TM

A Division of Aggarwal Educare

Aggarwal CLASSES

JEE (MAIN)

TEST PAPER

SUBJECT : PHYSICS,CHEMISTRY, MATHEMATICS *TEST CODE : TSJMT214*

ANSWER PAPER

TIME : 3 HRS *MARKS : 300*

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark(-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or “un- darken” a darkened bubble.

Part A - PHYSICS

Q.1 **A projectile is given an initial velocity of $(i+2j)$m/s where i is along the ground and j is along the vertical. If $g = 10\text{m/s}^2$ then the equation of its trajectory is**

(a) $y = x - 5x^2$ **(b)** $y = 2x - 5x^2$ **(c)** $4y = 2x - 5x^2$ **(d)** $4y = 2x - 25x^2$

Ans: (b)

Sol: Initial velocity $= (i+2j)$m/s

Magnitude of initial velocity, $u = \sqrt{(1)^2+(2)^2} = \sqrt{5}$m/s

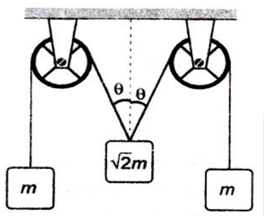
$$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta) \quad \left[\tan\theta = \frac{y}{x} = \frac{2}{1} = 2\right]$$

$$\therefore \quad y = x\times 2 - \frac{10(x)^2}{2(\sqrt{5})^2}[1+(2)^2]$$

$$= 2x - \frac{10(x^2)}{2\times 5}(1+4)$$

$$= 2x - 5x^2$$

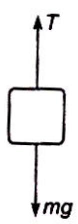
Q.2 **The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be**



(a) 0° **(b)** 30° **(c)** 45° **(d)** 60°

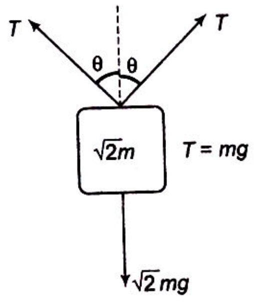
Ans: (c)

Sol: Free body diagram of m is



$T = mg$

Free body diagram of mass $\sqrt{2}m$ is



$$2T\cos\theta = \sqrt{2}mg$$

Dividing Eq. (ii) by Eq. (i), we get

$$\cos\theta = \frac{1}{\sqrt{2}}$$

or $\theta = 45^{\circ}$

Q.3 Which of the following sets have different dimensions?
(a) Pressure, Young's modulus, Stress
(b) Emf, Potential difference, Electric potential
(c) Heat, Work done, Energy
(d) Dipole moment, Electric flux, Elecric field

Ans: (d)

Sol: Dipole moment $=$ (charge) $\times$ (distance)

Electric flux $=$ (electric field) $\times$ (area)

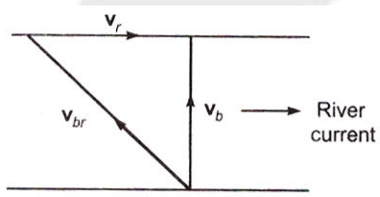
Hence, the correct option is (d).

Q.4 A boat which has a speed of 5 km/h in still water crosses a river of width 1 km along the shortest possible path in 15 min. The velocity of the river water in km/h is

(a) 1 **(b) 3** **(c) 4** **(d)** $\sqrt{41}$

Ans: (b)

Sol: Shortest possible path comes when absolute velocity boatman comes perpendicular to river current as shown figure.

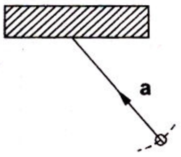


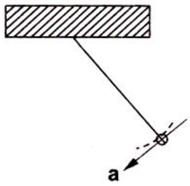
$$t = \frac{\omega}{v_b} = \frac{\omega}{\sqrt{v_{br}^2 - v_r^2}}$$

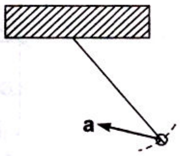
$$\therefore \quad \frac{1}{4} = \frac{1}{\sqrt{25 - v_r^2}}$$

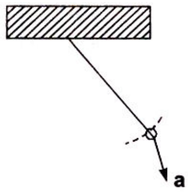
Solving the equation, we get $v_r = 3\,\text{km/h}$

Q.5 A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector a is correctly shown in

(a)


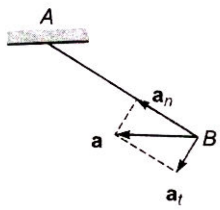
(b)


(c)


(d)


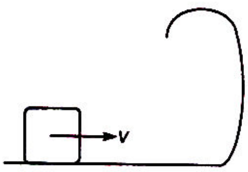
Ans: (c)

Sol: Net acceleration a of the bob in position B has two components.



(a) a_n = radial acceleration (towards BA)
(b) a_t = tangential acceleration (perpendicular to BA)
Therefore, direction of a is correctly shown in option (c).

Q.6 A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in

(a)


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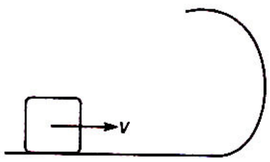
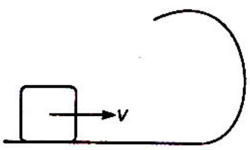
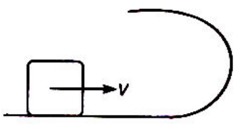
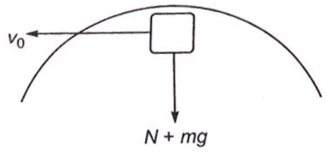


(c)


(d)


Ans: (a)

Sol: Since, the block rises to the same heights in all the fall four cases, from conservation of energy, speed of the block at higher point will be same in all four cases. Say it is v_0.
Equation of motion will be



$$N + mg = \frac{mv_0^2}{R}$$

or $$N = \frac{mv_0^2}{R} - mg$$

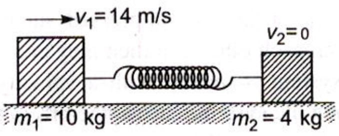
R (the radius of curvature) in first case is minimum.
Therefore, normal reaction N will be maximum in first case.

Q.7 Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is

(a) 30 m/s (b) 20 m/s (c) 10 m/s (d) 5 m/s

Ans: (c)

Sol:



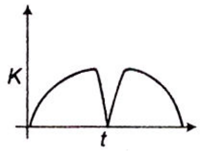
$$v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

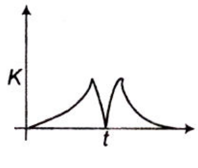
$$= \frac{10 \times 14 + 4 \times 0}{10 + 4}$$

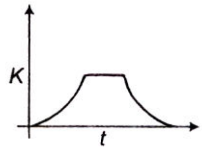
$$= \frac{140}{14}$$

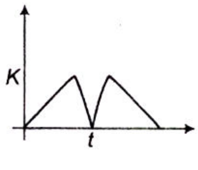
$$= 10 \text{m/s}$$

Q.8 A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches described the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale.

(a)


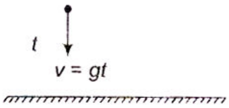
(b)


(c)


(d)


Ans: (b)

Sol: $t = 0$ •



$$K = \frac{1}{2} mg^2 t^2$$

$K \propto t^2$ Rherefore, K-1 graph is parabola.

During collision, retarding force is just like spring force $(F \propto x)$, therefore kinetic energy first decreases to elastic potential energy and then increaes.

Q.9 **A quantity X is given by $\varepsilon_0 \, L \frac{\Delta V}{\Delta t}$, where is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of**

(a) resistance **(b) charge** **(c) voltage** **(d) current**

Ans: (d)

Sol: $C = \frac{\Delta q}{\Delta V}$ or $\varepsilon_0 \frac{A}{L} = \frac{\Delta q}{\Delta V}$ or $\varepsilon_0 = \frac{(\Delta q)L}{A.(\Delta V)}$

$$X = \varepsilon_0 L \frac{\Delta V}{\Delta t} = \frac{(\Delta q)L}{A.(\Delta V)} L \frac{\Delta V}{\Delta t}$$

but $[\mathrm{A}] = [\mathrm{L}^2]$

$\therefore$ $\mathrm{X} = \frac{\Delta q}{\Delta t} = \text{current}$

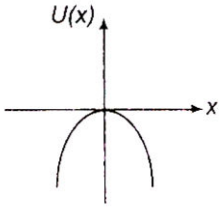
Q.10 **A cylinder rolls up an inclined plane, reaches some height and then rolls down (without slipping throught these motions). The directions of the frictional force acting on the cylinder are**

(a) up the incline while ascending and down the incline while descending
(b) up the incline while ascending as well as descending
(c) down the incline while ascending and up the incline while descending
(d) down the incline while ascending as well as descending

Ans: (b)

Sol: mg sin θ componet is always down the plane whether it is rolling up or rolling down. Thereffore, for no slipping, sense of angular acceleration should a;so be same in both the cases. Therefore, force of friction f always act upwards.

Q.11 **A particle is placed at the origin and a force $F = kx$ is acting on it (where, k is a positive constant). $U(0) = 0$, If the graph of U (x) versus x will be (where, U is the potential energy function).**

(a)


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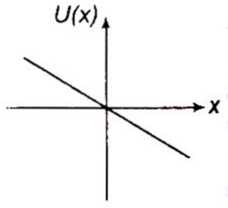
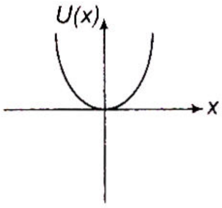
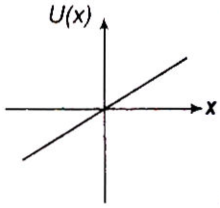


(c) 

(d) 

Ans: (a)

Sol: From $F=-\frac{dU}{dx}$

$$\int_0^{U(x)} dU = -\int_0^x F dx = -\int_0^x (kx)dx$$

$$\therefore \quad U(x) = -\frac{kx^2}{2} \quad \text{as } U(0)=0$$

Q.12 The moment of inertia of a uniform cylinder of length *l* and radius R about its perpendicular bisector is *I*. What is the ratio *l/R* such that the moment of inertia is minimum?

(a) $\frac{\sqrt{3}}{2}$ **(b)** 1 **(c)** $\frac{3}{\sqrt{2}}$ **(d)** $\sqrt{\frac{3}{2}}$

Ans: (d)

Sol: MI of a solid cylinder about its perpendicular bisector of length is

$$I = M\left(\frac{l^2}{12}+\frac{R^2}{4}\right)$$

$$\Rightarrow I = \frac{mR^2}{4}+\frac{ml^2}{12} = \frac{m^2}{4\pi\rho l}+\frac{ml^2}{12} \qquad [\because \rho\pi r^2 l = m]$$

For I to be maximum

$$\frac{dI}{dl} = -\frac{m^2}{4\pi\rho}\left(\frac{1}{l^2}\right)+\frac{ml}{6}=0 \Rightarrow \frac{m^2}{4\mu\pi\rho}=\frac{m^3}{6}$$

$$\Rightarrow \quad l^3 = \frac{3m}{2\pi\rho} \Rightarrow l = \left(\frac{3}{2}\right)^{1/3}\left(\frac{m}{\pi\rho}\right)^{1/3}$$

$$\rho = \frac{m}{\pi R^2 l} \Rightarrow R^2 = \frac{m}{\pi\rho l}$$

$$\Rightarrow R^2 = \frac{m}{\pi\rho}\left(\frac{2}{3}\right)^{1/3} = \left(\frac{m}{\pi\rho}\right)^{1/3}\left(\frac{2}{3}\right)^{1/3}$$

$$\Rightarrow \qquad R = \left(\frac{m}{\pi\rho}\right)^{1/3}\left(\frac{2}{3}\right)^{1/3}$$

$$\frac{l}{R}=\frac{\left(\frac{3}{2}\right)^{1/3}\left(\frac{m}{\pi\rho}\right)^{1/3}}{\left(\frac{m}{\pi\rho}\right)^{1/3}\left(\frac{2}{3}\right)^{1/3}}=\left(\frac{3}{2}\right)^{1/3}+\left(\frac{3}{2}\right)^{1/6}$$

$\therefore$ $$\frac{l}{R}=\sqrt{\frac{3}{2}}$$

Q.13 A small block slides without friction down an inclined plane starting from rest. Let s_n be the distance travelled from $t=n-1$ to $t=n$. Then, $\frac{s_n}{s_{n+1}}$ is

(a) $\frac{2n-1}{2n}$ **(b)** $\frac{2n+1}{2n-1}$ **(c)** $\frac{2n-1}{2n-1}$ **(d)** $\frac{2n}{2n+1}$

Ans: (c)

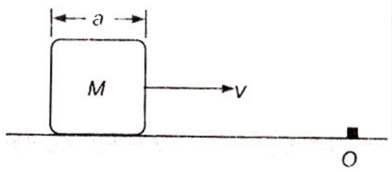
Sol: Distance travelled in t th second is, $s_t=u+at-\frac{1}{2}a$

Given, $u=0$

$\therefore$ $$\frac{s_n}{s_{n+1}}=\frac{an-\frac{1}{2}a}{a(n+1)-\frac{1}{2}a}=\frac{2n-1}{2n+1}$$

$\therefore$ Correct option is (c).

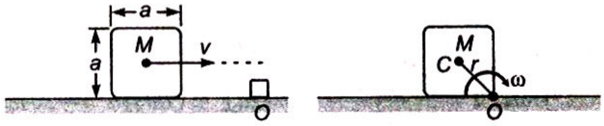
Q.14 A cubical block of side *a* moving with velocity v on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits O is



(a) $3v/4a$ **(b)** $3v/2a$ **(c)** $\sqrt{3}\sqrt{2}\,a$ **(d)** zero

Ans: (a)

Sol: $r=\sqrt{2}\frac{a}{2}$ or $r^2=\frac{a^2}{2}$



Net torque about O is zero.

Therefore, angular momentum (L) about O will be conserved, or $L_i=L_f$

$$Mv\left(\frac{a}{2}\right) = I_o\omega = \mathrm{M}(I_{CM} + Mr^2)\omega$$

$$= \left\{\left(\frac{Ma^2}{6}\right) + M\left(\frac{a^2}{2}\right)\right\}\omega$$

$$= \frac{2}{3}Ma^2\omega$$

$$\omega = \frac{3v}{4a}$$

Q.15 A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2/T_1 is

(a) 1 **(b) $\sqrt{2}$** **(c) 4** **(d) 2**

Ans: (d)

Sol: $T \propto \dfrac{1}{\sqrt{g}}$

$i.e., \dfrac{T_2}{R_1} = \sqrt{\dfrac{g_1}{g_2}}$

where, g_1 = acceleartion due to gravity on earth's surface $= g$

g_2 = acceleartion due to gravity at a height $h =$ R from earth's surface $= g/4$

$$\left[\text{Using } g(h) = \frac{g}{\left(1 + \dfrac{h}{R}\right)^2}\right]$$

$$\Rightarrow \quad \frac{T_2}{R_1} = \sqrt{\frac{g_1}{g/4}} = 2$$

Q.16 Assume tha a drop of liquid evaporates by decreases in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is S and L is its latent heat of vaporisation.

(a) $\dfrac{\rho L}{T}$ **(b) $\sqrt{\dfrac{T}{\rho L}}$** **(c) $\dfrac{T}{\rho L}$** **(d) $\dfrac{2T}{\rho L}$**

Ans: (d)

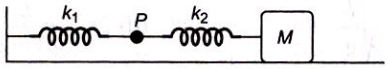
Sol: Decrease in surface energy = heat required in vaporisation

$\therefore \quad T(dS) = L(dm)$

$\therefore \quad T(2)(4\pi r)dr = L(4\pi r^2 dr)\rho$

$\therefore \qquad r = \dfrac{2T}{\rho L}$

Q.17 The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is



(a) $\dfrac{k_1 A}{k_2}$ **(b)** $\dfrac{k_2 A}{k_1}$ **(c)** $\dfrac{k_1 A}{k_1 + k_2}$ **(d)** $\dfrac{k_2 A}{k_1 + k_2}$

Ans: (d)

Sol: $x_1 + x_2 = A$ and $k_1 x_1 = k_2 x_2$ or $\dfrac{x_1}{x_2} = \dfrac{k_2}{k_1}$

Solving these equations, we get $x_1 = \left(\dfrac{k_2 A}{k_1 + k_2}\right) A$

Q.18 $Y(x,t) = \dfrac{0.8}{\left[(4x+5t)^2 + 5\right]}$ **represents a moving pulse where x and y are in metre and t is in second. Then,**

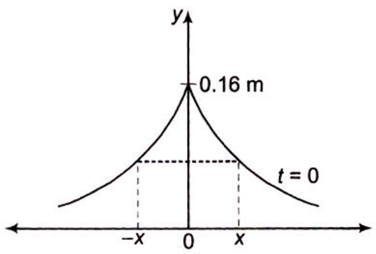
(a) pulse is moving in positive x-direction.
(b) in 2 s it will travel a distance of 2.5 m
(c) its maximum displacement is 0.16 m
(d) it is a symmertric pulse

Ans: (b,d)

Sol: The shape of pulse at would be as shown, in Fig. (a).

$$y(0,0) = \frac{0.8}{5} = 0.16m$$

From the figure it is clear that $y_{max} = 0.16m$



(a)

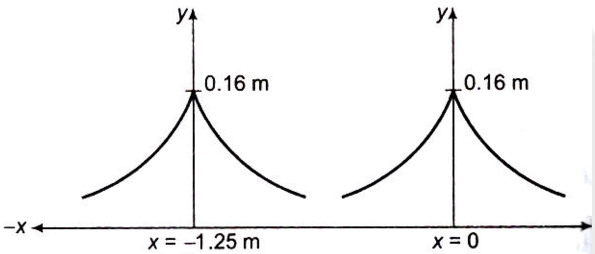
Pulse will be symmetric (Symmetry is checked about y_{max}) if at $t = 0$

$$y(x) = y(-x)$$

From the given equation,

and $\left.\begin{array}{l} y(c) = \dfrac{0.8}{16x^2+5} \\ y(-x) = \dfrac{0.8}{16x^2+5} \end{array}\right\}$ at $t = 0$

Therefore, pulse is symmetric.
Speed of pulse
At $t = 1s$ and $x = -1.25m$
value of y is again 0.16 m, i.e. pulse has travelled a distance of 1.25 m in 1 s in negative x-direction or we cans ay that the speed of pulse is 1.25 m/s and it is travelling in negative x-direction. Therefore, it will travel a distance of 2.5 m in 2s. The above statement can be better understood from Fig. (b).



(b)

Q.19 Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C.
The mass of the steam condensed in kg is
(a) 0.1230 (b) 0.065 (c) 0.260 (d) 0.135

Ans: (a)

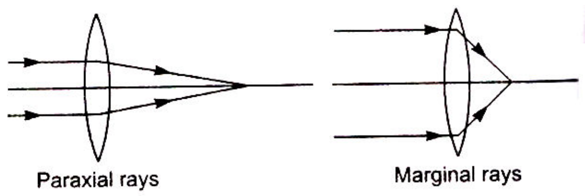
Sol: Heat required $Q_1 = (1.1 + 0.02) \times 10^3 \times 1 \times (80 - 15)$
$= 72800$ cal

Heat given it m (in kg) steam is condensed :
$$Q_2 = (m \times 540 \times 10^3) + m \times 1 \times 10^3 \times (100 - 80)$$
Equating $Q_1 = Q_2$, we get
$$m = 0.130 \text{ kg}$$

Q.20 Spherical aberration in athin lens can be reduced by
(a) using a monochromatic light
(b) using a doublet combination
(c) using a circular annular mark over the lens
(d) increasing the size of the lens

Ans: (c)

Sol: SPherical aberration is caused due to spherical nature of lens. Paraxial and marginal rays are focused at different places on the axis of the lens. Therefore, image so formed is blurred. This aberration can be reduced by qither stopping paraxial rays or marginal rays, which can be done by using a circular annular mark over the lens.



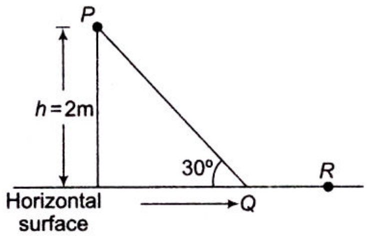
Q.21 The work, function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately?

Sol: $\lambda(\text{in } \overset{0}{A}) = \dfrac{12375}{W(eV)} = \dfrac{12375}{4.0}\overset{0}{A} = 3093\overset{0}{A}$

or $\quad \lambda = 309.3$ nm

$\quad = 310$ nm

Q.22 A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure, the coefficient of friction, between the particle and the rough track equals μ. The particle is released from rest, from the point O and it comes to rest at a point R. The energies lost by the ball, over the parts *PQ* and *QR* of the track, are equal to each other, and no energy is lost when particle changes direction from *PQ* to QR. The values of the coefficient of friction μ and the distance $x(= QR)$, are respectively close to



Sol: As energy loss is same, thus

$\mu\, mg\cos\theta.\,(PQ) = \mu\, mg.(QR)$

$\therefore \quad QR = (PQ)\cos\theta$

$\Rightarrow \quad QR = 4\times\dfrac{\sqrt{3}}{2}$

$\quad = 2\sqrt{3} = 3.5\ m$

Further, decrease in potential energy = loss due to friction

$\therefore\ mgh = (\mu\, mg\cos\theta)d_1 + (\mu mg)d_2 + \mu\times m\times 10\times 2\sqrt{3}$

$\Rightarrow \quad 4\sqrt{3}\ \mu = 2$

$\Rightarrow \quad \mu = \dfrac{1}{2\sqrt{3}} = 0.288 = 0.29$

Q.23 A satellite is launched into a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius (1.03)R. The period of the second satellite is larger than that of the first one by approximately

Sol: Orbital radius is increased by 3%.

Time period of satellite $T \alpha r^{3/2}$

$\therefore$ Percentage change in time period

$= \frac{3}{2}$(%change in orbital radius)

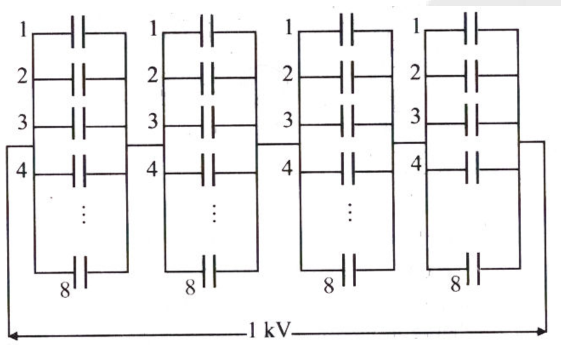
$= \frac{3}{2}(3\%) = 4.5\%$

Q.24 A capacitance of $2\mu F$ is required in an electrical circuit across a potential difference of 1.0 kV. A large number of $1\mu F$ capacitors are available which can withstand a potential difference of not more than 300V.
The minimum number of capacitors required to achive this is

Sol: i. To withstand potential difference of 1 kV minimum four capacitors are required in series.

ii. For C_{eq} to be $2\mu F$, 8 parallel combination are required.

$\therefore$ Minimum number of capacitors $= 8 \times 4 = 32$



Q.25 The molecules of a gas have 5 degrees of freedom. The heat absorbed by the gas if it performs 30 J of work while expanding at constant pressure is

Sol: $\gamma = 1 + \frac{2}{f}$

$= 1 + \frac{2}{5} = \frac{7}{5}$

Fraction of heat energy utilised in doing external work :

$\frac{\Delta W}{\Delta Q} = 1 - \frac{C_v}{C_p} = 1 - \frac{1}{\gamma}$

$= 1 - \frac{5}{7} = \frac{2}{7}$

$\therefore \quad \Delta Q = \frac{7}{2}(\Delta W)$

$= 105J$

Part - B - CHEMISTRY

Q.26 The hydration energy of Mg^{2+} is larger than that of

(a) Al^{3+} (b) Na^{+} (c) Be^{2+} (d) Mg^{3+}

Ans: (b)

Sol: Hydration energy depends on charge of ion and ionic radius. Higher the charge, greater the hydration energy. On the other hand, smaller the size, greter the hyrdartion energy. Charge is considered first for comparison. Hence, Mg^{2+} has higher hydration energy than Na^{+}.

Q.27 According to kinetic theory of gases, for a diatomic molecule

(a) the pressure exerted by the gas is proportional to mean square velocity of the molecule.

(b) the pressure exerted by the gas is proporational to the root mean velocity of the molecule.

(c) the root mean square velocity of the molecule is inversely proporational to the temperature.

(d) the mean translational kinetic energy of the molecule is propoational to the absolute temperature.

Ans: (d)

Sol: The mean translational kinetic energy $(\in)$ of an ideal gas is

$$\in = \frac{3}{2}K_B T; T = \text{Absolute temperature. } i.e. \in \propto T$$

Q.28 When the temperature is increased, surface tension of water

(a) increases (b) decreases

(c) remains constant (d) shows irregular behaviour

Ans: (b)

Sol: As temperature increases surface tension of liquid decreases.

Q.29 The following acids have been arranged in the order of decreasing acidic strength. Identify the correct order.

CIOH (I), BrOH (II), IOH (III)

(a)I > II > III (b) II > I > III (c) III > II > I (d) I > III > II

Ans: (a)

Sol: Amongst oxyacids of a given halogen, higher the oxidation number of halogen, stronger the acid, Hence

$HOC_l < HCLO_2 < HClO_3 < HClO_4$.The following acids have been arranged in the order of decreasing acidic strength. Identify the correct order.

CIOH (I), BrOH (II), IOH (III)

(a) I > II > III(b) II > I > III (c) III > II > I (d) I > III > II

Q.30 CH_3NH_2(0.1 mole, $K_b = 5\times10^{-4}$) is added to 0.08 mole of HCl and the solution is diluted to one litre, resulting hydrogen ion concentration is

(a) 1.6×10^{-11} (b) 8×10^{-11} (c) 5×10^{-5} (d) 8×10^{-2}

Ans: (b)

Sol:

$$CH_3NH_2 + HCl \longrightarrow CH_3NH_3^+ + Cl^-$$

Initial:	0.10	0.08	0	0
Final:	0.02	0	0.08	0.08

$$pOH = pK_b + log\frac{[CH_3NH_3^+]}{[CH_3NH_2]}$$

$$= -log\,(5\times10^{-4}) + log\frac{0.08}{0.02} = 3.9$$

$$pH = 14 - pOH = 10.1$$

$$[H^+] = 8\times10^{-11}$$

Q.31 Galvanisation is applying a coating of

(a) Cr (b) Cu (c) Zn (d) Pb

Ans: (c)

Sol: Zinc metal is the most stable metal to cover iron surfaces. The process of coating the iron surface by zinc is called galvanisation.

Q.32 Which series of reactions correctly represents chemical relations related to iron and its compound?

(a) $Fe \xrightarrow{\text{Dil. } H_2SO_4} FeSO_4 \xrightarrow{H_2SO_4, O_2} Fe_2(SO_4)_3 \xrightarrow{\text{Heat}} Fe$

(b) $Fe \xrightarrow{O_2,\text{Heat}} FeO \xrightarrow{\text{Dil. } H_2SO_4} FeSO_4 \xrightarrow{\text{Heat}} Fe$

(c) $Fe \xrightarrow{Cl_2,\text{Heat}} FeCl_3 \xrightarrow{\text{Heat,air}} FeCl_2 \xrightarrow{Zn} Fe$

(d) $Fe \xrightarrow{O_2,\text{Heat}} Fe_3O_4 \xrightarrow{CO,600^\circ C} FeO \xrightarrow{CO,700^\circ C} Fe$

Ans: (d)

Sol: The correct reactions are as follows :

(a) $Fe + \text{dil. } H_2SO_4 \longrightarrow FeSO_4 + H_2$

$$H_2SO_4 + 2FeSO_4 + \frac{1}{2}O_2 \longrightarrow Fe_2(SO_4)_3 + H_2O$$

$$Fe_2(SO_4)_3 \xrightarrow{\Delta} Fe_2O_3(s) + 3SO_3\uparrow$$

The given reaction is incorrect in question.

Q.33 The rate constant of a reaction depends on

(a) temperature

(b) initial concentration of the reactants

(c) time of reaction

(d) extent of reaction

Ans: (a)

Sol: The rate constant (k) of all chemical reactions depends on temperture.

$$k = Ae^{-E_a/RT}$$

where, A = *pre* – exponemtial,factor, E_a = activation energy.

Q.34 The molecular weight of benzoic acid in benzene as determined by depression in freezing point method corresponds to

(a) ionisation of benzoic acid
(b) dimerisation of benzoic acid
(c) trimerisation of benzoic acid
(d) solvation of benzoic acid

Ans: (b)

Sol: In benzene acid dimerises as :

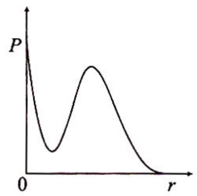
$$2C_6H_5COOH \underset{2}{\overset{1}{\rightleftharpoons}} (C_6H_5COOH)_2$$

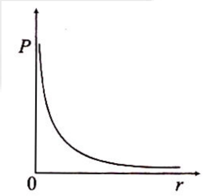
Q.35 If two compounds have the same empirical formula but different molecular formulae, they must have
(a) different percentage composition
(b) different molecular weight
(c) same velocity
(d) same vapour density

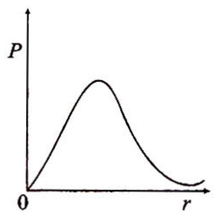
Ans: (b)

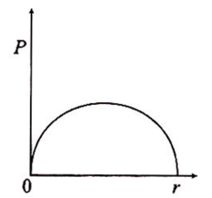
Sol: Compounds with same empirical formula but different molecular formula have same percentage composition of elements but different molecular weight.

Q.36 P is the probability of finding the 1s electron of hydrogen atom in a spherical shell of infinitesimal thickness, *dr*, at a distance r from the nucleus. The volume of this shell is $4\pi r^2 dr$. The qualitative sketch of the dependence of *P* on *r* is

(a) 

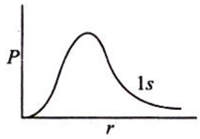
(b) 

(c) 

(d) 

Ans: (c)

Sol: This graph shows the probability of finding the electron within shell at various distances from the nucleus (radial probability). The curve shows the maximum, which means that the radial probability is greatest for a given distance from the nucleus.



This distance is equal to Bohr's radius $= a_0$

(a) It is for 2*s*-orbital.
(b) It is radial wave function for 1*s*.
(c) Correct
(d) Probability cannot be zero at a certain distance from nucleus.

Q.37 According to MO theory,

(a) O_2^+ is paramagnetic and bond order grater than O_2

(b) O_2^+ is paramagnetic and bond order less than O_2

(c) O_2^+ is damagnetic and bond order is less than O_2

(d) O_2^+ is damagnetic and bond order is more than O_2

Ans: (a)

Sol: $O_2^+(15e^-):\sigma 1s^2 \overset{*}{\sigma} 1s^2 \sigma 2s^2 \overset{*}{\sigma} 2s^2 \sigma 2p_x^2 \left| \begin{matrix} \pi 2p_y^1 \\ \pi 2p_z^2 \end{matrix} \right| \begin{matrix} \overset{*}{\pi} 2p_y^1 \\ \overset{*}{\pi} 2p_z^0 \end{matrix} \left| \overset{*}{\sigma} 2p_x^0 \right.$

Bond order $= \dfrac{10-5}{2} = 2.5$; paramagnetic

$O_2(16e^-) : \sigma 1s^2 \overset{*}{\sigma} 1s^2 \sigma 2s^2 \overset{*}{\sigma} 2s^2 \sigma 2p_x^2 \left| \begin{matrix} \pi 2p_y^1 \\ \pi 2p_z^2 \end{matrix} \right| \begin{matrix} \overset{*}{\pi} 2p_y^1 \\ \overset{*}{\pi} 2p_z^0 \end{matrix} \left| \overset{*}{\sigma} 2p_x^0 \right.$

Bond order $= \dfrac{10-6}{2} = 2$

Hence, (a) is the correct answer.

Q.38 A monoatomic ideal gas undergoes a process in which the ratio of p to V at any instant is constant and equals to 1. What is the molar heat capacity of the gas.

(a) $\frac{4R}{2}$ (b) $\frac{3R}{2}$ (c) $\frac{5R}{2}$ (d) 0

Ans: (a)

Sol: Given, $\dfrac{p}{V} = 1 \Rightarrow p = V$ ...(i)

Also from first law: $dq = C_v dT + pdV$

For one mole of an ideal gas : $pV = RT$

$\Rightarrow \quad pdV + Vdp = RdT$ ...(ii)

From (i) $\quad pdV = Vdp$

Substituting in Eq. (ii) gives

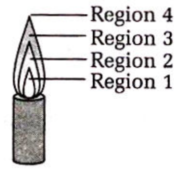
$2pdV = RdT$

$\Rightarrow \quad pdV = \dfrac{R}{2} dT$

$\Rightarrow \quad dq = C_V dT \dfrac{R}{2} dT$

$\Rightarrow \quad \int \dfrac{dq}{dT} = C_V + \dfrac{R}{2} = \dfrac{3}{2}R + \dfrac{R}{2} = 2R$

Q.39 The hottest region of Bunsen flame shown in the figure given below is



(a) region 2 **(b) region 3** **(c) region 4** **(d) region 1**

Ans: (a)

Sol: Region 1 (Pre-heating zone)
Region 2 (Primary combustion zone, hottest zone)
Region 3 (Internal zone)
Region 4 (Secondary reaction zone)

Q.40 A metal nitrate reacts with KI to give a black precipitate which on addition of excess of KI convert into orange colour solution. The cation of metal nitrate is

(a) Hg^{2+} **(b)** Bi^{3+} **(c)** Sn^{2+} **(d)** Pb^{2+}

Ans: (b)

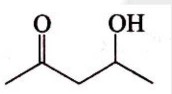
Sol: $Bi^{3+} + 3I^- \rightarrow \underset{\text{Black}}{BiI_3} \downarrow \xrightarrow[\text{excess}]{I^-} \underset{\substack{\text{Orange}\\ \text{solution}}}{[BiI_4]^-}$

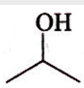
Q.41 The geometry of $Ni(CO)_4$ and $Ni(PPh_3)_2Cl_2$ are
(a) both square planar
(b) tetrahedral and square planar, respectively
(c) both tetrahedral
(d) square planar and tetrahedral, respectively

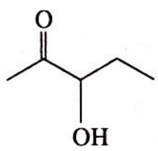
Ans: (c)

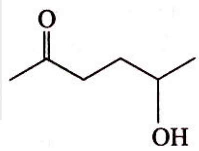
Sol: In $Ni(CO)_4$ Ni is in $3d^{10}$ state due to strong ligand field produced by CO. Hence, Ni is sp^3-hybridised and complex is tetrahedral. In $NiCl_2(PPh_3)_2$, Ni^{2+} has $3d^8$- configuration. Due to weak ligand field, Ni is sp^3-hybridised and complex is tertrahedral.

Q.42 Which one of the following will most readily be dehydrated in acidic condition?

(a) 

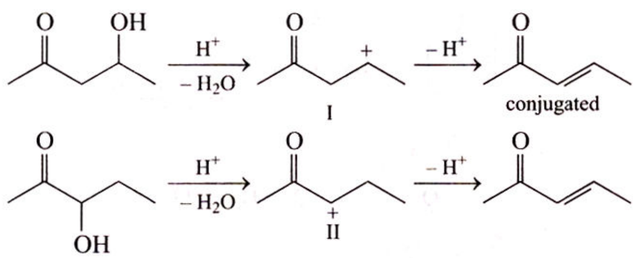
(b) 

(c) 

(d) 

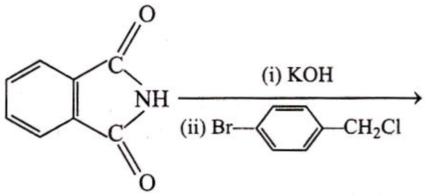
Ans: (a)

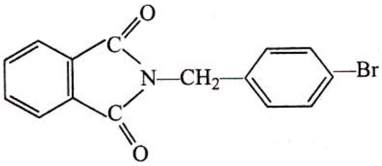
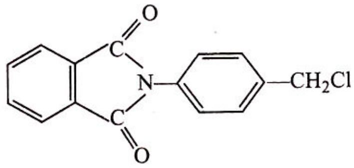
Sol:

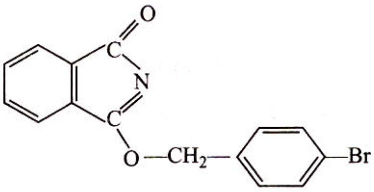
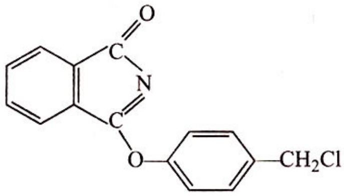


Although both reactions are giving the same product, carbocation I is more stable than II.

Q.43 The major product of the following reaction is

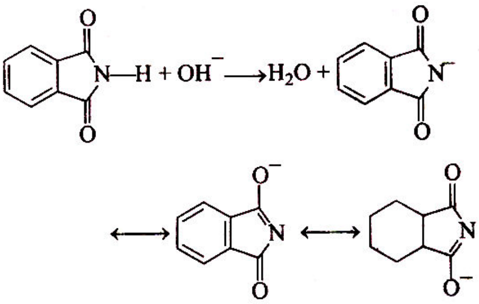


(a)  (b) 

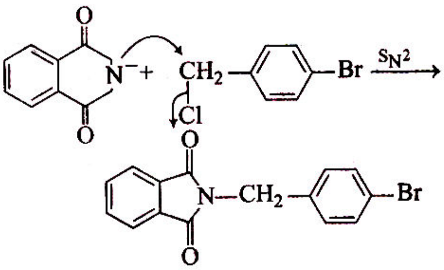
(c)  (d) 

Ans: (a)

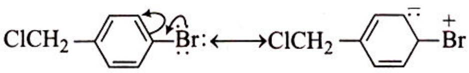
Sol: It is the first step pf Gabriel's phthalimide synthesis. The hydrogen bonded to nitrogen is sufficiently acidic due to two α-carbonyls.



The conjugate base formed above act as nucleophile in the subsequent step of reaction. As shown above, the nucleophile exist in three resonating form, one may think of oxygen being the donor atom in the nucleophilic attack. However, nitrogen act as donor as it is better donor than oxygen.



Bromine is not substituted in the above reaction as it is in resonance with benzene ring giving partial double bond charcter to $C-Br$ bond, hence difficult to break.



Q.44 In the following sequence of reaction

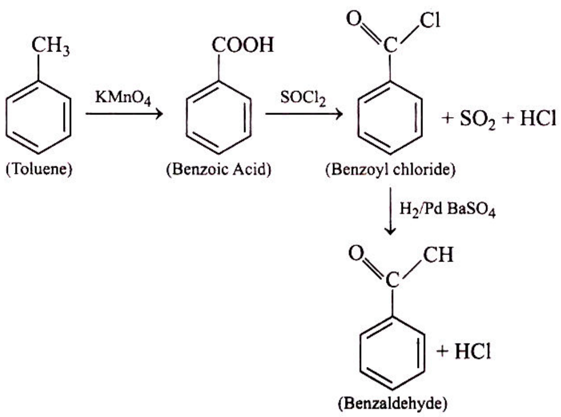
Toluene $\xrightarrow{KMnO_4}$ A $\xrightarrow{SOCl_2}$ B $\xrightarrow[BaSO_4]{H_2/Pd}$ C

(a) C_6H_5COOH **(b) $C_6H_5CH_3$**

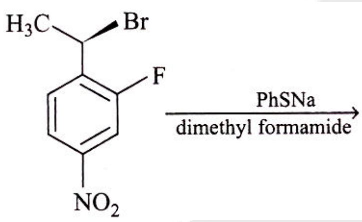
(c) $C_6H_5CH_2OH$ **(d) C_6H_5CHO**

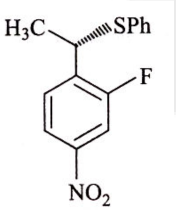
Ans: (d)

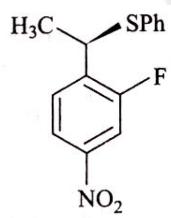
Sol: Toluene undergoes oxidation with $KMnO_4$, forms benzoic acid. In this conversion, alkyl part of toluene converts into carboxylic group. Further, benzoic acid reacts with thionyl chloride ($SOCl_2$) to give benzoyl chloride which upon reduction with $H_{2/}$Pd or $BaSO_4$ forms benzaldehyde (Rosenmund reduction) The conversion look like.

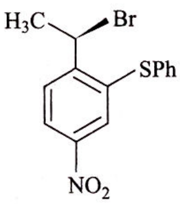


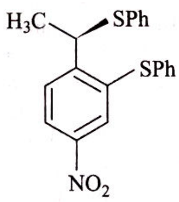
Q.45 The major product of the following reaction is



(a) 

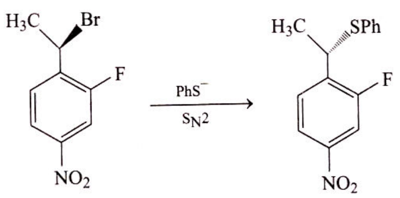
(b) 

(c) 

(d) 

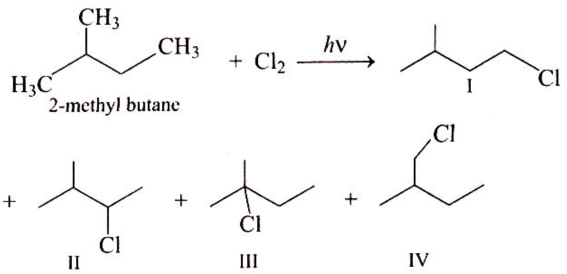
Ans: (a)

Sol: Nucleophile PhS^- substitute the Br^- through mechanism with inversion of configuration at α-C.



Q.46 How many chiral compounds are possible on mono chlorination of 2-methyl butane?

Sol:



Out of the four products formed above, II and IV are chiral, produced in pairs, giving total of six mono-chlorination products.

Q.47 The molarity of a solution obtained by mixing 750 mL of 0.5 M GCl with 250 mL of 2 M HCl will be?

Sol: From the formula, $M_f = \dfrac{M_1V_1 + M_2V_2}{V_1 + V_2}$

Given, $V_1 = 750\,m\text{L}$, $M_1 = 0.5\ M$

$V_2 = 250\,mL,\ M_2 = 2\,M$

$$= \frac{750 \times 0.5 + 250 \times 2}{750 + 250} = \frac{875}{1000} = 0.875\,\text{M}$$

Q.48 For the coagulation of 100 mL of cadmium sulphide solution, 5 mL of 1 M NaCl is required. Flocculation value of NaCl is _______?

Sol: 5 mL, 1M NaCl $\Rightarrow 5 \times 10^{-3}$ mol or 5 millimoles

100 mL Cds requires $\Rightarrow$ 5 millimoles

1000 mL CdS requires $\Rightarrow$ millimoles

Q.49 What volume of 6.0 M H_2SO_4 should be mixed with 10 L of 1.0 M H_2SO_4 to make 20.0 L of 3.0 M H_2SO_4 upon dilution to volume ?

Sol: apply $M_1 V_1 + M_2 V_2 = M_3 V_3$

$6V_1 + 10 \times 1 = 3 \times 20 \Rightarrow V_1 = \dfrac{50}{6} = 8.3\ \text{L}$

Q.50 Hydrogen has three isotopes, the number of possible diatomic molecules will be?

Sol: (b) $H^1H^1, H^1H^2, H^1H^3, H^2H^2, H^3H^3, H^2H^3$

Part - C - MATHEMATICS

Q.51 If the coefficients of x^3 and x^4 in the expansion of $(1+ax+bx^2)(1-2x)^{18}$ in powers of are both zero, then is equal to

(a) $\left(16, \frac{251}{3}\right)$ **(b)** $\left(14, \frac{251}{3}\right)$ **(c)** $\left(14, \frac{272}{3}\right)$ **(d)** $\left(16, \frac{272}{3}\right)$

Ans: (d)

Sol: To find the coefficient of x^3 and x^4, use the formula of coefficient of x^r in $(1-x)^n$ is $(-1)^r\,{}^nC_r$ and then simplify.

In expansion of $(1+ax+bx^2)(1-2x)^{18}$.

$$\text{Coefficient of } x^3 = \text{Coefficient of } x^3 \text{in } (1-2x)^{18}$$
$$+ \text{Coefficient of } x^2 \text{ in } a(1-2x)^{18}$$
$$+ \text{Coefficient of } x \text{ in } b(1-2x)^{18}$$
$$= -{}^{18}C_3.2^3 + a\,{}^{18}C_2.2^2 - b\,{}^{18}C_1.2$$

Given, coefficient of $x^3 = 0$

$$\Rightarrow \quad {}^{18}C_3.2^3 + a\,{}^{18}C_2.2^2 - b\,{}^{18}C_1.2 = 0$$

$$\Rightarrow \quad -\frac{18\times17\times16}{3\times2}.8 + a.\frac{18\times17}{2}.2^2 - b.18.2 = 0$$

$$\Rightarrow \quad 17a - b = \frac{34\times16}{3} \quad ...(i)$$

Similarly, coefficient of $x^4 = 0$

$$\Rightarrow \quad {}^{18}C_4.2^4 - a.{}^{18}C_3 2^2 + b.{}^{18}C_2.2^2 = 0$$

$$\therefore \quad 32a - 3b = 240 \quad ...(ii)$$

On solving Eqs, (i) and (ii), we get

$$a = 16, \quad b = \frac{272}{3}$$

Q.52 The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{vmatrix}x\\y\\z\end{vmatrix} = \begin{vmatrix}1\\0\\0\end{vmatrix}$ has exactly two distinct solutions, is

(a) 0 **(b) 2^9-1** **(c) 168** **(d) 2**

Ans: (a)

Sol: Since, $A\begin{vmatrix}x\\y\\z\end{vmatrix} = \begin{vmatrix}1\\0\\0\end{vmatrix}$ is linear equation in three variables and that could have only unique, no solution or infinitely many solution.

$\therefore$ It is not possible to have two solutions.

Hence, number of matrices A is zero.

Q.53 **If $\int_{\sin x}^{1} t^2 f(t)dt = 1 - \sin x, \forall x \in (0, \pi/2)$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is**

(a) 3 **(b) $\sqrt{3}$** **(c) 1/3** **(d) None of these**

Ans: (a)

Sol: $\int_{\sin x}^{1} t^2 f(t)dt = 1 - \sin x$, thus to find $f(x)$.

On differentiating both sides using Newton Leibnitz formula

i.e. $$\frac{d}{dx}\int_{\sin x}^{1} t^2 f(t)dt = \frac{d}{dx}(1-\sin x)$$

$\Rightarrow$ $\{1^2 f(1)\}.(0) - (\sin^2 x).f(\sin x).\cos x = -\cos x$

$\Rightarrow$ $$f(\sin x) = \frac{1}{\sin^2 x}$$

For $f\left(\frac{1}{\sqrt{3}}\right)$ is obtained when $\sin x = 1/\sqrt{3}$

i.e. $$f\left(\frac{1}{\sqrt{3}}\right) = (\sqrt{3})^2 = 3$$

Q.54 $\lim\limits_{x\to\pi/2} \dfrac{\cot x - \cos x}{(\pi - 2x)^3}$ **equals**

(a) $\frac{1}{24}$ **(b) $\frac{1}{16}$** **(c) $\frac{1}{8}$** **(d) $\frac{1}{4}$**

Ans: (b)

Sol: $$\lim_{x\to\pi/2} \frac{\cot x - \cos x}{(\pi - 2x)^3} = \lim_{x\to\pi/2} \frac{1}{8}.\frac{\cos x(1-\sin x)}{\sin x\left(\frac{\pi}{2}-x\right)^3}$$

$$= \lim_{h\to 0}\frac{1}{8}.\frac{\cos\left(\frac{\pi}{2}-h\right)\left[1-\sin\left(\frac{\pi}{2}-h\right)\right]}{\sin\left(\frac{\pi}{2}-h\right)\left(\frac{\pi}{2}-\frac{\pi}{2}+h\right)^3}$$

$$= \frac{1}{8}\lim_{h\to 0}\frac{\sin h(1-\cos h)}{\cos h.h^3}$$

$$= \frac{1}{8}\lim_{h\to 0}\frac{\sin h\left(2\sin^2\frac{h}{2}\right)}{\cos h.h^3}$$

$$=\frac{1}{4}\lim_{h\to 0}\frac{\sin h.\sin^2\left(\frac{h}{2}\right)}{h^3\cos h}$$

$$=\frac{1}{4}\lim_{h\to 0}\left(\frac{\sin h}{h}\right)\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right).\frac{1}{\cos h}.\frac{1}{4}=\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$$

Q.55 Consider an infinite geometric series with first team *a* and common ratio *r*. If its sum is 4 and the second term is 3/4, then

(a) $a = 4/7, r = 3/7$

(b) $a = 2, r = 3/8$

(c) $a = 3/2, r = 1/2$

(d) $a = 3, r = 1/4$

Ans: (d)

Sol: Since, sum $= 4$ and second term $=\frac{3}{4}$

It is given first trm a and common ratio r.

$\Rightarrow \quad \frac{a}{1-r}=4,\ ar=\frac{3}{4}$

$\Rightarrow \quad r=\frac{3}{4a}$

$\Rightarrow \quad \frac{a}{1-\frac{3}{4a}}=4$

$\Rightarrow \quad \frac{4a^2}{4a-3}=4$

$\Rightarrow \quad (a-1)(a-3)=0$

$\Rightarrow \quad a=1 \text{ or } 3$

When $a=1, r=3/4$

and when $a=3, r=1/4$

Q.56 Let $f(x)=\int e^x(x-1)(x-2)dx$. Then, *f* decreases in the interval

(a) $(-\infty,-2)$ **(b) $(-2-1)$** **(c) $(1,2)$** **(d) $(2,\infty)$**

Ans: (c)

Sol: Let $f(x)=\int e^x(x-1)(x-2)dx$

$\Rightarrow \quad f'(x)=e^x(x-1)(x-2)$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & 1 & & 2 & \end{array}$$

$\therefore\ f'(x) < 0$ *for* $1 < x < 2$

$\Rightarrow\ f(x)$ is decreasing for $x \in (1,2)$.

Q.57 Area of triangle formed by the lines $x + y = 3$ and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ is

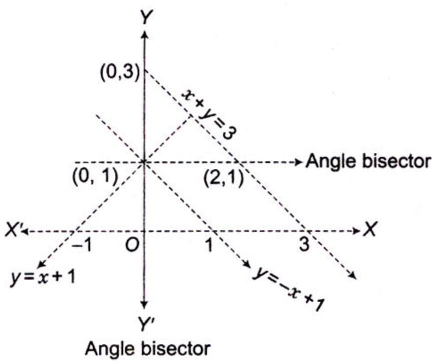
(a) 2 sq units **(b) 4 sq units** **(c) 6 sq units** **(d) 8 sq units**

Ans: (a)

Sol: Given, $x^2 - y^2 + 2y = 1$

$\Rightarrow \quad x^2 = (y-1)^2$

$\Rightarrow \quad x = y - 1$



From the graph, it is clear that equation of angle bisectors are

and $\quad x = -y + 1$

$\quad y = 1$

and $\quad x = 0$

$\therefore$ Area of region bounded by $x + y = 3, x = 0$

and $y = 1$ is

$\Delta = \frac{1}{2} \times 2 \times 2 = 2$ sq units

Q.58 The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the X-axis at Q. If M is the mid-point of the line segment PQ, then the locus of M intersects the latusrectum of the given ellipse at the points

(a) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ **(b)** $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{\sqrt{19}}{4}\right)$

(c) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ **(d)** $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

Ans: (c)

Sol: Given, $\frac{x^2}{16} + \frac{y^2}{4} = 1$

Here, $a = 4, b = 2$

Equation Normal

$$4x\sec\theta - 2y \operatorname{cosec}\theta = 12$$

$$M\left(\frac{7\cos\theta}{2}, \sin\theta\right) = (h,k) \qquad \text{[say]}$$

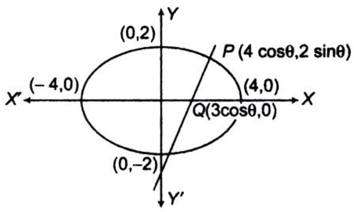
$$\therefore \quad h = \frac{7\cos\theta}{2} \Rightarrow \ = \frac{2h}{7} = \cos\theta \qquad \text{...(i)}$$

and $k = \sin\theta$ ...(ii)

On squaring and adding Eqs. (i) and (ii), we get

$$\frac{4h^2}{49} + k^2 = 1 \qquad [\because \cos^2\theta + \sin^2\theta = 1]$$

Hence, locus is $\frac{4x^2}{49} + y^2 = 1$ ...(iii)



For given ellipse, $e^2 = 1 - \frac{4}{16} = \frac{3}{4}$

$$\therefore \qquad e = \frac{\sqrt{3}}{2}$$

$$\therefore \qquad x = \pm 4 \times \frac{\sqrt{3}}{2} = \pm 2\sqrt{3} \quad [\because x = \pm ae] \quad \text{...(iv)}$$

On solving Eqs. (iii) and (iv), we get

$$\frac{4}{49} \times 12 + y^2 = 1 \Rightarrow y^2 = 1 - \frac{48}{49} = \frac{1}{49}$$

$$y = \pm\frac{1}{7}$$

$\therefore$ Required points $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$

Q.59 If arg $(z) < 0$, then arg $(-z) - \arg(z)$ equals

(a) π **(b)** $-\pi$ **(c)** $-\pi/2$ **(d)** $\pi/2$

Ans: (a)

Sol: Let $z = \cos\theta + i\sin\theta$

Org $(z) = \tan^{-1}\left|\frac{\sin\theta}{\cos\theta}\right| \qquad = \tan^{-1}(\tan\,\theta) = \theta$

Now,

$(-z) = -\cos\,\theta - i\,\sin\,\theta$

$(-z) = \cos\,(\lambda+\theta) + i\,\sin\,(\pi+\theta)$

Org $\quad (-z) = \tan^{-1}\left(\frac{\sin\,(\pi+\theta)}{\cos\,(\pi+\theta)}\right)$

Org $\quad (-z) = \tan^{-1}(\tan\,(\pi+\theta)) = \pi+\theta$

Org $\quad (-z) - org\,z = \pi+\theta-\theta = \pi$

Q.60 For the three events A, B and C, P (exactly one of the events A or B occurs) = P (exactly one of the events B or C occurs) = (exactly one of the events C or A occurs)

= P and P (all the three events occurs simultaneously) = p², where $0 < p < \frac{1}{2}$.

Then, the probability of atleast one of the three events A, B and C occurring is

(a) $\frac{3p+2p^2}{2}$ (b) $\frac{p+3p^2}{4}$ (c) $\frac{p+3p^2}{2}$ (d) $\frac{3p+2p^2}{4}$

Ans: (a)

Sol: We know that,

P (exactly one of A or B occurs)

$= P(A)+P(B)-2P(A\cap B)$

$\therefore \quad P(A)+P(B)-2P(A\cap B) = p \quad$ ...(i)

Similarly, $P(B)+P(C)-2P(B\cap C) = p \quad$ ...(ii)

On adding Eqs. (i), (ii) and (iii), we get

and $\quad P(C)+P(A)-2P(C\cap A) = p \quad$ ...(iii)

$2[P(A)+P(B)+P(C)-P(A\cap B)$

$-P(B\cap C)-P(C\cap A)] = 3p$

$\Rightarrow \quad P(A)+P(B)+P(C)-P(A\cap B)$

$-P(B\cap C)-P(C\cap A) = \frac{3p}{2} \quad$ ...(iv)

It also given that, $P(A\cap B\cap C) = p^2 \quad$ ...(v)

$\therefore$ P(at least one of the events A, B and C occurs)

$= P(A)+P(B)-P(C)-P(A\cap B)$

$-P(B\cap C)-P(C\cap A)]+P(A\cap B\cap C)$

$= \frac{3p}{2}+p^2 \qquad$ [from Eqs. (iv)and (v)]

$= \frac{3p+2p^2}{2}$

Q.61 The equation of the common tangent touching the circle $(x-3)^2+y^2=9$ and the parabola $y^2=4x$ above the X-axis is

(a) $\sqrt{3}y = 3x+1$ **(b)** $\sqrt{3}y = -(x+3)$

(c) $\sqrt{3}y = x+3$ **(d)** $\sqrt{3}y = -(3x+1)$

Ans: (c)

Sol: Any tangent $y^2 = 4x$ is of the form $y = mx + \frac{1}{m}$,

$(\because a = 1)$ and this touches the circle $(x-3)^2 + y^2 = 9$.

If $$\left|\frac{m(3) + \frac{1}{m} - 0}{\sqrt{m^2+1}}\right| = 3$$

$[\because$ centre of the circle is (3,0) and radius is 3].

$$\Rightarrow \quad \frac{3m^2+1}{m} = \pm 3\sqrt{m^2+1}$$

$$\Rightarrow \quad 3m^2+1 = \pm 3m\sqrt{m^2+1}$$

$$\Rightarrow \quad 9m^4 + 1 + 6m^2 = 9m^2(m^2+1)$$

$$\Rightarrow \quad 9m^4 + 1 + 6m^2 = 9m^4 + 9m^2$$

$$\Rightarrow \quad 3m^2 = 1$$

$$\Rightarrow \quad m = \pm\frac{1}{\sqrt{3}}$$

If the tangent touches the parabola and circle above the X-axis, then slope m should be positive.

$\therefore \quad m = \frac{1}{\sqrt{3}}$ and the equations is $y = \frac{1}{\sqrt{3}}x + \sqrt{3}$

or $\quad \sqrt{3}y = x + 3$.

Q.62 Let $P(a\sec\theta, b\tan\theta)$ and Q $(a\sec\phi, b\tan\phi)$, where $\theta + \phi = \frac{\pi}{2}$, be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

If (h, k) is the point of the intersection of the normals at P and Q, then k is equal to

(a) $\frac{a^2+b^2}{a}$ **(b)** $-\left(\frac{a^2+b^2}{a}\right)$ **(c)** $\frac{a^2+b^2}{b}$ **(d)** $-\left(\frac{a^2+b^2}{b}\right)$

Ans: (d)

Sol: Firstly, we obtain the slope of normal to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(a\sec\theta, b\tan\theta)$.

On differentiating w.r.t.x, we get

$$\frac{2x}{a^2}-\frac{2y}{b^2}\times\frac{dy}{dx}=0 \quad \Rightarrow \quad \frac{dy}{dx}=\frac{b^2}{a^2}\frac{x}{y}$$ (Slop of tangent)

Slope for normal at the point will be

$$-\frac{a^2 b\tan\theta}{b^2 a\sec\theta}=-\frac{a}{b}\sin\theta$$

$\therefore$ Equation of normal at $(a\sec\theta, b\tan\theta)$ is

$$(y-b\tan\theta)=-\frac{a}{b}\sin\theta(x-a\sec\theta)$$

$\Rightarrow \quad (a\sin\theta)x+by=(a^2+b^2)\tan\theta$

$\Rightarrow \quad ax+b\operatorname{cosec}\theta=(a^2+b^2)\sec\theta$ ...(i)

Similarly, equation of normal to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at

$(a\sec\phi, b\tan\phi)$ is $ax+b\,y\operatorname{cosec}\phi=(a^2+b^2)\sec\phi$ ...(ii)

On substracting Eq. (ii) from Eq. (i), we get

$b(\operatorname{cosec}\theta-\operatorname{cosec}\phi)y=(a^2+b^2)(\sec\theta-\sec\phi)$

$$\Rightarrow \quad y=\frac{a^2+b^2}{b}\cdot\frac{\sec\theta-\sec\phi}{\operatorname{cosec}\theta-\operatorname{cosec}\phi}$$

But $$\frac{\sec\theta-\sec\phi}{\operatorname{cosec}\theta-\operatorname{cosec}\phi}=\frac{\sec\theta-\sec\left(\frac{\pi}{2}-\theta\right)}{\operatorname{cosec}\theta-\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right)}$$ [$\because \phi+\theta=\pi/2$]

$$=\frac{\sec\theta-\operatorname{cosec}\theta}{\operatorname{cosec}\theta-\sec\theta}=-1$$

Thus, $y=-\left(\frac{a^2+b^2}{b}\right)$, i.e. $k=-\left(\frac{a^2+b^2}{b}\right)$

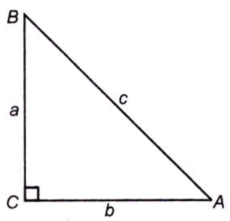
Q.63 The sides of a traingle are in the ratio $1:\sqrt{3}:2$, then the angles of the triangle are in the ratio

(a) 1 : 3 : 5 **(b) 2 : 3 : 2** **(c) 3 : 2 : 1** **(d) 1 : 2 : 3**

Ans: (d)

Sol: Let $a:b:c=1:\sqrt{3}:2 \Rightarrow c^2=a^2+b^2$

$\therefore$ Triangle is right angled at C.



or $\sqrt{C} = 90^\circ$

and $\dfrac{a}{b} = \dfrac{1}{\sqrt{3}}$

In ΔABC, $\tan A = \dfrac{a}{b} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad A = 30^\circ$

and $\quad b = 60^\circ$

$[\because A + B = 90^\circ]$

$\therefore$ Ratio of angles, $A:B:C = 30^\circ:60^\circ:90^\circ$

$= 1:2:3$

Q.64 Two adjacent sides of a parallelogram ABCD are given by The side AD is rotated by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$.. an acute angle α in the plane of the parallelogram so that AD becomes
AD' If AD' makes a right angle with the side AB, then the cosine of the angle is given by

(a) $\dfrac{8}{9}$ **(b)** $\dfrac{\sqrt{17}}{9}$ **(c)** $\dfrac{1}{9}$ **(d)** $\dfrac{4\sqrt{5}}{9}$

Ans: (b)

Sol: $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$

$\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$

Angle 'θ' between $\overrightarrow{AB}$ and $\overrightarrow{AD}$ is

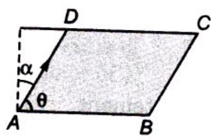
$$\cos(\theta) = \left| \frac{\overrightarrow{AB}.\overrightarrow{AD}}{|\overrightarrow{AB}||\overrightarrow{AD}|} \right|$$

$$= \left| \frac{-2 + 20 + 22}{(15)(3)} \right| = \frac{8}{9}$$

$\Rightarrow sin(\theta) = \dfrac{\sqrt{17}}{9}$

Since, $\alpha + \theta = 90^\circ$

$\therefore cos(\alpha) = cos(90^\circ - \theta) = sin(\theta) = \dfrac{\sqrt{17}}{9}$



Q.65 If $f : [0, \infty) \to [0, \infty)]$ and $f(x) = \dfrac{x}{1 + x'}$ then f is

(a) one-one and onto **(b) one-one but not onto**
(c) onto but not one-one **(d) one-one and onto**

Ans: (b)

Sol: Given, $f:[0,\infty)\to[0,\infty)]$

Here, domain is $[0,\infty)$ and codomain is $[0,\infty)$. Thus, to check one-one

Since, $f(x)=\dfrac{x}{1+x}\Rightarrow f'(x)\dfrac{1}{(1+x)^2}>0, \forall x\in[0,\infty)$

$\therefore f(x)$ is increasing in its domain. Thus $f(x)$ is one-one in its domain. To check onto (we find range)

Again $\quad y=f(x)=\dfrac{x}{1+x}$

$\Rightarrow \quad y+yx=x$

$\Rightarrow \quad x=\dfrac{y}{1-y}\Rightarrow \dfrac{y}{1-y}\geq 0$

Since, $x\geq 0$, therefore $0\leq y\leq 1$

i.e. Range $\neq$ Codomain

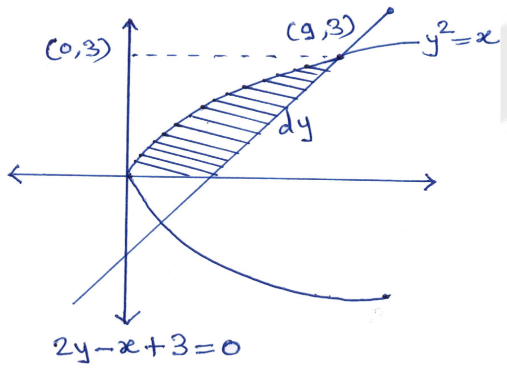
$\therefore f(x)$ is one-one but not onto.

Q.66 The area (in sq units) bounded by the curves $y=\sqrt{x}, 2y-x+3=0$, X-axis and lying in the first quadrant, is

(a) 9 **(b) 6** **(c) 18** **(d) $\dfrac{27}{4}$**

Ans: (a)

Sol:



Given curves are $x=y^2$ ...(i)

and $\quad 2y-x+3=0,$ ...(ii)

By solving equation (i) and (ii)

$x=9, \quad y=3$

$$A=\left|\int_0^3\left[y^2-[2y+3]\right]dy\right|$$

$$=\int_0^3(y^2-2y-3)dy$$

$= \left[\frac{y^3}{3} - y^2 - 3y\right]_0^3$

$= 9$ Sq units.

Q.69 The smallest postive root of the equation $x - x = 0$ lies in

(a) $\left(0, \frac{\pi}{2}\right)$ **(b)** $\left(\frac{\pi}{2}, \pi\right)$ **(c)** $\left(\pi, \frac{3\pi}{2}\right)$ **(d)** $\left(\frac{3\pi}{2}, 2\pi\right)$

Ans: (c)

Sol: Let $f(x) = \tan x - x$

We know, for $0 < x < \frac{\pi}{2}$

$\Rightarrow \quad \tan x > x$

$\therefore \quad f(x) = \tan x - x$ has no root in $(0, \pi/2)$

For $\pi/2 < x < \pi$, $\tan x$ is negative.

$\therefore \quad f(x) = \tan x - x < 0$

So, $\quad f(x) = 0$ has no root in $\left(\frac{\pi}{2}, \pi\right)$.

For $\frac{3\pi}{2} < x < 2\pi$ $\tan x$ is negative

$\therefore \quad f(x) = \tan x - x < 0$

So, $\quad f(x) = 0$ has no root in $\left(\frac{3\pi}{2}, 2\pi\right)$.

We have , $f(\pi) = 0 - \pi < 0$

and $\quad f\left(\frac{3\pi}{2}\right) = \tan\frac{3\pi}{2} - \frac{3\pi}{2} > 0$

$\therefore \quad f(x) = 0$ has at least one root between π and $\frac{3\pi}{2}$.

Q.70 Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then the equation of the circle through their points of intersection and the point (1, 1) is

(a) $x^2 + y^2 - 6x + 4 = 0$ **(b)** $x^2 + y^2 - 3x + 1 = 0$

(c) $x^2 + y^2 - 4y + 2 = 0$ **(d) None of the above**

Ans: (b)

Sol: The required equation of circle is, $S_1 + \lambda(S_2 - S_1) = 0$.

$\therefore \quad (x^2 + y^2 - 6) + \lambda(-6x + 14) = 0$

Also, passing through (1, 1).

$\Rightarrow \quad -4 + \lambda(8) = 0$

$\Rightarrow \quad \lambda = \frac{1}{2}$

$\therefore$ Required equation of circle is

$$x^2 + y^2 - 6 - 3x + 7 = 0$$

or $\quad x^2 + y^2 - 3x + 1 = 0$

Q.71 If $x\,dy = y(dx + ydy)$, $y(1) = 1$ and $y(x) > 0$. then, $y(-3)$ is equal to ?

Sol: Given, $x\,dy = y(dx + ydy),\ y > 0$

$\Rightarrow \quad x\,dy - ydx = y^2 dy$

$\Rightarrow \quad \left(\frac{xdy - ydx}{y^2}\right) = dy \Rightarrow d\left(\frac{x}{y}\right) = -dy$

On integrating both sides, we get

$$\frac{x}{y} = -y + c \qquad \text{...(i)}$$

Since, $\quad y(1) = 1 \Rightarrow x = 1,\ y = 1$

Now, Eq. (i) becomes, $\frac{x}{y} + y = 2$

Again, for $x = -3$

$\Rightarrow \quad -3 + y^2 = 2y$

$\Rightarrow \quad y^2 - 2y - 3 = 0$

$\Rightarrow \quad (y+1)(y-3) = 0$

As $y > 0$, take $y = 3$, neglecting $y = 1$.

Q.72 The letters of the word COCHIN are permuted and all the permutation are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN, is_______?

Sol: Arrange the letters of the word COCHIN as in the order of dictionary CCHINO.
Consider the words starting from C.
There are 5! such words. Number of words with the two C' s occupying first and second place = 4! .
Number of words starting with CH, CI, CN is 4! each.
Similarly, number of words before the first word starting with CO = 4! + 4! +4! = 96.
The word starting with CO found first in the dectionary is COCHIN. There are 96 words before COCHIN.

Q.73 If $P = (x, y)$, $F_1 = (3,0)$, $F_2 = (-3,0)$ and $16^2 + 25y^2 = 400$, then $PF_1 + PF_2$ equals ?

Sol: Given, $16^2 + 25y^2 = 400$ [given]

$\Rightarrow \quad \frac{x^2}{25} + \frac{y^2}{16} = 1$

Here, $\quad a^2 = 25,\ b^2 = 16$

But $\quad b^2 = a^2(1 - e^2)$

$\Rightarrow \quad 16 = 25(1-e^2)$

$\Rightarrow \quad \dfrac{16}{25} = 1 - e^2$

$\Rightarrow \quad e^2 = 1 - \dfrac{16}{25} = \dfrac{9}{25}$

$\Rightarrow \quad e = \dfrac{3}{5}$

Now, foci of the ellipse are $(\pm ae, 0) \equiv (\pm 3, 0)$

We have, $\quad 3 = a.\dfrac{3}{5}$

$\Rightarrow \quad a = 5$

Now, $PF_1 + PF_2 =$ Major axis $= 2a$

$= 2 \times 5 = 10$

Q.74 Let $a_1, a_2, ..., a_{10}$ be in Ap and h_1, h_2 equal to....,
h_{10} *be* in HP. If $a_1 = h_1 = 2$ and $a_{10} = 3$, then $a_4 h_7$ is ______?

Sol: Since, *a, b, c, d* are in AP.

$\Rightarrow \quad \dfrac{a}{abcd}, \dfrac{b}{abcd}, \dfrac{c}{abcd}, \dfrac{d}{abcd}$ are in AP.

$\Rightarrow \quad \dfrac{1}{bcd}, \dfrac{1}{cda}, \dfrac{1}{abd}, \dfrac{1}{abc}$ are in AP.

$\Rightarrow \quad bcd, cda, abd, abc$ are in HP.

$\Rightarrow \quad abc, abd, cda, bcd$ are in HP.

Q.75 Total number of solutions of $|\cot x| = \cot x + \dfrac{1}{\sin x}, x \in [0, 3\pi]$ is equal to?

Sol: $|\cot x| = \cot x + \dfrac{1}{\sin x}$

Let $\quad \cot x > 0 \quad \cot x = \cot x + \dfrac{1}{\sin x} = 0$

$\Rightarrow \dfrac{1}{\sin x} = 0$, which is not possible

Let $\quad \cot x \leq 0$

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ROUGH WORK

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