JEE (MAIN)

TEST PAPER

SUBJECT : PHYSICS,CHEMISTRY, MATHEMATICS | **TEST CODE : TSJMT213**

ANSWER PAPER

TIME : 3 HRS | **MARKS : 300**

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark(-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

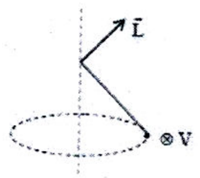
11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or “un- darken” a darkened bubble.

Part A - PHYSICS

Q.1 **A bob mass m attched to an inextensible string of length L is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the the vertical. About the point of suspension:**
(a) Angular momentum changes in direction but not in magnitude.
(b) Angular momentum changes both in direction and magnitude.
(c) Angular momentum is conserved.
(d) Angular momentum changes in magnitude but not in direction.

Ans: (a)

Sol: $\vec{L}$ changes in directions not in magnitude.



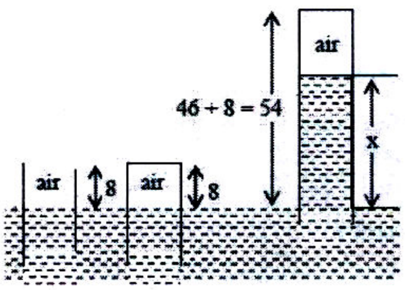
Q.2 **An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg).**
(a) 38 cm **(b) 6 cm** **(c) 16 cm** **(d) 22 cm**

Ans: (c)

Sol: $(76)(8) = (54 - x)(76 - x)$

$x = 38\,\text{cm}$

Length of air column $= 54 - 38 = 16\,\text{cm}$



Q.3 **Match List-I (Electromagnetic wave type) with List-II (Its association/application) and select the correct option from the choices given below the lists:**

	List-I		List-II
(a)	Infrrared waves	(i)	To treat muscylar strain
(b)	Radio waves	(ii)	For broadcasting
(c)	X-rays	(iii)	To detect fracture of bones
(d)	Ultraviolet rays	(iv)	Absorbed by the ozone layer of the atmosphere.

(a) (iii), (ii), (i), (iv) **(b) (i), (ii), (iii), (iv)**
(c) (iv), (iii), (ii), (i) **(d) (i), (ii), (iv), (iii)**

Ans: (b)

Sol: Infrared waves $\rightarrow$ To treat muscular strain
radio waves $\rightarrow$ for broadcasting
X-rays $\rightarrow$ To detect fracture of bones
Ultraviolet rays $\rightarrow$ Absorbed by the ozone layer of the atmosphere.

Q.4 A parallel plate capacitor is made of two circular plates separated by a distance of 5 nm and with a dielectric of dielectric constant 2.2 between them. When the field in the delectric 3×10^4 V/m is the charge density of the positive plate will be close to:

(a) $3\times10^4 C/m^2$ (b) $6\times10^4 C/m^2$ (c) $6\times10^{-7} C/m^2$ (d) $3\times10^{-7} C/m^2$

Ans: (c)

Sol: By formula of electric field between the plates of a capacitor $E=\frac{\sigma}{K\varepsilon_0}$

$$\Rightarrow \sigma = EK\varepsilon_0 = 3\times10^4\times2.2\times8.85\times10^{-12}$$
$$=6.6\times8.85\times10^{-8}$$
$$=5.841\times10^{-7}$$
$$=6\times10^{-7}C/m^2$$

Q.5 A student measured the length of a rod and wrote it as 3.5 cm. which instrument did he use to measure it?

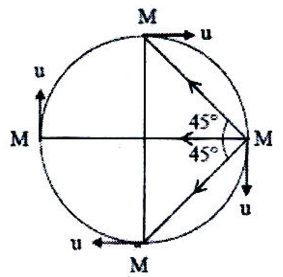
(a) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.
(b) A screw gague having 50 divisions in the circular scale and pitch as 1 mm.
(c) A meter scale.
(d) A vernier calliper where the 10 divisions in vernier scale matches with 9 divisions in main scale and main scale has 10 divisions in 1 cm.

Ans: (d)

Sol: Least cun of vernier calliper is $\frac{1}{10}mm = 0.1mm = 0.01cm$

Q.6 Four particles, each of mass M and equidistant from each other. move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particule is :

(a) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (b) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (c) $\sqrt{\frac{GM}{R}}$ (d) $\sqrt{2\sqrt{2}\frac{GM}{R}}$



Ans: (b)

Sol: Net force on any one particle.

$$=\frac{(GM)^2}{(2R)^2}+\frac{GM^2}{(R\sqrt{2})^2}\cos45^\circ+\frac{GM^2}{(R\sqrt{2})^2}\cos45^\circ$$

$$=\frac{(GM)^2}{R^2}\left[\frac{1}{4}+\frac{1}{\sqrt{2}}\right]$$

This force will be equal to centripetal force so

$$\frac{Mu^2}{R}=\frac{GM^2}{R^2}\left[\frac{1+2\sqrt{2}}{4}\right]$$

$$u=\sqrt{\frac{GM}{4R}\left[1+2\sqrt{2}\right]}=\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$$

Q.7 In a large building. there are 15 bulbs of 40 W. 5 bulbs of 100 W. 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be:

(a) 12 A (b) 14 A (c) 8 A (d) 10 A

Ans: (a)

Sol:

Item	No.	Power
40 W bulb	15	600 Watt
100 W bulb	5	500 Watt
80 W fan	5	400 Watt
1000 W heater	1	1000 Watt

Total Wattage = 2500 Watt

So current capacity $i=\frac{P}{V}=\frac{2500}{220}=\frac{125}{11}=11.36\cong 12\,\text{Amp.}$

Q.8 A particle moves with simple harmonic motion in a straight line. In fist τ s, after from it travels a distance a, and in next τ s it travels 2a, in same direction, then:

(a) amplitude of motion is 4a
(b) time period of oscillations is 6τ
(c) amplitude of motion is 3a
(d) time period of oscillations is 8τ

Ans: (b)

Sol:

$$\cos\omega\tau=\left(1-\frac{a}{A}\right)$$

$$\cos 2\omega\tau=\left(1-\frac{3a}{A}\right)$$

$$2\left(1-\frac{a}{A}\right)^2-1=1-\frac{3a}{A}$$

Solving the equation

$$\frac{a}{A}=\frac{1}{2}$$

$$A=2a$$

$$\cos\omega\tau=\frac{1}{2}$$

$$T=6\tau$$

Q.9 **The coercivity of a small magnet where the ferromagnet gets demagnetized is $3\times10^{3}\,Am^{-1}$ The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is:**

(a) 3A **(b) 6A** **(c) 30 mA** **(d) 60 mA**

Ans: (a)

Sol: $\mu_0 H = \mu_0 ni$

$3\times10^{3}\ \frac{100}{0.1}\times i \Rightarrow i = 3A$

Q.10 **During the propagation of electromagnetic waves in medium :**

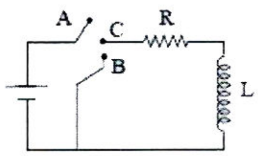
(a) Electric energy density is equal to the magnetic energy density.
(b) Both electric and magnetic energy densities are zero.
(c) Electric energy density is double of the magnetic energy density.
(d) Electric energy density is half of the magnetic energy density.

Ans: (a)

Sol: -------

Q.11 **In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time t = 0. Ratio of the voltage across resistance and the inductor at t = L/R will be equal to :**

(a) -1 **(b)** $\frac{1-e}{e}$ **(c)** $\frac{e}{1-e}$ **(d) 1**

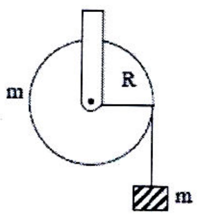


Ans: (d)

Sol: Since resistance and inductor are in parallel, so ratio will be 1.

Q.12 **A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder. with what acceleration will the mass fall on release?**

(a) $\frac{5g}{6}$ **(b) g** **(c)** $\frac{2g}{2}$ **(d)** $\frac{g}{2}$



Ans: (d)

Sol: For the mass m,

$mg - T = ma$

for the cylinder,

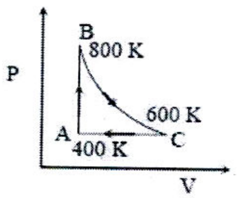
$$TR = mR^2 \frac{a}{R}$$

$\Rightarrow\ T = ma$

$\Rightarrow mg = 2ma$

$\Rightarrow a = g/2$

Q.13 One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K. 800 K and 600 Krespectively. Choose the correct statement:
(a) The change in internal energy in the process AB is -350R.
(b) The change in internal energy in the process BC is -500R.
(c) The change in internal energy in whole cyclic process is 250R.
(d) The change in internal energy in the process CA is 700R.



Ans: (b)

Sol: $$\Delta U_{AB} - nC_V(T_B - T_A) = 1 \times \frac{5R}{2}(800 - 400) = 1000R$$

$$\Delta U_{BC} - nC_V(T_C - T_B) = 1 \times \frac{5R}{2}(600 - 800) = -500R$$

$$\Delta U_{total} = 0$$

$$\Delta U_{CA} - nC_V(T_A - T_C) = 1 \times \frac{5R}{2}(400 - 600) = -500R$$

Q.14 From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u_2 and n is:

(a) $2gH = nu^2(n-2)$ **(b) $gH = (n-2)u^2$**

(c) $2gH = n^2u^2$ **(d) $gH = (n-2)^2u^2$**

Ans: (a)
Sol: Time to reach the maximum height

$$t_1 = \frac{u}{g}$$

If t_2 be the time taken to hit the ground

$$-H = ut - \frac{1}{2}gt_2^2$$

But $t_2 = nt_1$ (given)

$\Rightarrow -H = u\dfrac{nu}{g} - \dfrac{1}{2}g\dfrac{n^2u^2}{g^2}$

$\Rightarrow 2gH = nu^2(n-2)$

Q.15 A thin convex lens made from crown glass $\left(\mu = \dfrac{3}{2}\right)$ has local length f. When it is measured in two different liquids having refractive indices $\dfrac{4}{3}$ and $\dfrac{5}{3}$. it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is:

(a) $f_2 > f$ and f_1 becomes negative
(b) f_1 and f_2 both become negative
(c) $f_1 = f_2 < f$
(d) $f_1 > f$ and f_2 becomes negative

Ans: (d)

Sol: $\dfrac{f_m}{f} = \dfrac{(\mu-1)}{\left(\dfrac{\mu}{\mu_m}-1\right)}$

$\Rightarrow \dfrac{f_1}{f} = \dfrac{\left(\dfrac{3}{2}-1\right)}{\left(\dfrac{3/2}{4/3}-1\right)} = 4$

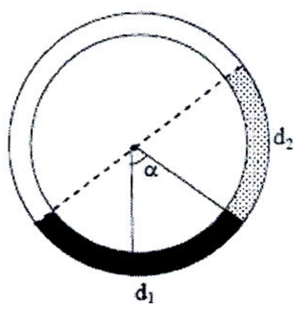
$\Rightarrow f_1 = 4f$

$\dfrac{f_2}{f} = \dfrac{\left(\dfrac{3}{2}-1\right)}{\left(\dfrac{3/2}{4/3}-1\right)} = -5$

$\Rightarrow f_2 < 0$

Q.16 There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90^o angle at centre. Radius joining their interface makes an angle α with vertical. ratio d_1/d_2 is

(a) $\dfrac{1+\tan\alpha}{1-\tan\alpha}$ **(b) $\dfrac{1+\sin\alpha}{1-\cos\alpha}$** **(c) $\dfrac{1+\sin\alpha}{1-\sin\alpha}$** **(d) $\dfrac{1+\cos\alpha}{1-\cos\alpha}$**



Ans: (a)

Sol: $P_A = P_B$

$P_o + d_1gR(\cos\alpha - \sin\alpha) = P_0 + d_2gR(\cos\alpha + \sin\alpha)$

$\Rightarrow \frac{d_1}{d_2} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1+\tan\alpha}{1-\tan\alpha}$

Q.17 A green light is incident from the water to the air - water interface at the critical angle (θ). Select the correct statement.

(a) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.

(b) The entire spectrum of visible light will come out of the water at an angle of 90^o to the normal

(c) The entire spectrum of visible light will come out of the water at an angle

(d) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

Ans: (d)

Sol: As frequency of visible light increases refractive index increases. With the increase of refractive index critical angle decreases. So that light having frequency grater than green will get total internal reflection and the light having frequency less than green will pass to air.

Q.18 Hydrogen (1H1), Deuterium (1H2), singly ionised Helium (2He4)+ and doubly ionised medium (3Li5)++ all have one electron around the nucleus. Consider an electron transition from n = 2 to n = 1. If the wave lengths oe emitted radiation are $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively then approximately which one of the following is correct?

(a) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

(b) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

(c) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

(d) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

Ans: (a)

Sol: $\frac{1}{\lambda} Rz^2 \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$\therefore \ \lambda = \frac{4}{3Rz^2}$

$\lambda_2 = \frac{4}{3R}$

$\lambda_3 = \frac{4}{12R}$

$\lambda_4 = \frac{4}{27R}$

$\Rightarrow \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

Q.19 A block of mass m is placed on a surface with a vertical cross section given by $y = x^3/6$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is :

(a) $\frac{1}{3}$**m** **(b)** $\frac{1}{2}$**m** **(c)** $\frac{1}{6}$**m** **(d)** $\frac{2}{3}$**m**

Ans: (c)

Sol: $mg\sin\theta = \mu mg\cos\theta$

$\tan\theta = \mu$

$\frac{dy}{dx} = \tan\theta = \mu = \frac{1}{2}$

$\frac{x^2}{2} = \frac{1}{2}, x = \pm 1$

$y = \frac{1}{6}m.$

Q.20 When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude F = ax + bx² where a and b are constants. The work done in stretching the unstretched rubber b and by L is :

(a) $\frac{aL^2}{2} + \frac{bL^2}{3}$ **(b)** $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$ **(c)** $aL^2 + bL^3$ **(d)** $\frac{1}{2}(aL^2 + bL^3)$

Ans: (a)

Sol: $F = ax + bx^2$

$dw = Fdx$

$W = \int_0^L (ax + bx^2)dx$

$W = \frac{aL^2}{2} + \frac{bL^3}{3}$

Q.21 The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is :
(For steel Young's modulus is $2\times10^{11}Nm^{-2}$ and coefficient of thermal expansion is $1.1\times10^{-5}K^{-1}$)

Sol: $0.10\times1.1\times10^{-5}\times100 = \frac{\frac{F}{A}}{2\times10^{11}}\times0.10$

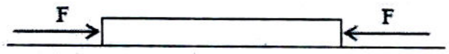
$\therefore \frac{F}{A} = \text{Pressure} = 1.1\times10^{-5}\times100\times2\times10^{11}$

$= 2.2\times10^{8}\,Pa$

Q.22 A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-a_z$ direction (see figure). For a field $\vec{B} = 3.0\times10^{-4}e^{-0.0x}\hat{a}_y$ find the power required to move the conductor at constant speed to $x = 2.0m$, $y = 0m$ in 5×10^{-3}s. Assume parallel motion along the x-axis.

Sol: $P = \frac{\text{Work Done}}{\text{Time}} == \frac{\int F dx}{t} = \frac{\int I\ell bB.dx}{t}$

$$= \frac{\int_0^2 (10)(3)(3\times 10^{-4} e^{-02\%})dx}{5\times 10^{-3}}$$

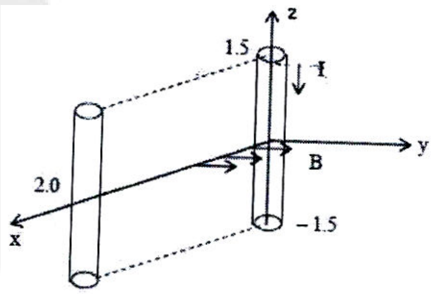


$$= \frac{9\times 10^{-3}}{5\times 10^{-3}}\left[\frac{e^{0.2x}}{-0.2}\right]_0^2 = 9\left[1-e^{-04}\right] = 2.97\ \text{W}$$

Q.23 The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)mA$, where the applied voltage V is in volts and the temperature T is in degree Kelvin. If a student makes an error measuring while measuring $\pm$ 0.01 V the current of 5 mA at 300 K, what will be the error in the value of current in mA?

Sol: $5 = e^{1000\frac{V}{T}} - 1$

$\Rightarrow e^{1000}\frac{V}{T} = 6$ ...(1)



Again, $I = e^{1000\frac{V}{T}} - 1$

$$\frac{dI}{dV} = e^{\frac{1000V}{T}}\frac{1000}{T}$$

$$dI = \frac{10000}{T}e^{1000\frac{V}{T}} - 1$$

Using (1)

$$\Delta I = \frac{1000}{T}\times 6\times 0.01 = \frac{60}{T} = \frac{60}{300} = 0.2mA.$$

Q.24 A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequences lie below 1250 Hz. The velocity of sound in air is 340 m/s.

Ans: (a)

Sol: In fundamental mode

$$\frac{\lambda}{4} = 0.85$$

$$\lambda = 4\times 0.85$$

$$f = v/\lambda = \frac{340}{4\times 0.85}$$

$$= 100\,\text{Hz}.$$

$\therefore$ Possible frequencies = 100 Hz, 300 Hz, 500hz, 700 Hz, 900 Hz, 1100 Hz below 1250 Hz.

Q.25 The radiation corresponding to $3 \to 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T If the radius of the largest circuar path followed by these electrons is 10.0 mm. the work function of the metal is close to :

Sol: $mv = qBR$

$$KE_{(max)} = \frac{(mv)^2}{2m} = 0.8eV$$

$$hv = 13.6\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\therefore\ W = hv - KE_{(max)}$$

$$= 13.6\frac{5}{36} - 0.8 = 1.1eV$$

Part - B - CHEMISTRY

Q.26 Which one of the following properties is not shown by NO ?
(a) It combines with oxygen to form nitrogen dioxide.
(b) It's bond order is 2.5
(c) It is diamagnetic in gaseous state
(d) It is a neutral oxide

Ans: (c)

Sol: NO is paramagnetic in gaseous state due to the presence of unpaired electron in its structure.

Q.27 If Z is a compressibility facor, van der Walls equation at low pressure can be written as:

(a) $Z = 1 - \frac{Pb}{RT}$ **(b)** $Z = 1 + \frac{Pb}{RT}$ **(c)** $Z = 1 + \frac{RT}{Pb}$ **(d)** $Z = 1 - \frac{a}{VRT}$

Ans: (d)

Sol: $$\left(P + \frac{n^2a}{V^2}\right)(V - nb) = nRT$$

For 1 mole, $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

$$PV = RT + Pb - \frac{a}{V} + \frac{ab}{V^2}$$

at low pressure, terms $Pb\&\frac{ab}{V^2}$ will be negligible as compared to RT.

So, $PV = RT - \frac{a}{V}$

$$Z = 1 - \frac{a}{RTV}$$

Q.28 The metal that cannot be obtained by electrolysis of an aqueous solution of its salts is :
(a) Cu **(b) Cr** **(c) Ag** **(d) Ca**

Ans: (d)

Sol: During the electrolysis of aqueous solution of s-block elements, H_2 gas is obtained at cathode.

Q.29 **Resistance of 0.2 M solution of an electrolyte is $50\,\Omega$. The specific constructance of the solution is $1.4\,\text{Sm}^{-1}$ The resistance of 0.5 M solution of the same electrolyte is $280\,\Omega$. The molar conductivity of 0.5 M solution of the electrolyte in S m² is :**

(a) 5×10^{3} **(b)** 5×10^{2} **(c)** 5×10^{-4} **(d)** 5×10^{-3}

Ans: (c)

Sol: $50=\frac{1}{K}\times\frac{\ell}{A}$

$50=\frac{1}{1.4}\times\frac{\ell}{A}$

$\frac{\ell}{A}=70\text{m}^{-1}$

$280=\frac{1}{K}\times70$

$K=\frac{1}{4}\text{Sm}^{-1}$

$A_m=\frac{1}{4}\times\left(\frac{1000}{M}\right)\left(10^{-2}\text{m}\right)^{3}$

$=\frac{1}{4}\times\frac{1000}{0.5}\times10^{-6}$

$=500\times10^{-6}$

$=5\times10^{-4}\text{Sm}^{2}\,\text{mol}^{-1}$

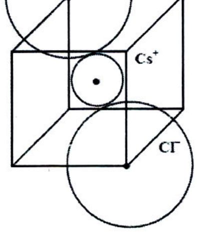
Q.30 **CsCl crystallises in body centred cubic lattice. If 'a' is its edge then which of the following expressions is correct?**

(a) $r_{Cs^+}+r_{Cl^-}=\frac{\sqrt{3}}{2}a$ **(b)** $r_{Cs^-}+r_{Cl^-}=\sqrt{3}a$

(c) $r_{Cs^+}+r_{Cl^-}=3a$ **(d)** $r_{Cs^{+-}}+r_{Cl^-}=\frac{3a}{2}$

Ans: (a)

Sol: In CsCl structure Cs^{+} ion is an contact with Cl^{-} ion at the nearest distance which is equal to $\sqrt{3}\frac{a}{2}$



Q.31 Consider separate solutions $C_2H_5OH(aq)$, $0.100\,M\,Mg_3(PO_4)_2$, $0.250\,M\,KBr_{(aq)}$ and $0.125\,M\,Na_3PO_{4(aq)}$ at $25°C$ of 0.500 M Which statement is true about these solutions, assuming all salts to be strong electrolytes?

(a) 0.125 M $Na_3PO_{4(aq)}$ has the highest osmotic pressure.

(b) 0.500 $C_2H_5OH_{(aq)}$ has the highest osmotic pressure.

(c) They all have the same osmotic pressure.

(d)0.100 M $Mg_3(PO_4)_{2(aq)}$ has the highest osmotic pressure.

Ans: (c)

Sol: For $C_2H_5OH, \pi = 1\times 0.5\times RT$

For $KBr, \pi = 2\times 0.25\times RT$

For $Mg_3(PO_4)_2, \pi = 5\times 0.1\times RT$

For $Na_3PO_4, \pi = 4\times 0.125\times RT$

So, all are isotonic solutions.

Q.32 In which of the following reactions H_2O_2 acts as a reducing agent?

(a) $H_2O_2 + 2H^+ + 2e^- \longrightarrow 2H_2O$ **(b)** $H_2O_2 - 2e^- \longrightarrow O_2 + 2H^+$

(c) $H_2O_2 + 2e^- \longrightarrow 2OH^-$ **(d)** $H_2O_2 + 2OH^- - 2e^- \longrightarrow O_2 + 2H_2O$

Ans: (b)

Sol: A reducing agent loses electrons during redox reaction
Hence (b, d) is correct.

Q.33 In S_N2 reactions, the correct order of reactivity for the following compounds : $CH_3Cl, CH_3CH_2Cl, (CH_3)_2CHCl$ and $(CH_3)_3CCl$ is:

(a) $CH_3CH_2Cl > CH_3Cl > (CH_3)_2CHCl > (CH_3)_3CCl$

(b) $(CH_3)_2CHCl > CH_3CH_2Cl > CH_3Cl > (CH_3)_3CCl$

(c) $CH_3Cl > (CH_3)_2CHCl > CH_3CH_2Cl > (CH_3)_3CCl$

(d) $CH_3Cl > CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3CCl$

Ans: (d)

Sol: Rate of S_N2 reaction $\alpha \dfrac{1}{\text{steric over crowding in transition state}}$

Q.34 The octahedral complex of a metal ion M3+ with four monodentate ligands L_1, L_2, L_3 and L_4 absorb wavelenghs in the region of red, green, yellow and blue, respectively. The increasing order of ligand strength of four ligands is :

(a) $L_3 < L_2 < L_4 < L_1$ **(b)** $L_1 < L_2 < L_4 < L_3$

(c) $L_4 < L_3 < L_2 < L_1$ **(d)** $L_1 < L_3 < L_2 < L_4$

Ans: (d)

Sol: Strong field ligands cause higher magnitude of crystal field splitting which is accompanied by the absorption of higher energy radiation, $\xrightarrow{\text{VIBGYOR}}$ decreasing energy.

Q.35 **For the estimation of nitrogen, 1.4 g of organic compound was digested by Kjeldahl method and the evolved ammonia was absorbed in 60 mL of $\frac{M}{10}$ sulphric acid. The unreacted acid required 20 ml of $\frac{M}{10}$ sodium hydroxide for complete neutralization. The percentage of nitrogen in the compound is :**

(a) 3% **(b) 5%** **(c) 6%** **(d) 10%**

Ans: (d)

Sol: $\% \text{ of N} = \dfrac{1.4 \times \text{milliequivalents of acid consumed}}{\text{mass of organic compound}}$

$$\text{Meq of acid consumed} = \left(60 \times \frac{1}{10} \times 2\right) - \left(20 \times \frac{1}{10} \times 1\right)$$

$$= 10$$

$$\therefore \% \text{ of N} = \frac{1.4 \times 10}{1.4} = 10\%$$

Q.36 **The equivalent conductance of NaCl at concentration C and at infinte are λ_C and λ_∞ respectively The correct relationship between s λ_C and λ_∞ is given as: (where the constant B is positive)**

(a) $\lambda_C = \lambda_\infty - (B)\sqrt{C}$ **(b)** $\lambda_C = \lambda_\infty + (B)\sqrt{C}$

(c) $\lambda_C = \lambda_\infty - (B)C$ **(d)** $\lambda_C = \lambda_\infty - (B)C$

Ans: (a)

Sol: According to Debye Huckel's Theory for a strong electrolyte.

$$\lambda_C = \lambda_\infty - (B)\sqrt{C}$$

Q.37 **For the reaction, $SO_{2(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons SO_{3(g)}$, if $K_p = K_C(RT)^x$ where the symbols have usual meaning then the value of x is : (assuming ideality)**

(a) $\frac{1}{2}$ **(b) 1** **(c) –1** **(d)** $-\frac{1}{2}$

Ans: (d)

Sol: For reaction,

$$SO_{2(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons SO_{3(g)}$$

$$\Delta n_g = -\frac{1}{2} = x$$

Q.38 **In the reaction,**

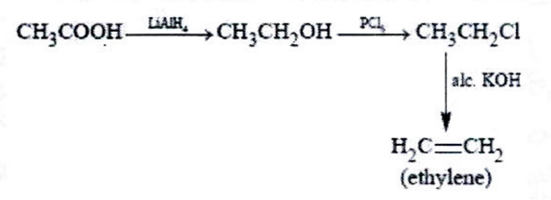
$$CH_3COOH \xrightarrow{LiAlH_4} A \xrightarrow{PCl_5} B \xrightarrow{alc.KOH} C.$$ **the product c is :**

(a) Ethylene **(b) Acetyl chloride**

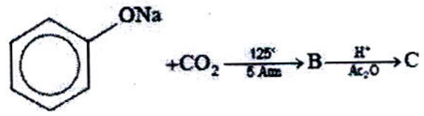
(c) Acetaldehyde **(d) Acetylene**

Ans: (a)

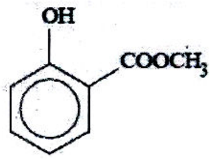
Sol:

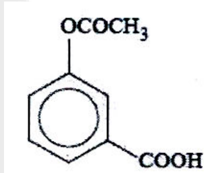


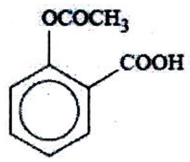
Q.39 **Sodium phenoxide when headted with CO_2 under pressure at 125 °C yields a product which on acetylation produces C.**

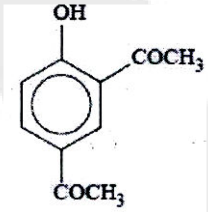


The major product C would be :

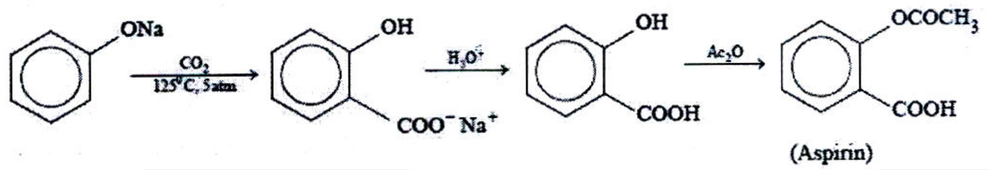
(a) 

(b) 

(c) 

(d) 

Ans: (c)

Sol: 

Q.40 **On heating an aliphatic primary amine with chloroform and ethanolic potassium hydroxide, the organic compound formed is :**

(a) an alkyl cyanide **(b) an alkayl isocyanide**

(c) an alkanol **(d) an alkanediol**

Ans: (b)

Sol: $RNH_2 + CHCl_3 + 3\underset{alc}{KOH} \longrightarrow \underset{\text{(alkyl isocyanide)}}{RNC} \uparrow + 3KCl + 3H_2O$

Q.41 **The correct statement for the molecule, CsI_3 is :**

(a) it contains Cs^{3+} and I^- ions.

(b) it contains Cs^{+}, I^- and lattice I_2 molecule.

(c) it is a covalent molecule.

(d) it contains Cs+ and I_3^-

Ans: (d)

Sol: $\Rightarrow$ Cs cannot show +3 oxidation state.
$\Rightarrow$ I_2 molecules are too large to be a accommodated in lattice.

Q.42 The equation which is balanced and represents the correct prosuct (s) is :

(a) $\left[Mg(H_2O)_8\right]^{2+} + (EDTA)^{4-} \xrightarrow{\text{excess NaOH}} \left[Mg(EDTA)\right]^{2+} + 6H_2O$

(b) $CuSO_4 + 4KCN \longrightarrow K_2\left[Cu(CN)_4\right] + K_2SO_4$

(c) $Li_2O + 2KCl \longrightarrow 2LiCl + K_2O$

(d) $\left[CoCl(NH_3)_5\right]^{+} + 5H^{+} \longrightarrow Co^{2+} + 5NH_4^{+} + Cl^{-}$

Ans: (d)

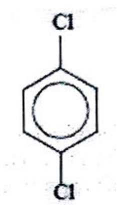
Sol: Equation - 1 is not balanced w.r.t. charge.

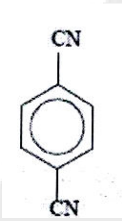
Equation - 2 gives $K_3[Cu(CN)_4]$ as product .

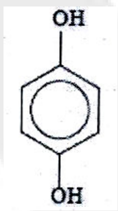
Equation - 3 reaction is unfavourable in the forward direction (K_2O is unstable while Li_2O is stable)

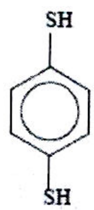
Equation - 4 is correct & balanced.

Q.43 For which of the following molecule significant $\mu \neq 0$?

(a) 

(b) 

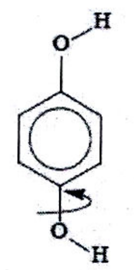
(c) 

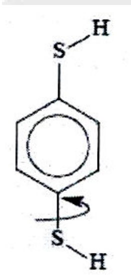
(d) 

(a) Only (c) **(b) (c) and (d)** **(c) Only (a)** **(d) (a) and (b)**

Ans: (b)

Sol:





During to infinte possible condormations in the above cases (of which only one has zero μ): a weighted μ will finally exist.

Q.44 For the non - stoichiometric reaction $2A + B \longrightarrow C + D$ the following kinetic data were obtained in three separate experiments all at 298 K.

Initial concentration (A)	Initial concentration (B)	Initial rate of formation of $C(mol\,L^{-}S^{-})$
0.1 M	0.1 M	1.2×10^{-3}
0.1 M	0.2 M	1.2×10^{-3}
0.2 M	0.1 M	2.4×10^{-3}

The rate law for the formation of C is :

(a) $\frac{dc}{dt} = k[A][B]^2$ (b) $\frac{dc}{dt} = k[A]$

(c) $\frac{dc}{dt} = k[A][B]$ (d) $\frac{dc}{dt} = k[A]^2[B]$

Ans: (b)

Sol: $R = k[A]^x[B]^y$

$1.2\times10^{-3} = k[0.1]^x[0.1]^y$

$1.2\times10^{-3} = k\times[0.1]^x[0.2]^y$

$2.4\times10^{-3} = k[0.2]^x[0.1]^y$

Solving $x = 1,\ y = 0$

$R = k[A]$

Q.45 Which series of reactions correctly repersents chemical reactions related to ion and its compound?

(a) $Fe \xrightarrow{Cl_2,heat} FeCl_3 \xrightarrow{heat,air} FeCl_2 \xrightarrow{Zn} Fe$

(b) $Fe \xrightarrow{O_2,heat} Fe_3O_4 \xrightarrow{CO,600^oC} FeO \xrightarrow{CO,700^oC} Fe$

(c) $Fe \xrightarrow{dil\,H_2SO_4} FeSO_4 \xrightarrow{H_2SO_4,O_2} Fe_2(SO_4)_3 \xrightarrow{heat} Fe$

(d) $Fe \xrightarrow{O_2,heat} FeO \xrightarrow{dil\,H_2SO_4} FeSO_4 \xrightarrow{heat} Fe$

Ans: (b)

Sol: In Eq. (1) $FeCl_3$ cannot be reduced when heated in air.
In Eq. (3) $Fe_2(SO_4)_3$ cannot convert to Fe on heating instead oxide (s) will be formed.
In Eq. (4) $FeSO_4$ cannot be converted to Fe on heating to Fe on heating; instead oxide(s) will be formed.
Hence Eq. (2) is correct.

Q.46 The osmotic pressure of blood is 7.65 atm at 37° C. How much glucose should be used per litre for an intravenous injection that is to have the same osmotic pressure as blood ?

Sol: $n = \frac{\pi V}{RT}$

$= \frac{(7.65\text{ atm})(1.00\text{L})}{(0.0821\text{ L atm / mol k})(310\text{ K})} = 0.301\text{ Mol}$

Mass of glucose $= (0.3\text{ mol})\times(180\text{g / mol}) = 54.2\text{ g}$

Q.47 A system is provided 50 J of heat and work done on the system is 10 J. The change in internal energy during the process is :

Sol: 60

Q.48 What is the maximum mass (in grams) of NO that could be obtained from 15.5 g of N_2 O_4 and 4.68 g of N_2 H_4 when they react ? tha balenced quation is______?

Sol: The stoichiometric mass ratio of $\frac{N_2O_4}{N_2H_4} = 5.75$

Supplied mass ratio of $\frac{N_2O_4}{N_2H_4} = \frac{15.5}{4.68} = 3.3 < 5.75$

$\Rightarrow$ N_2O_4 is the limiting reactant.

$\therefore$ 184 g N_2O_4 gives 180 g NO

$\therefore$ 15.5 g N_2O_4 will gives $\frac{180}{184} \times 18.5 = 15.2\,\text{g NO}$

Q.49 What is likely to be principal quantum number for a circular orbit of diameter 20 nm of the hydrogen atom if we assume Bohr orbit to be the same as that represented by the principal quantum number?

Sol: Radius $= 0.529 \frac{n^2}{z} \overset{o}{A} = 10 \times 10^{-9}\,\text{m}$

So $n^2 = 189$ or $n \approx 14$

Q.50 What volume of 6.0 M H_2SO_4 should be mixed with 10 L of 1.0 M H_2SO_4 to make 20.0 L of 3.0 M H_2SO_4 upon dilution to volume ?

Sol: apply $M_1 V_1 + M_2 V_2 = M_3 V_3$

$6V_1 + 10 \times 1 = 3 \times 20 \Rightarrow V_1 = \frac{50}{6} = 8.3$ L

Part - C - MATHEMATICS

Q.51 The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line

(a) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

(b) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$

(c) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$

(d) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

Ans: (a)

Sol: Line is parallel to plane

Image of (1, 3, 4) is $(-3, 5, 2)$

Q.52 If the coefficients of x^3 and x^4 in the expansion of in powers of x are both zero, then (a, b) is equal to

(a) $\left(6, \frac{251}{3}\right)$ (b) $\left(14, \frac{251}{3}\right)$ (c) $\left(14 \frac{272}{3}\right)$ (d) $\left(16, \frac{272}{3}\right)$

Ans: (d)

Sol: $1(1-2x)^{18} + ax(1-2x)^{18} + bx^2(1-2x)^{18}$

Coefficient of $x^3 : (-2)^3\, {}^{18}C_3 + a(-2)^2\, C_2 + b(-2)\, {}^{18}C_1 = 0$

$$\frac{4\times(17\times16)}{(3\times2)}-2a.\frac{17}{2}+b=0 \qquad \text{...(i)}$$

Coefficient of $x^4 : (-2)^4\ {}^{18}C_4 + a(-2)^3\ {}^{18}C_3 + b(-2)^2\ {}^{18}C_1 = 0$

$$(4\times20)-2a.\frac{16}{3}+b=0 \qquad \text{....(ii)}$$

From equation (i) and (ii), we get

$$4\left(\frac{17\times8}{3}-20\right)+2a\left(\frac{16}{3}-\frac{17}{2}\right)=0$$

$$4\left(\frac{17\times8-60}{3}\right)+\frac{2a(-19)}{6}=0$$

$$a=\frac{4\times76\times6}{3\times2\times19}$$

$$\Rightarrow a=16$$

$$\Rightarrow b=\frac{2\times16\times16}{3}-80=\frac{272}{3}$$

Q.53 If $a\in R$ and the equation $-3(x-[x])^2+2(x-[x])+a^2=0$ (where [x] denotes the greatest integer integer $\leq x$) has no integral solution, then all possible values of a lie in the interval

(a) $(-1,0)\cup(0,1)$ (b) (1, 2) (c) $(-2,-1)$ (d) $(-\infty,-2)\cup(2,\infty)$

Ans: (a)

Sol: $a^2=3t^2-2t$

For non-integral solution

$0<a^2<1$

$a\in(-1,0)\cup(0,1)$

Q.54 If $[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}]=\lambda\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ then is equal to

(a) 2 (b) 3 (c) 0 (d) 1

Ans: (d)

Sol: $[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}]=\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$

$\lambda=1.$

Q.55 A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45o. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30o. Then the speed (in m/s) of the bird is

(a) $40\left(\sqrt{2}-1\right)$ (b) $40\left(\sqrt{3}-\sqrt{2}\right)$ (c) $20\sqrt{2}$ (d) $20\left(\sqrt{3}-1\right)$

Ans: (d)

Sol: $\tan 30^0 = \frac{20}{20+x} = \frac{1}{\sqrt{3}}$

$20 + x = 20\sqrt{3}$

$x = 20\left(\sqrt{3}-1\right)$

$\Rightarrow$ 20 speed is $20\left(\sqrt{3}-1\right)$m/sec

Q.56 The statement $\sim(P \leftrightarrow \sim q)$ is

(a) equivalent to $p \leftrightarrow q$ **(b) equivalent to $\sim P \leftrightarrow q$**

(c) a tatutology **(d) a fallacy**

Ans: (a)

Sol:

P	q	$\sim q$	$P \leftrightarrow \sim q$	$\sim(P \leftrightarrow \sim q)$	$P \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Q.57 The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to

(a) $(x-1)e^{x+\frac{1}{x}}+c$ **(b) $xe^{x+\frac{1}{x}}+c$**

(c) $(x+1)e^{x+\frac{1}{x}}+c$ **(d) $-xe^{x+\frac{1}{x}}+x$**

Ans: (b)

Sol: $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$

$= \int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx + \int x\left(1-\frac{1}{x^2}\right)e^{\left(x+\frac{1}{x}\right)}dx$

$= \int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx + xe^{\left(x+\frac{1}{x}\right)} - \int e^{\left(x+\frac{1}{x}\right)}dx$

$= xe^{\left(x+\frac{1}{x}\right)} + c$

Q.58 If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z+\frac{1}{2}\right|$

(a) is equal to $\frac{5}{2}$ **(b) lies in the interval (1, 2)**

(c) is strictly greater than $\frac{5}{2}$ **(d) is strictly grater than $\frac{3}{2}$ but less than $\frac{5}{2}$**

Ans: (b)

Sol: $|z| \geq 2$

$$\left|z+\frac{1}{2}\right| \geq \left||z|-\left|\frac{1}{2}\right|\right| \geq \left|2-\frac{1}{2}\right| \geq \frac{3}{2}.$$

Hence, minimum distance between z and $\left(-\frac{1}{2},0\right)$ is $\frac{3}{2}$

Q.59 If A is an non-singular matrix such that AA' = A'A and B = $A^{-1}A'$. then BB' equals

(a) I + B **(b) I** **(c) B^{-1}** **(d) $(B^{-1})'$**

Ans: (b)

Sol: $B = A^{-1}A' \Rightarrow AB = A'$

$ABB' = A'B' = (BA)' = (A^{-1}A'A)' = (A^{-1}AA') = A.$

$\Rightarrow BB' = I.$

Q.60 If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$.then $g'(x)$

(a) $1+x^5$ **(b) $5x^4$** **(c) $\frac{1}{1+\{g(x)\}^5}$** **(d) $1+\{g(x)\}^5$**

Ans: (d)

Sol: $f(g(x)) = x$

$f'(g(x))g'(x) = 1$

$g'(x) = 1+(g(x))^5$

Q.61 If $\alpha, \beta \neq 0$, and

$f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$ and then K is equal to

(a) $\alpha\beta$ **(b) $\frac{1}{\alpha\beta}$** **(c) 1** **(d) -1**

Ans: (c)

Sol:

$$\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \alpha-1 & \alpha \\ 1 & \alpha^2-1 & \beta^2-1 \end{vmatrix}$$

$= ((\alpha-1)(\beta^2-1)-(\beta-1)(\alpha^2-1))^2$

$= (\alpha-1)^2(\beta-1)^2(\alpha-\beta)^2 \Rightarrow k = 1$

Q.62 Let $f_k(x) = \dfrac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals

(a) $\frac{1}{6}$ **(b) $\frac{1}{3}$** **(c) $\frac{1}{4}$** **(d) $\frac{1}{12}$**

Ans: (d)

Sol: $\frac{1}{4}\left(\sin^4 x + \cos^4 x\right) - \frac{1}{6}\left(\sin^6 x + \cos^6 x\right)$

$= \dfrac{3\left(\sin^4 x + \cos^4 x\right) - 2\left(\sin^6 x + \cos^6 x\right)}{12}$

$= \dfrac{3\left(1 - 2\sin^2 x\cos^2 x\right) - 2\left(1 - 3\sin^2 x\cos^2 x\right)}{12}$

$= \dfrac{1}{12}$

Q.63 Let α and β be the roots of equation $px^2 + qx + r =, p \neq 0.$ are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is

(a) $\frac{\sqrt{61}}{9}$ **(b) $\frac{2\sqrt{17}}{9}$** **(c) $\frac{\sqrt{34}}{9}$** **(d) $\frac{2\sqrt{13}}{9}$**

Ans: (d)

Sol: $\frac{1}{\alpha} + \frac{1}{\beta} = 4$

$2q = p + r$

$\Rightarrow -2(\alpha + \beta) = 1 + \alpha\beta$

$\Rightarrow -2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \frac{1}{\alpha\beta} + 1$

$\Rightarrow \frac{1}{\alpha\beta} = -9$

Equation having roots α, β is $9x^2 + 4x - 1 = 0$

$\alpha, \beta = \dfrac{-4 \pm \sqrt{16 + 36}}{2 \times 9}$

$|\alpha - \beta| = \dfrac{2\sqrt{13}}{9}$

Q.64 Let the population of rabbits surviving at *a* time *t* be governed by the differential equation $\frac{dp(1)}{dt}=\frac{1}{2}p(t)-200$. If $p(0)=100$, then $p(f)$equals

(a) $400-300e^{t/2}$ **(b)** $300-200e^{-t/2}$ **(c)** $600-500e^{t/2}$ **(d)** $400-300e^{-t/2}$

Ans: (a)

Sol: $\frac{dp}{dt}=\frac{p-400}{2}$

$\frac{dp}{p-400}=\frac{1}{2}dt$

$In\left|p-400\right|=\frac{1}{2}t+c$

at $t=0, p=100$

In 300 = c

$In\left|\frac{p-400}{300}\right|=\frac{t}{2}$

$\Rightarrow \left|p-400\right|=300e^{t/2}$

$\Rightarrow 400-p=300e^{e/2}(\text{as } p<400)$

$\Rightarrow p=400-300e^{t/2}$

Q.65 Let C be the circle with centre at (1, 1) and radius - 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to

(a) $\frac{\sqrt{3}}{\sqrt{2}}$ **(b)** $\frac{\sqrt{3}}{2}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{1}{4}$

Ans: (d)

Sol: According to the figure

$(1+y)^2=(1-y)^2+1(y>0)$

$\Rightarrow y=\frac{1}{4}$

Q.66 The area of the region described by $A=\left\{(x,y): x^2+y^2\le 1 \text{ and } y^2\le 1-x\right\}$ is

(a) $\frac{\pi}{2}+\frac{4}{3}$ **(b)** $\frac{\pi}{2}-\frac{4}{3}$ **(c)** $\frac{\pi}{2}-\frac{2}{3}$ **(d)** $\frac{\pi}{2}+\frac{2}{3}$

Ans: (a)

Sol: $A=\frac{1}{2}\times\pi+2\int_0^1\sqrt{1-xdx}$

$=\frac{\pi}{2}+\frac{4}{3}.$

Q.67 Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax+2ay+c=0$ and $5bx+2by+d=0$ lies in the fourth quadrant and its equidistant from the two axes then

(a) $2bc-3ad=0$ **(b)** $2bc+3ad=0$

(c) $3bc-2ad=0$ **(d)** $3bc+2ad=0$

Ans: (c)

Sol: Let point of intersection is $(h,-h)$

$$\Rightarrow \begin{cases} 4ax+2ay+c=0 \\ 5bx+2by+d=0 \end{cases}$$

$$\text{So}, -\frac{c}{2a}=\frac{d}{3b}$$

$$3bc-2ad=0$$

Q.68 Let PS be the median of the tringle with vertices $P(2,2)$. $Q(6,-1)$ and $R(7,3)$. The equation of the line passing through $(1-1)$ and parallel to PS is

(a) $4x-7y-11=0$ **(b)** $2x+9y+7=0$

(c) $4x+7y+3=0$ **(d)** $2x-9y-11=0$

Ans: (b)

Sol: $S\left(\frac{13}{2},1\right), P(2,2)$

$$\text{Slope}=-\frac{2}{9}$$

$$\text{Equation will be } \frac{y+1}{x-1}=-\frac{2}{9}$$

$$9y+9+2x-2=0$$

$$2x+9y+7=0$$

Q.69 $\lim\limits_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$ **is equal to**

(a) $\frac{\pi}{2}$ **(b)** 1 **(c)** $-\pi$ **(d)** π

Ans: (d)

Sol: $\lim\limits_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$

$$=\lim_{x\to 0}\frac{\sin(\pi-\pi\sin^2 x)}{x^2}$$

$$=\lim_{x\to 0}\frac{\sin(\pi\sin^2 x)}{\pi\sin^2 x}\times\frac{\pi\sin^2 x}{x}=\pi$$

Q.70 If $X = \{4^n - 3n - 1 : n \in N]$ and $Y = [9(n-1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to

(a) N **(b) $Y - X$** **(c) X** **(d) Y**

Ans: (d)

Sol: Set X contains elements of the form

$4^n - 3n - 1 = (1+3)^n - 3n - 1$

$= 3^n + {}^nC_{n-1}3^{n-1} \ldots\ldots {}^nC_2 3^1$

$= 9(3^{n-2} + {}^nC_{n-1}3^{n-1} \ldots\ldots + {}^nC_2)$

Set X has natural numbers which are multiples of 9 (not all)

Set Y has all multiples of 9

$X \cup Y = Y$

Q.71 A five digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4 and 5 without repetation. The total number of ways this can be done is _____?

Sol: Since, a five digit number is formed by using the digits, 0, 1, 2, 3, 4 and 5, divisible by 3 i.e. only possible when sum of digit is multiple of three which gives two cases.

Cases I using digit 0, 1, 2, 4, 5 the number of

ways $= 4 \times 4 \times 3 \times 2 \times 1 = 96$.

Case II Using digit 1, 2, 3, 4, 5 the number of

ways $= 5 \times 4 \times 3 \times 2 \times 1 = 120$.

$\therefore$ Total number formed $= 120 + 96 = 216$.

Q.72 The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \cos^2 t\, dt}{x \sin x}$ is _____?

Sol: $\lim_{x \to 0} \frac{\int_0^{x^2} \cos^2 t\, dt}{x \sin x}$

Using L' Hospital rule,

$$= \lim_{x \to 0} \frac{\cos^2(x^2) \cdot 2x - 0}{x \cos x + \sin x}$$

$$= \lim_{x \to 0} \frac{2\cos^2(x^2)}{\cos x + \frac{\sin x}{x}}$$

$$= \frac{\lim_{x \to 0} 2\cos^2(x^2)}{\lim_{x \to 0}\left[\cos x + \frac{\sin x}{x}\right]} = \frac{2}{2} = 1$$

Q.73 If f (x) = $\begin{cases} 1 + |x|, x < 1 \\ \lfloor |x| \rfloor, \quad x \geq -1 \end{cases}$, where $[\cdot]$ denotes the greatest integer function, Then f (f (-3, 3) is_____?

Sol: f (f (-3.3)) = f (1 + | - 3.3 |) = f (4.3) = [4.3] = 4

Q.74 The greatest value of a non-vegative real number λ for which both the equations $2x^2$ + (λ - 1)x + 8 = 0 and x^2 - 8x + λ + 4 = 0 have real roots is _____?

Sol: Roots of the equation $Zx^2 + (\lambda - 1)x + 8 = 0$ are real

$\Rightarrow (\lambda - 1)^2 - 64 \geq 0$
$\Rightarrow \lambda - 2\lambda - 63 \geq 0$
$\Rightarrow (\lambda - 9)(\lambda + 7) \geq 0$
$\Rightarrow \lambda \leq -7$ or $\lambda \geq 9$

Roots of the equation x2 - 8x + λ + 4 = 0

$\Rightarrow 64 - (\lambda + 4)^2 \geq 0$

$\Rightarrow \lambda^2 - 8\lambda - 48 \leq 0$

$\Rightarrow -4 \leq \lambda \leq 12.$

$\Rightarrow -4 \leq \lambda \leq 12$

From (i) and (ii), we have $9 \leq \lambda \leq 12$

Hence the greatest value of λ is 12.

Q.75 The number of integral values of m, for which the x-coordinate of the point of intersection of the lines 3x + 4y = 9 and y = mx + 1 is also integer is _____?

Sol: Solving 3x + 4y = 9, y = mx + 1, we $x = \frac{5}{3 + 4m}x$ get is an integer if 3 + 4 m = 1, -1, 5, -5

$\therefore \quad m = \frac{-2}{4}, \frac{-4}{4}, \frac{2}{4}, \frac{-8}{4}$

So, m has 2 integral values.

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ROUGH WORK

ROUGH WORK