature.
(IV) For 1 mol of all ideal gases, kinetic energy is same at same temperature.
(a) All are correct **(b) I and IV are correct**
(c) IV is correct **(d) None of these**

Ans: (c)

Sol: If the gas is not ideal, then its molecule will possess potential energy. Hence, statements (I) is wrong. The rms speed of different gases at same temperature depends on its molecular weight $\left(v_{rmd} \propto \frac{1}{\sqrt{M}}\right)\cdot$ Hence, statement (II) is also wrong. Kinetic energy of 1 g gas depends on the molecular weight $\left(E_{gm} \propto \frac{1}{M}\right)\cdot$ Hence, statement (III) is also wrong. But KE of 1 mol of ideal gas does not depends on the molecular weight $\left(E = \frac{3}{2}RT\right)\cdot$Hence, (IV) is correct.

Q.8 The kinetic energy and potential energy of a particle executing SHM will be equal, when displacement is (amplitude $= a$)

(a) $a/2$ **(b)** $a\sqrt{2}$ **(c)** $\frac{a}{\sqrt{2}}$ **(d)** $\frac{a\sqrt{2}}{3}$

Ans: (c)

Sol: According to the proble, Kinetic energy $=$ Potential energy

$$\Rightarrow \quad \frac{1}{2}m\omega^2(a^2 - y^2) = \frac{1}{2}m\omega^2 y^2$$

$$\Rightarrow \quad a^2 - y^2 = y^2 \qquad \therefore \quad y = a\sqrt{2}$$

Q.9 When two sound waves with a phase difference of $\pi/2$ and each having amplitude A and frequency ω are superimposed on each other, then maximum amplitude and frequency of resultant wave is

(a) $\frac{A}{\sqrt{2}}; \frac{\omega}{2}$ **(b)** $\frac{A}{\sqrt{2}}; \omega$ **(c)** $\sqrt{2}A; \frac{\omega}{2}$ **(d)** $\sqrt{2}A; \omega$

Ans: (d)

Sol: Resultant amplitude

$$= \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\phi} \qquad = \sqrt{A^2 + A^2 + 2A^2\cos\frac{\pi}{2}} = \sqrt{2}A$$

Q.10 Two spherical conductors B and C having equal radii and carrying equal charges on them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that B but uncharged is brought in contact with B, then brought in contact with C and finally removed away from both.

The new force of repulsion between B and C is

(a) F/4 **(b) 3F/4** **(c) F/8** **(d) 3F/8**

Ans: (d)

Sol: When A (which is of some size as that of B) is brought in contact with B, the charge on both becomes q/2. When A, now containing a charge q/2, is brought in contact with C, charge on each becomes $\frac{(q/2)+q}{2}=\frac{3q}{4}$.

The new force of repulsion between B and C. $F'=\frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{2}\right)\left(\frac{3q}{4}\right)}{d^2}=\frac{3F}{8}$

Q.11 This question contains Statement -1 and Statement -2. Of the four choices given after the statement, choose the one that first desecribes the two statements.

Statements- 1 : For a chargd particle moving from pint P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Satatement 2 : The net work done by a conservative force on a object moving along a closed loop is zero.

(a) Statement -1 is true, Statement-2 is false

(b) Statement-1 is true, Statement-2 is true; Statement-2 is correct explanation of Statement-1

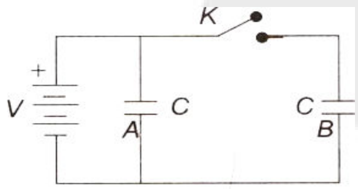
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.

(d) Statement-1 is false, Statement-2 is true.

Ans: (b)

Sol: Work done by conservation force does not depend on the path. Electrostatic force is a conservation force.

Q.12 As shown in Figure two identical capacitors are connected to a battery of V volts in parallel. When capacitors are fully charged, their stored energy is U_1. if the K is opened and material of dielectric constant $K = 3$ is inserted in each capacitor, their stored energy is now U_2. U_1/U_2 will be



(a) $\frac{3}{5}$ **(b)** $\frac{5}{3}$ **(c) 3** **(d)** $\frac{1}{3}$

Ans: (a)

Sol: Initially, potential difference across both the capacitor is same hence energy of the system is

$$U_1=\frac{1}{2}CV^2+\frac{1}{2}CV^2=CV^2$$

In the second case, when key K is opened and dielectric medium is filled between the plates, capacitance of both the capacitor becomes, 3C, while potential difference across A is V and potential differenc across B is V/3. Hence, energy of the system now is

$$U_2 = \frac{1}{2}(3C)V^2 + \frac{1}{2}(3C)\left(\frac{V}{3}\right)^2 = \frac{10}{6}CV^2 \quad \text{So, } \frac{U_1}{U_2} = \frac{3}{5}$$

Q.13 A charged particle moves through a magnetic field perpendicular to its direction. Then

(a) Both momentum and kinetic energy of the particle are not constant.

(b) Both momentum and kinetic energy of the particle are constant

(c) The kinetic energy changes but the moment is constant

(d) The momentum changes but kinetic energy is constant

Ans: (d)

Sol: $\frac{1}{2}mV^2 = KE$

$\because$ Speed does not change; the KE remains same. Due to change in direction of motion, momentum changes.

Q.14 The plane of dip circle is set in the geographic meridian and the apparent dip is θ_1. It is then set in a vertical plane perpendicular to the geographic meridian. Now, The apparent dip is θ_2. The angle of declination α at that place is

(a) $\tan\alpha = \sqrt{\tan\theta_1 \tan\theta_2}$ **(b) $\tan\alpha = \sqrt{(\tan\theta_1)^2 + (\tan\theta_2)^2}$**

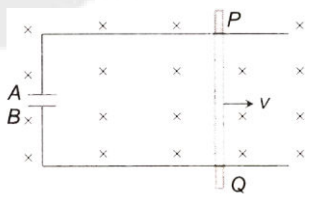
(c) $\tan\alpha = \frac{\tan\theta_1}{\tan\theta_2}$ **(d) $\tan\alpha = \frac{\tan\theta_2}{\tan\theta_1}$**

Ans: (c)

Sol: $\tan\theta_1 = \frac{\tan\delta}{\cos\alpha}$ or $\cos\alpha = \frac{\tan\delta}{\tan\theta_1}$

Again, $\tan\theta_2 = \frac{\tan\delta}{\sin\alpha}$ or $\sin\alpha = \frac{\tan\delta}{\tan\theta_2}$

Q.15 A conducting rod PQ of length $L = 1.0$ m is moving with a uniform speed $v = 2$ m/s in a uniform magentic field $B = 4.0$ T directed into the paper. A capacitor of capacity $C = 10\ \mu F$ is connected as shown in figure. Then



(a) $q_A = +80\ \mu C$ and $q_B = -80\mu C$

(b) $q_A = -80\ \mu C$ and $q_B = +80\mu C$

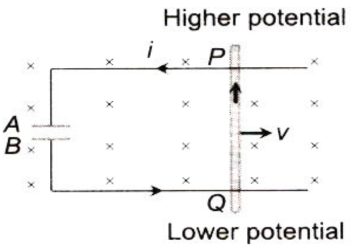
(c) $q_A = 0 = q_B$

(d) Charge stored in the capacitor increases exponentially with time

Ans: (a)

Sol: $Q = CV = C\ (Bvl) = 10 \times 10^{-6} \times 4 \times 2 \times 1 = 80\mu C$

According to Felming's right hand rul induced current flows form Q to P. Hence, P is the heigher potential and Q is at lower potential. Therfore, A is positively charged and B is negatively charged.



Q.16 Primary voltage is V_p, resistance of the primary winding is R_p. Turns in primary and secondary are respectively N_p and N_s. then secondary current in terms of primary voltage and secondary voltage, respectively, will be

(a) $\dfrac{V_pN_p}{R_pN_s}, \dfrac{V_sN_P^2}{R_pN_s^2}$ **(b)** $\dfrac{V_pN_p^2}{R_pN_s}, \dfrac{V_s^2N_P^2}{R_pN_s^2}$ **(c)** $\dfrac{V_pN_p}{R_p^2N_s}, \dfrac{V_sN^2}{R_p^2N_s^2}$ **(d)** $\dfrac{V_pN_p^2}{R_pN_s^2}, \dfrac{V_s^2N_p}{R_p^2N_s}$

Ans: (a)

Sol: $\dfrac{i_s}{i_p} = \dfrac{N_p}{N_s}$

Now, according to the information given in the problem, i_p can be calculated by using the formula.

$$V = iR \Rightarrow i_s = \frac{V_p}{R_p} \times \frac{N_p}{N_s}$$

(This is the secondary current in terms of V_p) Now, to reaarange the result obtained above, in terms of secondary voltage, we must replace the term of V_p in the above result by V_s. We know that

$$\frac{V_p}{V_s} = \frac{N_p}{N_s};\ V_p = \frac{V_sN_p}{N_s}.$$

Substituting this in (i) $i_s = \dfrac{V_sN_p^2}{R_pN_s^2}$.

Q.17 Light from the constellation Vigro is observed to increase in wavelength by 0.4 %. With respect to Earth the constellation is

(a) Moving away with velocity 1.2×10^6 m/s

(b) Coming closer with velocity 1.2×10^6 m/s

(c) Moving away with velocity 4×10^6 m/s

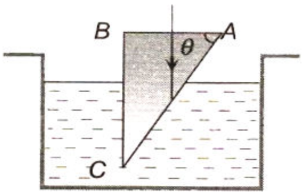
(d) Coming closer with velocity 4×10^6 m/s

Ans: (a)

Sol: By using $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c}$; Where $\dfrac{\Delta\lambda}{\lambda} = \dfrac{0.4}{100}$ and $c = 3 \times 10^8$ m/s

$$\Rightarrow \quad \frac{0.4}{100} = \frac{v}{3 \times 10^8} \quad \Rightarrow \quad v = 1.2 \times 10^6 \text{ m/s}$$

Q.18 A glass prism of refractive index 1.5 is immersed in water $(\mu = 4/3)$ figure. A light beam incident normally on the face *AB* is totally reflected to reach the face *BC* if

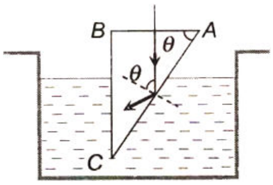


(a) $\sin\theta > 8.9$ **(b)** $2/3 < \sin\theta < 8/9$

(c) $\sin\theta \le 2/3$ **(d)** $\cos\theta \ge 8/9$

Ans: (a)

Sol: From figure; It is clear that



Total internal reflection takes place at AC, only if $\theta > C$

$\Rightarrow \quad \sin\theta > \sin C \quad \Rightarrow \sin\theta > \frac{1}{{}_{\omega}\mu_g} \quad \Rightarrow \sin\theta > \frac{1}{9/8} \quad \Rightarrow \quad \sin\theta > \frac{8}{9}$

Q.19 An electron is acceleration through a potential difference of V volt. It has a wavelength λ associated with it. Through what potential difference and electron must be accelerated so that its de-Broglie wavelength is the same as that of a proton ? Take mass of proton to be 1837 times larger than the mass of electron.

(a) V volt **(b) 1837 V volt** **(c) $\frac{V}{1837}$ volt** **(d) $\sqrt{1837}$ V volt**

Ans: (c)

Sol: $\lambda = \frac{h}{\sqrt{2mqV}}$ $\quad mV =$ constant

$1837\ V' = 1\ V'$ or $V' = \frac{V}{1837}V$

Q.20 The electric potential between a proton and an electron is given by $V = V_0$ r, where r is a constant. Assuming Bohr's model to be applicable, write variationof r, with n, n being the principle quantum number

(a) $r_n \propto n$ **(b)** $r_n \propto 1/n$ **(c)** $r_n \propto n^2$ **(d)** $r_n \propto 1/n^2$

Ans: (a)

Sol: $|F| = \left|\frac{-du}{dr}\right| = \frac{mv^2}{r} \quad \Rightarrow \quad v = \sqrt{\frac{v_0 r}{m}}$

$mv_n r_n = \frac{nh}{2\pi} \qquad r_n \propto n$

Q.21 An LC current contains inductance $L = 1\mu H$ and capacitance $C = 0.01\mu H$. The wavelength of electromagnetic wave generated is nearly?

Sol: $v = \frac{1}{2\pi\sqrt{LC}} \quad \lambda = \frac{c}{v}$

$= c \cdot 2\pi \sqrt{LC}$ $= 3 \times 10^8 \times 2\pi \sqrt{1 \times 10^{-6} \times 0.01 \times 10^{-6}}$

$= 3 \times 10^8 \times 2 \times 3.14 \times 10^{-7}$

$= 188.4$ m

Q.22 A wire of length L and three identical cells of negligible internal resistances are connected in series. Due to this current, the temperature of the wire is raised by ΔT in time *t*. A number *N* of similar cells is now connected in series with a wire of the same material and cross-section but of length 2L. The temeperature of wire is reaised by same amount ΔT in the same time *t*. The value of N is______?

Sol: Heat $= mS\ \Delta T = t^2 Rt$

Case I : Length (L) $\Rightarrow$ Resistance $= R$ and mass $= m$

Case II : Length (2L) $\Rightarrow$ Resistance $=$ 2R and mass $= 2m$

So, $\frac{m_1 S_1 \Delta T_1}{m_2 S_2 \Delta T_2} = \frac{i_1^2 R_1 t_1}{i_2^2 R_2 t_2}$ $\Rightarrow$ $\frac{mS\Delta T}{2mS\Delta T} = \frac{i_1^2 Rt}{i_2^2 Rt}$

$\Rightarrow$ $i_1 = i_2 \Rightarrow \frac{(3E)^2}{12} = \frac{(NE)^2}{2R} \Rightarrow N = 6$

Q.23 Three equal resistors connected in series across a source of emf together dissipate 10 W. If the same resistors are connected in parallel across the same emf, then the power dissipiated will be?

Sol: In series consumed power, $P_s = P/n$, While in parallel consumed power, $P_p = np$. $\Rightarrow$

$P_p = n^2 \cdot P_s$ $\Rightarrow$ $P_p = (3)^2 \times 10 = 90W$

Q.24 A-2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes an uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10.000 N/m. The spring compresses by (in cm)

Sol: Let spring is compressed by x. Loss in KE of block $=$ Gain in PE of spring + Work done against friction

$\Rightarrow$ $\frac{1}{2} \times 2 \times 4^2 = \frac{1}{2} \times 1000\ x^2 + 15x$ Solve to get $x = 5.5$

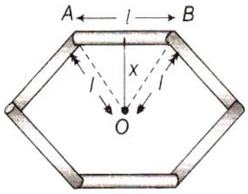
Q.25 The moment of inertia of rod of length *l* about an axis passing through its centre of mass and perpendicular to rod is I. The moment of inertia of hexagonal shape formed by six such rods about an axis passing through its centre of mass and perpendicular to its plane will be ?

Sol: Moment of inertia or rod AB about its centre and perpendicular to the length $= \frac{ml^2}{12} = l$ $\Rightarrow$

$ml^2 = 12l$

Now moment of inertia of the rod about the axis which is passing through O and perpendicular to the plane of hexangon

$I_{rod} = \frac{ml^2}{12} + nx^2$ [From the theorem of parallel axes]



$$= \frac{ml^2}{12} + m\left(\frac{\sqrt{3}}{2}l\right)^2 = \frac{5ml^2}{6}$$

Now the moment of inertia of system.

$$I_{system} = 6 \times I_{rod} = 6 \times \frac{5ml^2}{6}$$

$$= 6ml^2 = 5(12l) = 160I \qquad [\text{As } ml^2 = 12I]$$

Part - B - CHEMISTRY

Q.26 The mass of Mg_3N_2 produced if 48 g of Mg metal is reacted with 34 g NH_3 gas is :

$Mg + NH_3 \rightarrow Mg_3N_2 + H_2$

(a) $\frac{200}{3}$ **(b) $\frac{100}{3}$** **(c) $\frac{400}{3}$** **(d) $\frac{150}{3}$**

Ans: (a)

Sol: $Mg + NH_3 \rightarrow Mg_3N_2 + H_2$

Mole $\frac{48}{24} = 2$; $\frac{34}{17} = 2$ L. R. Mg

Mass of Mg_3N_2 $= \frac{1}{3} \times 2 \times (3 \times 24 + 28) = \frac{200}{3}$

Q.27 Assuming pure 2*s* and 2*p* orbitals of carbon are used in forming, CH_4, molecule, which of the following statements is false ?

(a) Three $C-H$ bonds will be at right angle

(b) Once $C-H$ bond will be weaker than other three $C-H$ bonds

(c) The shape of molecule wll be tetrahedral

(d) The angle $C-H$ bond formed by $\overline{s-s}$ overlapping will be uncertain with respect to other three bonds.

Ans: (c)

Sol: C : $2s^2$ $2p^2$

	$2s^2$	$2p^2$		
C (in ground state)	↑↓	↑	↑	
C(in excited state)	↑	↑	↑	↑
	2s	2*p*		

In the unhybridized state of carbon, 2p orbitals are 90^o to one another and each one will overlap with 1s orbital of three hydrogen atoms, thus three $C-H$ bonds are formed which are 90^o to one another. For the fourth hydrogen atom, its 1s orbital may overlap with, the non-directional 2*s* orbital of the carbon. This σ-bond will be stronger than σ $C-H$ bonds formed by 2-1*s* overlap. In such situation, CH4 moleucle can never have tetrahedral geometry.

Q.28 **In the reaction :** $CS_{2(l)} + 3O_{2(g)} \rightarrow CO_{2(g)} + 2SO_{2(g)}$; $\Delta H = -265$ **kcal**
The enthalpis of formation of CO_2 and SO_2 are both negative and are in the ratio 4 : 3. The enthalpy of formation of CS_2 is +26 kcal/mol. Calculate the entahlpy of formation of SO_2.
(a) -90 kcal/ mol **(b) -52 kcal/mol** **(c) -78 kcal/mol** **(d) -71.7 kcal/mol**

Ans: (d)

Sol: $CS_{2(l)} + 3O_{2(g)} \rightarrow CO_{2(g)} + 2SO_{2(g)}$; $\Delta H = -265\ kcal$

Let $\Delta H_f(CO_2, g) = -4x$ $\Delta H_f(SO_2, g) = -3x$

$$\Delta H_{reaction} = \Delta H_f(CO_2, g) + 2\Delta H_f(SO_2, g) - \Delta H_f(CS_2, 1)$$

$-265 = -4x - 6x - 26$ $x = +23.9$

$\therefore$ $\Delta H_f(SO_2, g) = 3x = -71.7\ kcal/mol$

Q.29 **Ferrous oxide has a cubic structure, and the edge length of the unit cell is 5.0 A. Assumingn the density of ferrous oxide to be 3.84 g/cm³, the number of Fe^{2+} and O^{2-} ions present in each unit cell will be : (use $N_A = 6 \times 10^{23}$)**
(a) 4 Fe^{2+} and 4 Q^{2-} **(b) 2 Fe^{2+} and 2 O^{2-}** **(c) 1 Fe^{2+} and O^{2-}** **(d) 3 Fe^{2+} and 4 O^{2-}**

Ans: (a)

Sol: Mass of unit cell $= d \times V = (5 \times 10^{-8})^3 \times 3.84 \times 6 \times 10^{23}$

$= 288$ a.m.u.

Mass of formula unit of FeO $= 56 + 16 = 72$ a.m.u.

$\therefore$ Number of formula unit $= \frac{288}{72} = 4$

Q.30 **The EMF of given cell** $Pt, \underset{(P_1)}{H_2} | H^+ || H^+ | \underset{(P_2)}{H_2}, Pt$ **is :**

(a) $\frac{RF}{F} \log \frac{P_1}{P_2}$ **(b)** $\frac{RF}{2F} \log \frac{P_1}{P_2}$

(c) $\frac{RF}{F} \log \frac{P_2}{P_1}$ **(d) None of these**

Ans: (b)

Sol: $\underset{P_1}{H_2} \longrightarrow 2H^+ + 2e$ $2H^+ + 2e \longrightarrow \underset{P_2}{H_2}$

$$E_{Cell} = E^o_{OP_H} + E^o_{RP_H} + \frac{RT}{2F} \log_e \frac{[P_{H_2}]_1}{[P_{H_2}]_2} = \frac{RT}{2F} \log_e \frac{P_1}{P_2}$$

Q.31 **Inversion of cane sugar proceeds with half life of 500 min at pH $= 5$ for any concentration of sugar. However, if pH $= 6$, the half life changes to 50 min. The rate law expression for sugar inversion can be written as :**

(a) $r = K[sugar]^2 [H^+]^0$ **(b)** $r = K[sugar]^1 [H^+]^0$

(c) $r = K[sugar]^1 [H^+]^1$ **(d)** $r = K[sugar]^0 [H^+]^0$

Ans: (c)

Sol: $pH = 5$ $pH = 6$

$[H^+] = 10^{-5}$ $[H^+] = 10^{1-n}$

$$\frac{(t_{1/2})}{(t_{1/2})} = \left(\frac{10^{-5}}{10^{-6}}\right)^{1-n} = \frac{1}{10} = (10)^{1-n}$$

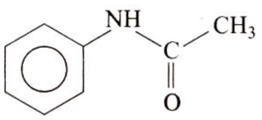
$$10^{-1} = (10)^{1-n} \quad \Rightarrow \quad 1-n = -1 \Rightarrow n = 2$$

Q.32 Point of false statement :
(a) Brownian movement and Tyndall effect are shown by colloidal systems
(b) Gold number is a measure of the protective power of a lyophilic colloidal
(c) The colloidal solution of a liquid in liquid is called gel
(d) Hardy-Schulze rule is related with coagulation

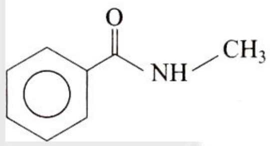
Ans: (c)

Sol: The colloidal solution of liquid in liquid is called emulsion, and not gel.

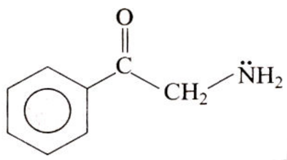
Q.33 The correct basic strength order is :



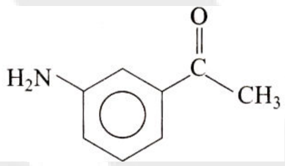
I



II



III



IV

(a) I > II > IV > III **(b) IV > III > II > I**
(c) III > II > IV > I **(d) III > IV > II > I**

Ans: (d)

Sol: The basicity order will be inversely proportinal to resonance stability of lone pair.

Q.34 Two isomers can be metamers if they have :
(a) equal distribution of alkyl group on either side of the functional group
(b) unequal distribution of alkyl group on either side of the functional group
(c) different functional group
(d) different positions of an atom or group on the rings of same side

Ans: (b)

Sol: unequal distribution of alkyl group on either side of the functional group

Q.35 Rank the following organometallic compounds in the increasing order of nuclephilicity
(A) H_3CMgBr **(B) $(CH_3)_2Cd$** **(C) CH_3Na** **(D) CH_3Li**
(a) B < C > D < A **(b) B < A < D < C** **(c) C < D < A < B** **(d) D < A < B < C**

Ans: (b)

Sol: -------------------

Q.36 The most suitable method for the separation of a 1 : 1 mixture of ortho and *para-nitrophenols* is :
(a) distillation **(b) sublimation**
(c) crystallization **(d) chromatography**

Ans: (a)

Sol: *o*- and *p*- nitrophenols are separated by steam distillation. *o*-Nitrophenol is steam volatile, while *p*-isomer is not.

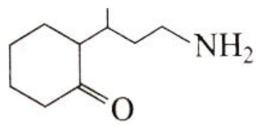
Q.37 A substance $C_6H_{12}O$ does not react with Fehling's solution but gives positive reactions for a carbonyl group. It also gives positive iodoform reaction. Which of the following structure will best correspond to the above statements ?

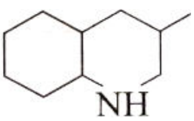
(a) $CH_3CH_2CH(CH_3)COCH_3$
(b) $CH_3CH_2COCH_2CH_2CH_3$
(c) $CH_3CH_2COCH(CH_3)_2$
(d) $CH_3CH_2CH(CH_3)CH_2CHO$

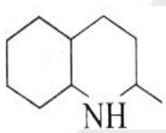
Ans: (a)

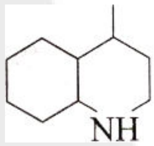
Sol: Ketones having CH_3CO- group give iodoform test.

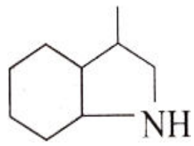
Q.38 Reductive amination of A form :

(A) : 

(a) 

(b) 

(c) 

(d) 

Ans: (c)

Sol: ------------------

Q.39 A mixture containing primary, secondary, and tertiary amines is treated with diethyl oxalate. Choose the correct statement.

(a) The distilate of the mixture after treatment mainly contains 1° amine
(b) 3° amine do not react with diethyl glyoxalate
(c) This is Hinsberg method of separating 1°, 2°, and 3° amines
(d) 3° amine is removed by filtration

Ans: (b)

Sol: ------------------

Q.40 An organic compound contains C = 36%, H = 6%, and rest oxygen. Its empirical formula is :

(a) CH_2O **(b) $C_2H_3O_3$** **(c) CH_2O_2** **(d) $C_2H_2O_2$**

Ans: (a)

Sol:

Element	%	Number of moles	Simple ratio
C	36	$36/12 = 3$	$3/3 = 1$
H	6	$6/1 = 6$	$6/3 = 2$
O	58	$58/16 = 3.62$	$3.62/3 = 1$

Therefore, emperical formula $= CH_2O$

Q.41 Sucrose molecule is made up of :

(a) a glucopyranose and fructopyranose
(b) a glucopyranose and fructofuranose
(c) a glucofuranose and a fructopyranose
(d) a glucofuranose and a fructofuranose

Ans: (b)

Sol: Surcrose is composed of α-D- glucopyranose unit and a β-D- fructofuranose unit.

These units are joined by α- glycosidic linkage between C-1 of the glucose unit and C-2 of the fructose unit.

Q.42 High-density polyethylene (HDPE) can be prepared from ethylene by :
(a) Ziegler- Natta process **(b) Heating with peroxides**
(c) Condensing in sealed tubes **(d) Condensing with styrenes**

Ans: (a)

Sol: HDPE is prepared by coordination polymerization which occurs through the intermediate formation of coordination complexes. For example, ethylene first forms a coordination complex with the transition metal titanium by donating its π- electrons. The π complex thus formed then reacts stepwise with a large number of ethylene molelcules, ultimately leading to the formation of a polymer. The polythene so obtained has high density (0.97 g/cm^3) and higher melting point (403 K) as compare to LDPE (density : 0.92 g/ cm^3) and melting point : 384 K.

Q.43 (i) $A + Na_2CO_3 \rightarrow B + C$, **(ii) $A \xrightarrow{CO_2}$ (Milky) C**
The chemical formulas of A and B are respectively :
(a) NaOH and $Ca(OH)_2$ **(b) $Ca(OH)_2$ and NaOH**
(c) NaOH and CaO **(d) CaO and $Ca(OH)_2$**

Ans: (b)

Sol: $\underset{(A)}{Ca(OH)_2} + Na_2CO_3 \longrightarrow \underset{(B)}{2NaOH} + \underset{(C)}{CaCO_3}$ $\quad \underset{(A)}{Ca(OH)_2} \xrightarrow{CO_2} \underset{C(milky)}{CaCO_3}$

Q.44 Na_2CO_3 can be manufactured by Solvey's process but K_2CO_3 cannot be prepared because :
(a) K_2CO_3 is more soluble
(b) K_2CO_3 is less soluble
(c) $KHCO_3$ is more soluble than $NaHCO_3$
(d) $KHCO_3$ is less soluble than $NaHCO_3$

Ans: (c)

Sol: -----------------

Q.45 In nitroprusside ion, the iron and NO exist, respectively as Fe^{II} and NO^+ rather than Fe^{III} and NO. These forms can be differentiated by :
(a) estimating the concentration of iron
(b) measuring the concentration of CN^-
(c) measuring the solid state magnetic moment
(d) thermally decomposing the compound

Ans: (c)

Sol: The existance of Fe^{2+} and NO^+ in nitroprusside ion $[Fe(CN)_5NO]^{2-}$ can be established by measuring the magnetic moment of the solid compound which should correspond to $(Fe^{2+} = 3d^6)$ four unpaired electrons.

Q.46 Vapor pressure of a solution of 5 g of non-electrolyte in 100 g of water at a particular temperature is 2985 N/m^2. The vapor pressure of pure water is 3000 N/m^2. The molecular weight of the solute is :

Sol: $$\frac{P^o - P_s}{P^o} = \frac{(W_2 / M_2)}{(W_1 / M_1)} = \frac{3000-2985}{3000} = \frac{(5 / M_2)}{(100/18)} \quad \text{or} \quad M_2 = 180$$

Q.47 ow many grams of CaC_2O_4 (molecular weight $=128$) on dissolving in distilled water will give a saturated $[K_{sp}(CaC_2O_4) = 2.5 \times 10^{-9} mol^2 / L^2]$

Sol: Solubility of $CaC_2O_4 = \sqrt{K_{sp}} = \sqrt{2.5 \times 10^{-9}} = 5 \times 10^{-5} mol/L$

$$= 5 \times 10^{-5} \times 128 = 640 \times 10^{-5} = 0.0064\ g$$

Q.48 What is likely to be principal quantum number for a circular orbit of diameter 20 nm of the hydrogen atom if we assume Bohr orbit to be the same as that represented by the principlal quantum number ?

Sol: Radius $= 0.529 \frac{n^2}{z} \overset{0}{A} = 10 \times 10^{-9}\ m$

So, $n^2 = 189$ or $n = 14$

Q.49 A sample of O_2 gas is collected over water at 23°C at a baromatric pressure of 751 mm Hg (vapor pressure of water at 23°C is 21 mm Hg.). The partial pressure of O\ gas in the sample collector is :

(a) 21 mm Hg **(b) 751 mm Hg** **(c) 0.96 atm** **(d) 1.02 atm**

Ans: (c)

Sol: Pressure of O_2 (dry) $= 751 - 21 = 730$ mm Hg $= \frac{730}{760} = 0.96$ atm

Q.50 A metal electrode has a reduction potential of 0.136 V when measured against a standard calomel electrode (E^0 calomel (oxid^n) = -0.244 V). The potential of metal electrode against SHE is_____?

Sol: $M^{n+} + ne^- \longrightarrow M\ E^0 = X$

$2Hg + 2Cl^- \longrightarrow Hg_2Cl_2 + 2e^-, E^0 = -0.244\,V$

$E^0_{M^{n+}|M} + E^0_{Cal.} = 0.136$ V $E^0_{M^{n+}|M} + (-0.244\ V) = 0.136$

$E^0_{M^{n+}|M} = 0.380$ V $E^0_{SHE} = 0.000\,V$

$E^0_{cell\ (against SHE)} = E^0_{SHE} + E^0_{M^{n+}|M} = 0.000 + 0.380\ V = 0.380\ V$

Part - C - MATHEMATICS

Q.51 If the 4th term in the expansion of $\left(ax + \frac{1}{x}\right)$ is $\frac{5}{2}$, then the values of *a* and *n* are :

(a) $\frac{1}{2}$, 6 **(b) 1, 3** **(c) $\frac{1}{2}$, 3** **(d) cannot be found**

Ans: (a)

Sol: It is given that the fourth term in the expansion of $\left(ax + \frac{1}{x}\right)^n$ is $\frac{5}{2}$, therefore

$${}^nC_3\,(ax)^{n-3}\left(\frac{1}{x}\right)^3 = \frac{5}{2} \qquad \Rightarrow \qquad {}^nC_3, a^{n-3}\ x^{n-6} = \frac{5}{2}$$

We must have x^0 for which $n - 6 = 0$. Hence, $n = 6$ and we get ${}^6C_3\ a^3 = \frac{5}{2}$.

$$\Rightarrow \qquad a^3 = \frac{1}{8} \Rightarrow a = \frac{1}{2}$$

Q.52 If a, b, c are consecutive positive integers and log $(1+ac)=2K$ then the value of K is

(a) log b **(b) log a** **(c) 2** **(d) 1**

Ans: (a)

Sol: Use $a=x-1, b=x, c=x+1$

Q.53 If a, b, c are consecutive positive integers and log $(1+ac)=2K$ then the value of K is

(a) log b **(b) log a** **(c) 2** **(d) 1**

Ans: (a)

Sol: Use $a=x-1, b=x, c=x+1$

Q.54 A light ray coming along the line $3x+4y=5$ gets reflected from the line $ax+by=1$ and goes along the line $5x+12y=10$ then

(a) $a=\frac{64}{115}, b=\frac{112}{15}$ **(b)** $a=\frac{14}{15}, b=-\frac{8}{115}$

(c) $a=\frac{64}{115}, b=-\frac{8}{115}$ **(d)** $a=\frac{64}{15}, b=\frac{14}{15}$

Ans: (c)

Sol: $ax+by=1$ will be one of the bissection of the given lines. Equation of bisectors of the given lines are $\frac{3x+4y-5}{5}=\pm\left(\frac{5x-12y-10}{13}\right)$

$\Rightarrow \quad 64x-8y=115 \quad 14x+112y=15$

$\Rightarrow \quad a=\frac{64}{115}, b=-\frac{8}{115}$ or $a=\frac{14}{15}, b=\frac{112}{15}$

Q.55 Let AB be a chord of the circle $x^2+y^2 = r^2$ subtending a right angle at the centre. Thus the locus of the centroid of the triangle PAB as P moves on the circle is

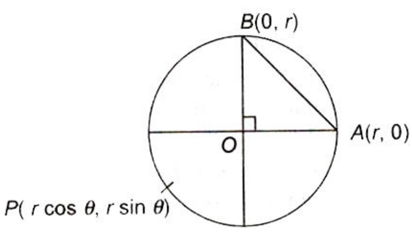
(a) a parabola **(b) a circle**

(c) an ellips **(d) a pair of the straight line**

Ans: (b)

Sol: Let (h, k) be the centroid of ΔPAB. Then

$$h=\frac{r+0+r\cos\theta}{3} \quad k=\frac{0+r+r\cdot\sin\theta}{3}$$



$\Rightarrow \quad 3h=r(1+\cos\theta), 3k=r(1+\sin\theta)$

$\Rightarrow \quad \left(\frac{3h}{r}-1\right)^2+\left(\frac{3h}{r}-1\right)^2=\cos^2\theta+\sin^2\theta=1$

Therefore, locus of (h, k) is $\left(\frac{3x}{r}-1\right)^2+\left(\frac{3y}{r}-1\right)^2=1$

$\Rightarrow \quad (3x-r)^2+(3y-r)^2=r^2$

$\Rightarrow \quad \left(x-\frac{r}{3}\right)^2+\left(y-\frac{r}{3}\right)^2=r^2/9$ Which is a circle.

Q.56 Let A and B be two distinct points on the parabola $y^2=4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be

(a) $-\frac{1}{r}$ (b) $\frac{1}{r}$ (c) $\frac{2}{r}$ (d) none of these

Ans: (c)

Sol: $A=(t_1^2, 2t_1)$; $B=(t_2^2, 2t_2)$

$$\text{center}=\left[\frac{t_1^2+t_2^2}{2}, (t_1+t_2)\right] t_1+t_2=\pm r$$

$$m=\frac{2(t_1-t_2)}{t_1^2+t_2^2}=\frac{2}{t_1+t_2}=\pm\frac{2}{r}$$

Q.57 The equation of the ellipse (referred to its axes as the axes of x and y respectively) Whose foci are $(\pm 2, 0)$ and eccentricity 1/2, is

(a) $\frac{x^2}{12}+\frac{y^2}{16}=1$ (b) $\frac{x^2}{16}+\frac{y^2}{12}=1$ (c) $\frac{x^2}{16}+\frac{y^2}{8}=1$ (d) none of these

Ans: (b)

Sol: Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\cdot$

Given, $e=\frac{1}{2}$

Also foci of ellips are $(\pm ae, 0)=(\pm 2, 0) \Rightarrow ae=2 \Rightarrow a=4$

Now, $b^2=a^2(1-e^2) \Rightarrow b^2=12$

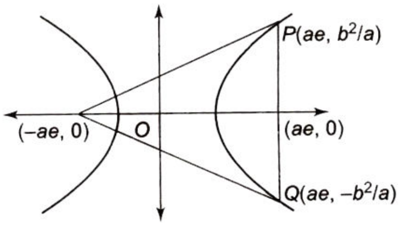
Thus, the required ellipse is $\frac{x^2}{16}+\frac{y^2}{12}=1.$

Q.58 If the latus rectum of a hyperbola through one focus subtends 60° **angle at the other focus, then its ecentricity is**

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) $\sqrt{6}$

Ans: (b)

Sol:



$$\tan 30^o = \frac{b^2 la}{2ac}$$

$\Rightarrow \quad \frac{2}{\sqrt{3}} e = e^2 - 1 \qquad \Rightarrow \sqrt{3}e^2 - 2e - \sqrt{3} = 0$

$\Rightarrow \quad e = \frac{2 \pm \sqrt{4+12}}{2\sqrt{3}} = \frac{2 \pm 4}{2\sqrt{3}} \qquad \Rightarrow e = \frac{3}{\sqrt{3}} = \sqrt{3}$

Q.59 Let S be a non-empty subset of R. Consider the following statements :
P : There is a rational number $x \in S$ such that $x > 0$ which of the following statements is the negation of the statement P ?

(a) $x \in S$ and $x \le 0 \Rightarrow x$ is not rational

(b) There is a rational number $x \in S$ such that $x \le 0$

(c) There is no rational number $x \in S$ such that $x \le 0$.

(d) Every rational number $x \in S$ satisfies $x \le 0$.

Ans: (d)

Sol: P : There is rational number $x \in S$ such that $x > 0$

P : Every rational number $x \in S$ satisfies $x \le 0$.

Q.60 The function f : R → R defined by $f(x) = x(x-1)(x-2)(x-3)$ is

(a) one -one but not onto **(b) onto but not one-one**
(c) both one -one and onto **(d) neither one-one nor onto**

Ans: (b)

Sol: We have $f(x) = (x-1)(x-2)(x-3)$ and $f(1) = f(2) = f(3) = 0 \Rightarrow f(x)$

For each $y \in R$, there exists $x \in R$ such that $f(x) = y$.

Therefore, f is onto. Hence $f : R \to R$ is onto by not one-one.

Q.61 $\lim_{x \to \frac{\pi}{2}} \frac{\sin x}{\cos^{-1}\left[\frac{1}{4}(3\sin x - \sin 3x)\right]}$**, Where [ ] denotes the greatest integer function, is**

(a) $\frac{2}{\pi}$ **(b) 1** **(c)** $\frac{4}{\pi}$ **(d) does not exist**

Ans: (c)

Sol: Let $\frac{\pi}{2^x} = p \Rightarrow 2^{x-1} = \frac{\pi}{2p}$ as $x \to \infty, \ p \to 0$

$$l = \lim_{p \to 0} \frac{\pi}{2p}(\sin p + \tan p) = \frac{\pi}{2} \lim_{p \to 0}\left(\frac{\sin p}{p} + \frac{\tan p}{p}\right) = \pi$$

Q.62 **If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is**

(a) $\frac{1+x}{1+\log x}$ **(b)** $\frac{1-\log x}{1+\log x}$ **(c) not defined** **(d)** $\frac{\log x}{(1+\log x)^2}$

Ans: (d)

Sol: We have $x^y = \log e^{x-y}$

$\Rightarrow \quad \log x^y = \log e^{x-y} \qquad \Rightarrow y \log x = (x-y)$

$\Rightarrow \quad y \log x = x - y \qquad \Rightarrow y = \frac{x}{1+\log x}$

$$\therefore \quad \frac{dy}{dx} = \frac{(1+\log x)\cdot 1 - x\cdot\left(0+\frac{1}{x}\right)}{(1+\log x)^2}$$

$$= \frac{1+\log x - 1}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$$

Q.63 **Let** $f(x) = \begin{cases} (x-1)\sin\left(\frac{1}{x-1}\right), & \text{if } x \neq 1 \\ 0, & \text{if } x = 1. \end{cases}$ **Then which one of the following is true ?**

(a) *f* is neither differentiable at $x = 0$ and at $x = 1$
(b) *f* is differentiable at $x = 0$ and at $x = 1$
(c) *f* is differentiable at $x = 0$ but not at $x = 1$
(d) *f* is differentiable at $x = 1$ but not at $x = 0$

Ans: (a)

Sol: $f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} \qquad \Rightarrow \qquad f'(1)$

$$= \lim_{h\to 0}\frac{(1+h-1)\sin\left(\frac{1}{1+h-1}\right) - 0}{h}$$

$$= \lim_{h\to 0}\frac{h}{h}\sin\left(\frac{1}{h}\right)$$

$$\Rightarrow \quad f'(1) = \lim_{h\to 0}\sin\left(\frac{1}{h}\right)$$

Therefore, f is not differentiable at $x = 1$

Similarly, $f'(0) = \lim_{h\to 0}\frac{f(h)-f(0)}{h}$

$$\Rightarrow \quad f'(0) = \lim_{h\to 0}\frac{(h-1)\sin\left(\frac{1}{h-1}\right) - \sin(1)}{h}$$

Q.64 **If the curve $y = ax^2 - 6x + b$, passes through (0, 2) and has its tangent parallel to the x-axis at $x = 3/2$, then the values of a and b are respectively**

(a) 2 and 2 **(b) -2 and -2** **(c) -2 and 2** **(d) 2 and -2**

Ans: (a)

Sol: $y = ax^2 - 6x + b$ passes through (0, 2)

Again $\frac{dy}{dx} = 2ax - 6$

At $x = \frac{3}{2}, \frac{dy}{dx} = 3a - 6$

Since tangent is parallel to the x-axis, therefore, $dy/dx = 0 \Rightarrow 3a - 6 = 0 \Rightarrow a = 2.$

Q.65 **Let $f(x) = \frac{a}{x} + x^2 \cdot$ If it has a maximum at $x = -3$ then a is**

(a) -1 **(b) -16** **(c) 1** **(d) none of these**

Ans: (d)

Sol: $f'(x) = -\frac{1}{x^2} + 2x$ For $f'(x) = 0, x^3 = \frac{a}{2}$

For $x = -3, a = -54$

Now, $f'(x) = \frac{2a}{x^3} + 2 \Rightarrow f''(-3) = \frac{-108}{(-3)^3} + 2 > 0$

Hence $f(x)$ cannot have maxima at $x = -3$.

Q.66 $\int \frac{dx}{\cos x - \sin x}$ **is equal to**

(a) $\frac{1}{\sqrt{2}} \log \left|\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)\right| + c$ **(b)** $\frac{1}{\sqrt{2}} \log \left|\cot\left(\frac{x}{2}\right)\right| + c$

(c) $\frac{1}{\sqrt{2}} \log \left|\tan\left(\frac{x}{2} - \frac{3\pi}{8}\right)\right| + c$ **(d)** $\frac{1}{\sqrt{2}} \log \left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + c$

Ans: (a)

Sol: $I = \int \frac{d}{\cos x - \sin x}$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\cos x \cdot \frac{1}{\sqrt{2}} - \sin x \frac{1}{\sqrt{2}}} \qquad = \frac{1}{\sqrt{2}} \int \frac{dx}{\sin\left(\frac{\pi}{4} - x\right)}$$

$$= \frac{1}{\sqrt{2}} \int \text{cosec}\left(\frac{\pi}{4} - x\right) dx \qquad = \frac{1}{\sqrt{2}} \log \left|\tan\left(\frac{\pi}{8} - \frac{\pi}{2}\right)\right| + c$$

$$= \frac{1}{\sqrt{2}} \log \left|\tan\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right| + c$$

Q.67 **Let $f : (-1, 2) \to [0, \infty]$ be a continuous function such that $f(x) = f(1 - x)$ for all**

$x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} xf(x)\,dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x-axis. Then,

(a) $R_1 = 2R_2$ (b) $R_1 = 3R_2$ (c) $2R_1 = R_2$ (d) $3R_1 = R_2$

Ans: (c)

Sol: $R_1 = \int_{-1}^{2} xf(x)\,dx$

$$= \int_{-1}^{2} (2-1-x)(2-1-x)\,dx \qquad = \int_{-1}^{2} (1-x)f(1-x)\,dx$$

$$= \int_{-1}^{2} (1-x)f(x)\,dx$$

Hence, $2R_1 = \int_{-1}^{2} f(x)\,dx = R_2$.

Q.68 An object falling from rest in the air is subject not only to the gravitational force but also to the air resistance. Assume that the air resistance is proportional to the velocity with constant of proportionality as k > 0, and acts in a directions opposite to motion ($g = 9.8\ m/\sec^2$). Then velocity cannot exceed

(a) 9.8/k m/sec (b) 98/k m/sec (c) $\frac{k}{9.8}$ m/sec (d) none of these

Ans: (a)

Sol: Let V(t) be the velocity of the object at time t.

Given $\frac{dV}{dt} = 9.8 - kV \quad \Rightarrow \quad \frac{dV}{9.8 - kV} = dt$

Integrating, we get $\log(9.8 - kV) = -kt + C$

$\Rightarrow 9.8 - kV = \text{Constant } e^{-kt}$.

But $V(0) = 0$. Constant $= 9.8$ Thus, $9.8 - kV = 9.8$

$e^{-kt} \Rightarrow kV = 9.8(1 - e^{-et}) \qquad \Rightarrow V(t) = \frac{9.8}{k}(1 - e^{-kt}) < \frac{9.8}{k}$

For all t. Hence V(t) cannot exceed $\frac{9.8}{k}$.

Q.69 If $\sin(x + 20^o) = 2\sin x \cos 40^o$ where $x \in \left(0, \frac{\pi}{2}\right)$ then which of the following dose not hold good

(a) $\cot\frac{x}{2} = (2 + \sqrt{3})$ (b) $\operatorname{cosec} 4x = 2\sqrt{3}$

(c) $\sec\frac{x}{2} = \sqrt{6} - \sqrt{2}$ (d) $\tan 4x = \sqrt{3}$

Ans: (d)

Sol: $\sin x \cos 20^o + \cos x \sin 20^o = 2\sin x \cos 40^o$

$\sin 20^o \cos x = \sin x\,(2\cos 40^o - \cos 20^o)$

$$\tan x = \frac{\sin 20^o}{2\cos 40^o - \cos 20^o} \qquad = \frac{\sin 20^o}{\cos 40^o + \cos 40^o - \cos 20^o}$$

$$= \frac{\sin 20^o}{\cos 40^o + 2\sin 30^o \sin(-10^0)} \qquad = \frac{\sin 20^o}{\sin 50^o - \sin 10^o}$$

$$= \frac{\sin 20^o}{2\cos 30^o \sin 20^o}$$

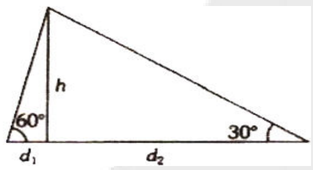
$$\tan x = \frac{1}{\sqrt{3}} \qquad \Rightarrow \qquad x = 30^o$$

Q.70 The base of cliff is circular. From the extremities of a diameter of the base that angles of elevation of the top of the cliff are 30° and 60°. If the height of the cliff is 500 m. then the diameter of the base of the cliff is

(a) $1000/\sqrt{3}\ m$ (b) $2000/\sqrt{3}\ m$ (c) $1000\sqrt{2}\ m$ (d) $2000\sqrt{2}\ m$

Ans: (a)

Sol: $d_2 = h \cot 30^o = 500\sqrt{3},\ d_1 = \frac{500}{\sqrt{3}}$



Diameter $D = 500\sqrt{3} + \frac{500}{3}\sqrt{3} = \frac{2000}{\sqrt{3}}\ m$

Q.71 Consider the frequency distribution of the given numbers

Value	1	2	3	4
Frequency	**5**	**4**	**6**	***f***

If the mean is known to be 3, then the value of *f* is

Sol: Since mean = 3

$$\therefore \quad \frac{1.5 + 2.4 + 3.6 + 4.f}{5 + 4 + 6 + f} = 3$$

$$5 + 8\ 18 + 4f = 3(15 + f)$$

$$31 + 4f = 45 + 3f \Rightarrow f = 14$$

Q.72 Two persons A and B get together once a week to play a game. They always play 4 games. From past experience Mr. A wins 2 of the 4 games just as often as he win 3 of the 4 games. If Mr. A does not alway win or always lose, then the probability that Mr. A wins any one game is (given the probability of A's wining a game is a non-zero constant less than one).

Sol: ${}^4C_2\, p^2\, (1-p)^2 = {}^4C_3\, p^3 (1-p) \qquad 6(1-p) = 4p$

$$\Rightarrow \qquad p = \frac{3}{5} = 0.6$$

Q.73 The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is ______?

Sol: The letter of word COCHIN in alphabetic order are C, C, H, I, N, O. Fixing first letter C and keeping C at second place, rest four can be arranged in 4! ways.

Q.74 $\lim_{x\to 0} \dfrac{x^n - \left[\sin (x)^n\right]}{x-(\sin x)^n}$ **is non-zero finite, then *n* must be equal to ?**

Sol: For n = 0, we have $\lim_{x\to 0} \dfrac{1-\sin 1}{x-1} = \sin 1 - 1$

$$\text{For } n = 1, \lim_{x\to 0} \frac{x-\sin x}{x-\sin x} = 1$$

$$\text{For } n = 2, \lim_{x\to 0} \frac{x^2-\sin^2 x}{x-\sin^2 x} = \lim_{x\to 0} \frac{1-\dfrac{\sin^2 x}{x^2}}{\dfrac{1}{2}-\dfrac{\sin^2 x}{x^2}} = 0$$

For n = 3 also the given limit is 0. Hence n = 0 or 1.

Q.75 The number of real solutions of the equation $|x|^2 - 3|x| + 2 = 0$ is _____?

Sol: Since, $|x|^2 - 3|x| + 2 = 0$

$\Rightarrow$ $(|x|-1)\,(|x|-2) = 0$

$\Rightarrow$ $|x| = 1, 2$

$\therefore$ $x = 1, -1, 2, -2$

Hence, four real solutions exist.

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ROUGH WORK