A Division of Aggarwal Educare
Aggarwal CLASSES™

JEE (MAIN)

TEST PAPER

SUBJECT : PHYSICS,CHEMISTRY, MATHEMATICS ***TEST CODE : TSJMT212***

ANSWER PAPER

TIME : 3 HRS ***MARKS : 300***

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark(-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or “un- darken” a darkened bubble.

Part A - PHYSICS

Q.1 From the options given, the most likely reason to maintain the national standard of time interval second through four cesium atomic clocks is
(a) having only 1 clock reduces precision.
(b) having only 1 clock reduces accuracy.
(c) having only 1 clock reduces accuracy and precision.
(d) having four clocks reduce errors due to random electronic transitions.

Ans: (B)

Sol: Countries maintain a network of atomic clocks which are inter compared and kept synchronized to maintain an accuracy of 10^{-9} sec per day. These clocks collectively define a continous and stable time scale.

Q.2 The vector sum of two velocities is perpendicular to their vector differences. In this case, the velocities
(a) are equal to each other in magnitude.
(b) are not equal to each other in magnitude.
(c) are at an angle of 90^0 with each other.
(d) are equal to each other in direction.

Ans: (A)

Sol: Let the two velocities be $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$

$\therefore$ according to given condition we have,

$$\left(\overrightarrow{v_1}+\overrightarrow{v_2}\right)\cdot\left(\overrightarrow{v_1}-\overrightarrow{v_2}\right)=0 \qquad \text{...(orthogonality condition)}$$

$$\therefore \left(\overrightarrow{v_1}\cdot\overrightarrow{v_1}\right)-\left(\overrightarrow{v_1}\cdot\overrightarrow{v_2}\right)+\left(\overrightarrow{v_2}\cdot\overrightarrow{v_1}\right)-\left(\overrightarrow{v_2}\cdot\overrightarrow{v_2}\right)=0$$

$$\therefore \left(\overrightarrow{v_1}\cdot\overrightarrow{v_1}\right)-\left(\overrightarrow{v_2}\cdot\overrightarrow{v_2}\right)=0$$

$$\therefore \left(\overrightarrow{v_1}\cdot\overrightarrow{v_1}\right)=\left(\overrightarrow{v_2}\cdot\overrightarrow{v_2}\right)$$

$$\therefore v_1^2\cos\theta=v_2^2\cos\theta$$

$$\therefore v_1^2=v_2^2$$

$$\therefore v_1=v_2$$

Q.3 A body is thrown vertically upwards. If air resistance is to be taken into account, then the time during which the body rises is
(a) equal to the time of fall.
(b) less than the time of fall.
(c) greater than the time of fall.
(d) twice the time of fall.

Ans: (B)

Sol: Let the initial velocity of ball be u.

Time of rise $t_1=\dfrac{u}{g+a}$ and

height reached $=\dfrac{u^2}{2(g+a)}$

Time of fall t_2 is given by,

$$\frac{1}{2}(g-a)t_2^2 = \frac{u^2}{2(g+a)}$$

$$\therefore \quad t_2 = \frac{u}{\sqrt{(g+a)(g-a)}} = \frac{u}{(g+a)}\sqrt{\frac{g+a}{g-a}}$$

$$\therefore \quad t_2 > t_1 \text{ because } (g+a) > (g-a)$$

Q.4 A body of mass M at rest explodes into three pieces, two of which of mass M/3 each, are thrown-off in perpendicular directions with velocities of 6 m/s and 8 m/s respectively. The third piecewill be thrown-off with a velocity of

(a) 15 m/s (b) 10 m/s (c) 2.8 m/s (d) 3.3 m/s

Ans: (B)

Sol: Momentum of one piece $= \frac{M}{3} \times 6$

Momentum of the other piece$= \frac{M}{3} \times 8$

$$\therefore \quad \text{Resultant momentum} = \sqrt{4M^2 + \frac{64M^2}{9}} = \frac{10M}{3}$$

The third piece should also have the same momentum. Let its velocity be v, then

$$\frac{10M}{3} = \frac{M}{3} \times v \text{ or } v = 10\,m/s$$

Q.5 What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop?

(a) $\sqrt{3gR}$ (b) $\sqrt{5gR}$ (c) $\sqrt{gR}$ (d) $\sqrt{2gR}$

Ans: (B)

Sol: ----

Q.6 A particle of mass m is driven by a machine that delivers a constant power k watt. If the particle starts from rest the force on the particle at time t is

(a) $\sqrt{\frac{mk}{2}}t^{\frac{-1}{2}}$ (b) $\sqrt{mkt}^{\frac{-1}{2}}$ (c) $\sqrt{2mkt}^{\frac{-1}{2}}$ (d) $\frac{1}{2}\sqrt{mkt}^{\frac{-1}{2}}$

Ans: (A)

Sol: $$\text{Power} = \frac{\text{Energy}}{\text{time}}$$

$$\therefore \quad P = mv\frac{dv}{dt}$$

$$F = \frac{P}{v} = m\frac{dv}{dt}$$

also, $P = k$ watt

$$\therefore \quad \frac{1}{2}\frac{mv^2}{t} = k$$

$$\therefore \quad v^2 = \frac{2kt}{m} \text{ or } v = \sqrt{\frac{2kt}{m}}$$

$$\text{Hence}, F = \frac{k}{\sqrt{\frac{2kt}{m}}} = \sqrt{\frac{mk}{2}}t^{\frac{-1}{2}}$$

Q.7 Two rotating bodies A and B of masses m and 2 m with moments of inertia I_A and $I_B(I_B>I_A)$ have equal kinetic energy of rotation. If L_A and L_B be their angular momentum respectively, then

(a) $L_A > L_B$ (b) $L_A = \frac{L_B}{2}$ (c) $L_A = 2L_B$ (d) $L_B > L_A$

Ans: (D)

Sol: K.E. of rotating body is given by,

$$K.E. = \frac{1}{2}I\omega^2$$

Here,

$$(K.E.)_A = (K.E.)_B \quad \text{....(Given)}$$

$$\therefore \quad \frac{1}{2}I_A\omega_A{}^2 = \frac{1}{2}I_B\omega_B{}^2$$

As $I_B > I_A$,

$\omega_B < \omega_A$

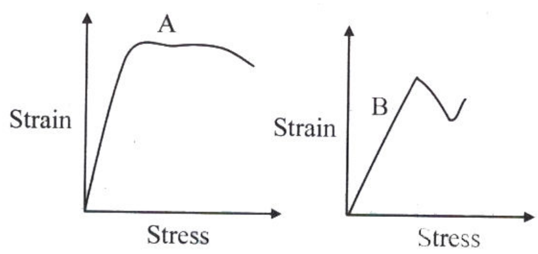
$$\text{Also, } K.E.=\frac{1}{2}L\omega \quad \text{....}(\because L = I\omega)$$

$$\therefore \quad \frac{1}{2}L_A\omega_A = \frac{1}{2}L_B\omega_B$$

$\therefore$ as $W_B > W_A$

$\therefore$ $L_B > L_A$

Q.8 The stress-strain graphs for materials A and B are shown in figure. The graphs are drawn to the same scale. Select the CORRECT statement



(a) $Y_A>Y_B$ and material A has weaker strength than material B.
(b) $Y_A<Y_B$ and material A has stronger strength than material B.
(c) $Y_A<Y_B$ and material A has weaker strength than material B.
(d) $Y_A>Y_B$ and material A has stronger strength than material B.

Ans: (D)

Sol: Slope of graph of material 'A' is greater than that of 'B' and material 'A' can bear greater stress before the wire breaks.

Q.9 A solid spherical object of radius r and density σ is dropped in identical viscous fluid lakes on 5 different planets. On which of the planets terminal velocity

attained by the object will be maximum? (Assume value of g_{planet} doesn't change inside the lake)

(a) Planet of mass 3×10^{31} kg and radius 7×10^{3} km.

(b) Planet of mass 2×10^{32} kg and radius 14×10^{4} Km

(c) Planet of mass 2×10^{23} kg and radius 3×10^{3} Km

(d) Planet of mass 2×10^{35} kg and radius 3×10^{6} Km

Ans: (D)

Sol: $v=\dfrac{2r^2(\rho-\sigma)g}{9\eta}$

$\text{Given}: \rho \text{ and } \eta \text{ are constant}$

$\therefore v\ \alpha\, g_{planet}=\sqrt{\dfrac{GM_{planet}}{R_{planet}}}$

i.e., $v\alpha\sqrt{\dfrac{M_{planet}}{R_{planet}}}$

$\Rightarrow$ Planet in option (D) will have highest value of g_{planet}. Hence, the given object will attain maximum terminal velocity on planet in option (D).

Q.10 A reversible engine and an irreversible engine are operating between the same temperatures. The efficiency of

(a) both the engines will be 100%.

(b) reversible engine will be 100%.

(c) reversible engine will be greater.

(d) irreversible engine will be greater.

Ans: (C)

Sol: ---

Q.11 A spring of length 'L' and force constant 'k' is stretched to obtain extension 'l' .It is further stretched to obtain extension 'l_1'. The work done in second stretching is

(a) $\frac{1}{2}kl_1^2$ **(b) $\frac{1}{2}k(l^2+l_1^2)$** **(c) $\frac{1}{2}kl_1(2l+l_1)$** **(d) $\frac{1}{2}k(l_1^2 - l^2)$**

Ans: (C)

Sol: $W_1=\dfrac{1}{2}kl^2$

$W_1+W_2=\dfrac{1}{2}k(l+l_1)^2$

$W_2=(W_1+W_2)-W_1$

$=\dfrac{1}{2}k(l+l_1)^2-\dfrac{1}{2}kl^2$

$=\dfrac{1}{2}k[l^2+l_1^2+(2l)l_1-l^2]$

$=\dfrac{1}{2}k(l_1+2l)l_1$

Q.12 A transverse wave is described by the equation, $y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$. The maximum particle velocity is equal to twice the wave velocity, if

(a) $\lambda = \frac{\pi y_0}{4}$ (b) $\lambda = \frac{\pi y_0}{2}$ (c) $\lambda = \pi y_0$ (d) $\lambda = 2\pi y_0$

Ans: (C)

Sol: $y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$

$\therefore \quad \frac{dy}{dt} = y_0 2\pi f \cos 2\pi\left(ft - \frac{x}{\lambda}\right)$

$\therefore \quad \left|\frac{dy}{dt}\right|_{max} = y_0 2\pi f \qquad ...\left[\because \text{For} \left|\frac{dy}{dt}\right|_{max}, \cos 2\pi\left(ft - \frac{x}{\lambda}\right) = 1\right]$

$\because$ Wave velocity $= f\lambda$

According to the given condition, $y_0 2\pi f = 2f\lambda$

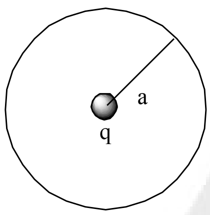
$\therefore \quad \lambda = \pi y_0$

Q.13 The electric field in a certain region is acting radially outward and is given by E = Ar. A charge contained in a sphere of radius 'a' centred at the origin of the field, will be given by

(a) $4\pi\varepsilon_0 Aa^2$ (b) $A\varepsilon_0 a^2$ (c) $4\pi\varepsilon_0 Aa^3$ (d) $\varepsilon_0 Aa^3$

Ans: (C)

Sol:



Let charge enclosed in the sphere of radius a be q. According to Gauss' theorem.

$\oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$

$E(4\pi r^2) = \frac{q}{\varepsilon_0}$

$4\pi Ar^3 = \frac{q}{\varepsilon_0} \qquad ....(\because E = Ar)$

$\therefore q = 4\pi\varepsilon_0 Aa^3 \qquad ....(\because r = a)$

Q.14 Which of the following characteristics of electrons determines the current in a conductor?
(a) Drift velocity alone.
(b) Thermal velocity alone.
(c) Both drift velocity and thermal velocity.
(d) Neither drift nor thermal velocity.

Ans: (A)

Sol: ---

Q.15 A moving coil sensitive galvanometer gives at once much more deflection. To control its speed of deflection
(a) a small copper wire should be connected across its terminals.
(b) a high resisitance is to be connected across its terminals
(c) a magnet should be placed near the coil.
(d) the body of galvanometer should be earthed.

Ans: (C)

Sol: Magnet provides damping.

Q.16 The power factor of a CR circuit is $\frac{1}{\sqrt{2}}$ .If the frequency of ac signal is halved, then the power factor of the circuit becomes

(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{1}{\sqrt{7}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{11}}$

Ans: (A)

Sol: For CR circuit, power factor is given by

$$\cos\phi = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{R}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}}$$

$$\therefore \quad (\cos\phi)_1 = \frac{R}{\sqrt{R^2 + \frac{1}{(\omega_1 C)^2}}} \quad ...(i)$$

$$\therefore \quad \frac{1}{\sqrt{2}} = \frac{R}{\sqrt{R^2 + \frac{1}{(\omega_1 C)^2}}}$$

$$\therefore \quad \frac{1}{2} = \frac{R^2}{R^2 + \frac{1}{(\omega_1 C)^2}}$$

$$\therefore \quad R^2 + \frac{1}{(\omega_1 C)^2} = 2R^2$$

$$\therefore \quad R^2 = \frac{1}{(\omega_1 C)^2} \quad ...(ii)$$

Now,

$$(\cos\phi)_2 = \frac{R}{\sqrt{R^2 + \frac{1}{(\omega_2 C)^2}}}$$

$$\text{But}, \omega_2 = \frac{\omega_1}{2}$$

$\therefore \quad (\cos\phi)_2 = \dfrac{R}{\sqrt{R^2 + \dfrac{4}{(\omega_1 C)^2}}}$ ...(iii)

Dividing equation (iii) by equation (i),

$$\frac{(\cos\phi)_2}{(\cos\phi)_1} = \frac{R}{\sqrt{R^2 + \dfrac{4}{(\omega_1 C)^2}}} \times \frac{\sqrt{R^2 + \dfrac{1}{(\omega_1 C)^2}}}{R}$$

$$\therefore \quad (\cos\phi)_2 = \cos\phi \times \sqrt{\frac{R^2 + \dfrac{1}{(\omega_1 C)^2}}{R^2 + \dfrac{4}{(\omega_1 C)^2}}}$$

Using equation (ii),

$$(\cos\phi)_2 = \frac{1}{\sqrt{2}}\sqrt{\frac{R^2 + R^2}{R^2 + 4R^2}}$$

$$= \frac{1}{\sqrt{2}}\sqrt{\frac{2R^2}{5R^2}}$$

$$\therefore \quad (\cos\phi)_2 = \frac{1}{\sqrt{5}}$$

Q.17 A plane polarised light is incident normally on a tourmaline plate. Its E vectors make an angle of 60^0 with the optic axis of the plate. Find the percentage difference between initial and final intensities.

(a) 25% **(b) 50%** **(c) 75%** **(d) 90%**

Ans: (C)

Sol: Using law of Malus,

$I_2 = I_1 \cos^2\theta$

$\therefore \; I_2 = I_1 \cos^2 60^0$

$\therefore \; I_2 = I_1 \left(\dfrac{1}{2}\right)^2 = \dfrac{I_1}{4} = 0.25\ I_1$

Hence intensity becomes 25% of initial i.e., it decreases by 75%.

Q.18 Order of magnitude of density of thorium nucleus is ($m_p = 1.67 \times 10^{-27}$ kg)

(a) 10^{20}kg/m^3 **(b) 10^{17}kg/m^3** **(c) 10^{14}kg/m^3** **(d) 10^{11}kg/m^3**

Ans: (B)

Sol: The order of magnitude of mass and volume of thorium nucleus will be

$m \approx A(1.67 \times 10^{-27}\,\text{kg})$ (A is atomic number)

$$V = \frac{4}{3}\pi r^3 \approx \frac{4}{3}\pi[(1.25 \times 10^{-15}\,\text{m})A^{1/3}]^3$$

$$\approx (8.2 \times 10^{-45}\,\text{m}^3)A$$

Hence, $\rho = \frac{m}{V} = \frac{A(1.67\times10^{-27}\,kg)}{(8.2\times10^{-45}\,m^3)A}$

$\approx 2.0\times10^{17}\,kg/m^3$

Q.19 When only carrier is transmitted, antenna current observed is 10 A. When it is modulated with 500 Hz sine wave antenna, current becomes 12A. Find % modulation.

(a) 70% **(b) 62%** **(c) 94%** **(d) 84%**

Ans: (C)

Sol: $\left(\frac{I_t}{I_c}\right) = \sqrt{1+\frac{m_a^2}{2}}$

Given : $I_t = 12A$ and $I_c = 10A$

$\therefore m_a = \sqrt{2\left[\left(\frac{I_t}{I_c}\right)^2 - 1\right]} = \sqrt{2[(1.2)^2 - 1]}$

$= 0.9380 = 94\%$

Q.20 For CE transitors amplifiers, the audio signal voltage across the collector resistance of $2\,k\Omega$ is 4V. If the current amplification factor of the transistor is 100 and the base resistance is 1 $k\Omega$, then the input signal voltage is

(a) 15mV **(b) 10 mV** **(c) 20 mV** **(d) 30 mV**

Ans: (C)

Sol: Given,

$R_{in} = R_B = 1k\Omega$

$R_{out} = R_C = 2k\Omega$

$V_{out} = 4V$

$\beta = 100$

We know,

$A_v = \beta \times \text{resistance gain}$

$\therefore A_v = \beta\times\frac{R_C}{R_B} = 100\times\frac{2k\Omega}{1k\Omega} = 200$

Also, $A_v = \frac{V_{out}}{V_{in}}$

$\therefore \frac{V_{out}}{V_{in}} = 200$

$\therefore \frac{4}{V_{in}} = 200$

$\therefore V_{in} = \frac{4}{200} = 20\text{ mV}$

Q.21 The de Broglie wavelength of an electron accelerated to a potential of 400 V is approximately?

Sol: $\lambda = \frac{12.27}{\sqrt{V}} \overset{0}{A}$

$= \frac{1.227 \times 10^{-9}}{\sqrt{400}}$

$= 0.061 \times 10^{-9}\, m$

$= 0.06 nm$

Q.22 A thin plastic sheet of refractive index 1.6 is used to cover one of the slits of a double slit arrangement. The central point on the screen is now occupied by what would have been the 7^{th} bright fringe before the plastic was used. If the wavelength of light is 600 nm, what is the thickness (in μm) of the plastic?

Sol: Fringe shift is given by

$S = \frac{D(\mu-1)t}{d} = \frac{\beta(\mu-1)t}{\lambda}$

$\because$ the central bright point has shifted to 7^{th} bright fringe after plastic she was used,

$\therefore S = 7\beta$

$\therefore 7\beta = \frac{\beta(\mu-1)t}{\lambda}$

$\therefore t = \frac{7\lambda}{(\mu-1)}$

$\therefore t = \frac{7 \times 600 \times 10^{-9}}{(1.6-1)}$

$\therefore t = 7 \mu m$

Q.23 A 20 cm long bar magnet is placed in magnetic meridian with its north pole pointing south. The neutral point is observed at a distance of 30 cm from its centre. Then the pole strength of the magnet is [Horizontal component of earth's field is 0.3 G.]

Sol: As $B = B_H$

$\frac{\mu_0}{4\pi} \frac{2Mr}{(r^2 - l^2)^2} = B_H$

$M = \frac{B_H (r^2 - l^2)^2}{2r(\mu_0 / 4\pi)}$

$= \frac{0.3 \times 10^{-4} (0.3^2 - 0.1^2)^2}{2 \times 0.3 \times 10^{-7}}$

$\therefore M = 3.2\ Am^2$

$\text{Also}, m = \frac{M}{2l} = \frac{3.2}{0.2} = 16 Am$

Q.24 The excess pressure inside a soap bubble is thrice the excess pressure inside a second soap bubble. The volume of the first bubble is n times the volume of the second, where n is_____?

Sol: $P_1 = \frac{4T}{R_1}, P_2 = \frac{4T}{R_2}$ ...(i)

Now, $P_1 = 3P_2$

$\therefore \dfrac{1}{R_1} = \dfrac{3}{R_2}$ ...[from(i)]

$\therefore \dfrac{R_1}{R_2} = \dfrac{1}{3}$

Now, $V_1 = \dfrac{4}{3}\pi R_1{}^3$ and $V_2 = \dfrac{4}{3}\pi R_2{}^3$

But $V_1 = nV_2$ ....(Given)

$\therefore \quad \dfrac{4}{3}\pi R_1{}^3 = n\dfrac{4}{3}\pi R_2{}^3$

$\therefore \quad R_1{}^3 = nR_2{}^3$

$\therefore \quad n = \left(\dfrac{R_1}{R_2}\right)^3 = \left(\dfrac{1}{3}\right)^3 = \dfrac{1}{27} = 0.037$

Q.25 Two identical rods of copper and iron are coated with wax uniformly. When one end of each is kept at temperature of boiling water, the length upto which wax melts are 6.2 cm and 3.1 cm respectively. If thermal conductivity of copper is 0.9, then thermal conductivity of iron is_____?

Sol:

$$\frac{K_1}{K_2} = \frac{l_1^2}{l_2^2}$$

$$\therefore \quad K_2 = \frac{K_1 l_2^2}{l_1^2} = \frac{0.9 \times (3.1)^2}{(6.2)^2} = 0.225$$

Part - B - CHEMISTRY

Q.26 The ratio of the frequency of radiation with wavelength 400 nm to the frequency of radiation with wavlength 760 nm is ______________

(a) 3:2 (b) 2:1 (c) 3:1 (d) 4:3

Ans: (B)

Sol: $v_1 = \dfrac{c}{\lambda_1} = \dfrac{3\times10^8\,ms^{-1}}{400\times10^{-9}\,m}$

$= 7.5\times10^{14}\,s^{-1}$

$v_2 = \dfrac{c}{\lambda_2} = \dfrac{3\times10^8\,ms^{-1}}{760\times10^{-9}\,m}$

$= 3.95\times10^{14}\,s^{-1}$

Ratio $\dfrac{v_1}{v_2} = \dfrac{7.5\times10^{14}\,s^{-1}}{3.95\times10^{14}\,s^{-1}}$

$= 1.89 : 1 \cong 2 : 1$

Q.27 Which of the following atoms don't have valence eletrons in 4d-subshell?

(a) Mo (b) Tc (c) Ru (d) As

Ans: (D)

Sol: As (Astatine) is a p-block element, hence valence electrons do not enter d-orbital.

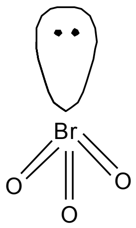
Q.28 The geometry of BrO_3^- ion according to VSEPR theory will be ________________ .

(a) trigonal planar **(b) trigonal pyramidal**

(c) see-saw **(d) square planar**

Ans: (B)

Sol:



Q.29 When heat is absorbed during the reaction, it is denoted as ________________ and when work is done by the system on the surroundings, it is denoted as ________________ .

(a) positive; positive **(b) negative; negative**

(c) positive; negative **(d) negative; positive**

Ans: (C)

Sol: ----

Q.30 Under given conditions, which of the following reactions can reach a state of equilibrium?

(a) Formation of rust on old bicycles.

(b) Decomposition of calcium carbonate in an open vessel

(c) Decompositioin of N_2O_4 in a closed vessel.

(d) Reaction between hydrogen and iodine in an open vessel.

Ans: (C)

Sol: A state of equilibrium can be reached only in a closed surrounding where the reactants/ products cannot escape the vessel. Equilibrium cannot be reached in a reaction carried out in open vessel.

Q.31 Which of the following is TRUE for a Daniel cell?

(a) Electron flow from cathode to anode.

(b) Electrons flow through the salt bridge.

(c) Current flows from anode to cathode.

(d) Oxidation occurs at the anode.

Ans: (D)

Sol: ----

Q.32 Which of the following is TRUE for a Daniel cell?

(a) Electron flow from cathode to anode.

(b) Electrons flow through the salt bridge.

(c) Current flows from anode to cathode.

(d) Oxidation occurs at the anode.

Ans: (D)

Sol: ----

Q.33 Assertion: But-2-ene shows geometrical isomerism.

Reason: Both the C-atoms have $-CH_3$ groups attached to them.

(a) Assertion and Reason are true. Reason is the correct explanation of Assertion.
(b) Assertion and Reason are true. Reason is not the correct explanation of Assertion.
(c) Assertion is true. Reason is false.
(d) Assertion is false. Reason is true.

Ans: (B)

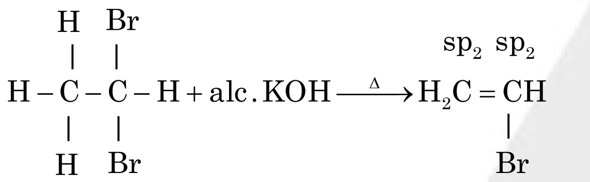
Sol: But-2-ene shows geometrical isomerism because the atoms or subsituents attached to the same carbon atom (of the double bond) must be different.

Q.34 The type of hybridization of the carbon atoms in the product, formed by heating ethylidene dibromide with alcoholic potash is ________.

(a) sp **(b) sp^3** **(c) sp^2** **(d) dsp^2**

Ans: (C)

Sol:



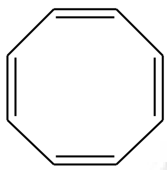
1,1 – Dibromoethane Vinyl bromide
(Ethylidene dibromide)

Q.35 According to Huckel rule, cyclooctatetraene shows __________ character.

(a) antiaromatic **(b) aromatic** **(c) non-aromatic** **(d) none of these**

Ans: (C)

Sol:



Cyclooctatertraene

It has 8π-electrons which is not a Huckel number and is non-planar. Hence, it is non-aromatic in nature.

Q.36 A mixed oxide has oxide ions arranged in ccp array, 'A' occupies 1/3 of all the octahedral voids and 'B' occupies 1/2 of all the tetrahedral voids. The formula of this oxide is

(a) $A_4B_9O_6$ **(b) AB_3O_3** **(c) AB_4O_2** **(d) $A_2B_3O_6$**

Ans: (B)

Sol: Number of oxide ions in ccp array, n=4

Number of 'A' atoms occupying octahedral voids$=\frac{1}{3}\times n=\frac{4}{3}$

Number of 'B' atoms occupying tetrahedral voids$==\frac{1}{2}\times 2n=\frac{1}{2}\times 2\times 4=4$

Ratio of A:B:O

$=\frac{4}{3}:4:4$

=4:12:12
=1:3:3

$\therefore$ Formula of the compound is AB_3O_3.

Q.37 When initial concentration of a reactant is doubled in a reaction, its half-life period is not affected. The order of the reaction is ________.

(a) zero **(b) first**
(c) second **(d) more than zero but less than first**

Ans: (B)

Sol: The half-life of first order reaction is given by equation

$$t_{1/2} = \frac{0.693}{k}$$

The equation implies that the half-life of a first order reaction is constant and is independent of the reactant concentration . Since in given reaction, change in concentration does not affect its half-life; it must be a first order reaction.

Q.38 Which of the following indicates positive deviation from Raoult's law?

(a) When 10 cc of a liquid is mixed with 10 cc of other liquid, the volume of resulting solution is 19 cc.
(b) Two liquids produce warm solution on mixing.
(c) Mixing of chloroform with acetone.
(d) mixing of carbon disulphide with acetone.

Ans: (D)

Sol: Option (A), (B) and (C) show negative deviation from Raoult's law whereas option (D) shows positive deviation from Raoult's law.

Q.39 When aqueous $CuSO_4$ solution is electrolyzed using platinum electrode, the blue colour of the solution disappear with the liberation of gas at electrode. The colourless solution is ________.

(a) Cu_2SO_4 **(b) $CuSO_4$** **(c) H_2SO_4** **(d) $PtSO_4$**

Ans: (C)

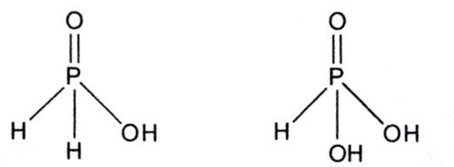
Sol: During electrolysis of $CuSO_4$, Cu^{2+}gets discharged at cathode and OH^- at anode. Thus, solution becomes acidic due to excess of H^+. These H^+ ions combine with SO_4^{2-} to form H_2SO_4

Q.40 Which is CORRECT statement for the given acids?

(a) Phosphinic acid is a monoprotic acid while phosphonic acid is a diprotic acid.
(b) Phosphonic acid is a diprotic acid while phosphonic acid is a monoprotic acid.
(c) Both are diprotic acids.
(d) Both are triprotic acids.

Ans: (A)

Sol:



Phosphinic acid Phosphonic acid

Phosphinic acid has one P - OH bond, and it can provide one proton, hence it is monoprotic acid. Whereas phosphonic acid has two P - OH bonds and it is diprotic acid.

Q.41 Both methane and ethane can be prepared in one step from ______.

(a) C_2H_4 **(b) CH_3OH** **(c) CH_3Br** **(d) CH_3CH_2OH**

Ans: (C)

Sol: $$\underset{\text{Methyl bromide}}{CH_3-Br}+2[H]\xrightarrow[\Delta]{\text{Zn-Cu couple and alcohol}}\underset{\text{Methane}}{CH_4}+HBr$$

$$\underset{\text{Methyl bromide}}{CH_3\text{-}Br}+2Na+\underset{\text{Methyl bromide}}{Br\text{-}CH_3}\xrightarrow{\text{dry ether}}\underset{\text{Ethane}}{CH_3\text{-}CH_3}+2NaBr$$

Q.42 Lower ethers are volatile liquids which on evaporation produce cooling. Therefore, they are used as ______.

(a) lubricants for motors

(b) refrigerants

(c) oinments and creams

(d) all of these

Ans: (B)

Sol: Due to their highly volatile nature, lower ethers, on evaporation produce cooling and are hence used as refrigerants. A mixture of ethers and dry ice (solid CO_2) gives a temperature as low as -110^{0}C.

Q.43 The stability of arenediazonium salts is due to ______.

(a) its tendency to eliminate a nitrogen molecule

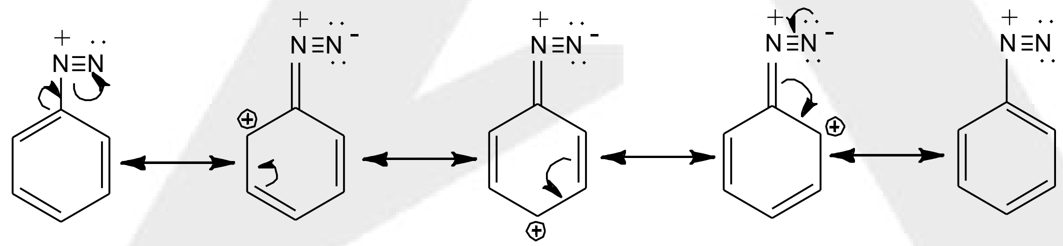
(b) dispersal of negative charge over the benzene ring

(c) its tendency to form a stable salt with halogens

(d) dispersal of positive charge over the benzene ring

Ans: (D)

Sol: Resonance provides stability to arenediazonium ion.



Q.44 The following statements are TRUE about crystal field theory EXCEPT ______ .

(a) It does not explain π bonding in complexes.

(b) Satisfactory explanation is not provided for the fact that water is a stronger ligand than OH^-

(c) Its considers s, p and d orbitals of the central metal.

(d) Partial covalent nature of metal ligand bond is not explained.

Ans: (C)

Sol: In crystal field theory, only d orbitals of the central metal are considered. There is no explanation for s and p orbitals.

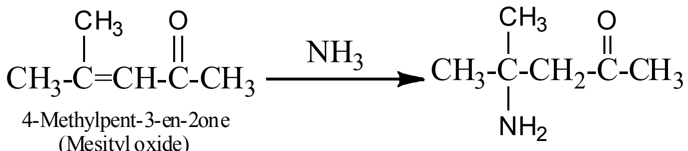
Q.45 Action of ammonia on 4-methylpent-3-en-2-one forms ______.

(a) hemiacetal **(b) diacetone amine**

(c) urotropine **(d) pinacol**

Ans: (B)

Sol:



4-Methylpent-3-en-2one (Mesityl oxide)

4-Amino-4-methylpentan-2-one (Diacetone amine)

Q.46 How many grams of caustic potash is required to completely neutralize 25.2 g HNO_3?

Sol: $HNO_3 + KOH \rightarrow KNO_3 + H_2O$

$\frac{25.2}{63} = 0.4$ mole;

0.4 mole of $HNO_3 = 0.4$ mole of KOH

$\therefore$ $0.4 \times$ Molecular mass (KOH)

$= 0.4 \times 56 = 22.4$ g of KOH

Q.47 Praveen went to a hill station where he experienced some breathing problems due to low density of oxygen. If at NTP density of oxygen is 1.520 g L^{-1}, what is the difference in densities, if the temperature at the hill station is 3^0C and pressure is 705 mm. The hill is at a height of 2173 m.

Sol:

$d = \frac{PM}{RT}$ OR $\frac{d_1}{d_2} = \frac{P_1}{T_1} \times \frac{T_2}{P_2}$

$P_1 = 760$ mm, $T_1 = 298$ K, $P_2 = 705$ mm

$T_2 = 276$ K

$d_1 = 1.52$ g L^{-1}, $d_2 = ?$ $d_2 - d_1 = ?$

$\frac{1.52}{d_2} = \frac{760}{298} \times \frac{276}{705}$

$d_2 = \frac{319336.8}{209760} = 1.522$

$d_2 - d_1 = 1.522 - 1.52$

$d_2 - d_1 = 0.002$

Q.48 1 g of activated charcoal at STP undergoes monolayer of adsorption of NH_3. If 1 g of charcoal has surface area of $2 \times 10^2\ m^2$ and effective surface area of NH_3 molecule is $0.172 nm^2$, the moles of NH_3 adsorbed on 20 g of charcoal is ________.

Sol: Surface area of charcoal available for adsorption= $20 \times 2 \times 10^2 m^2 = 4 \times 10^3 m^2$

Effective surface area of 1 NH_3 molecule

$= 0.172 nm^2$

$= 0.172 \times (10^{-9})^2 = 0.172 \times 10^{-18} m^2$

Number of molecules of NH_3 adsorbed

$$= \frac{4 \times 10^{3}}{0.172 \times 10^{-18}} = 2.326 \times 10^{22} \text{ molecules}$$

$$= \frac{2.326 \times 10^{22}}{6.022 \times 10^{23}}$$

molecules of nh3 adsorbed.

$$= 0.0386 \, \text{mol}$$

Q.49 The possible number of tetrapeptides from four different amino acids is ____________.

Sol: The number of possible arrangement can be 4!

$4! = 4 \times 3 \times 2 \times 1 = 24$

Q.50 How much faster would a reaction proceed at 25°C than if the activation energy is 65 kJ?

Sol:

$$\log\left(\frac{K_2}{K_1}\right) = \frac{E_a}{2.303R}\frac{(T_2 - T_1)}{T_1T_2} = \frac{65 \times 10^{3} \times (298 - 273)}{2.303 \times 8.3 \times 298 \times 273}$$

$$\frac{K_2}{K_1} = 11$$

Part - C - MATHEMATICS

Q.51 Consider the inequation $\frac{(2x+1)(x+4)}{x^2} \geq 0$.

Which of the following is correct?

(a) All negative integers satisfy the inequation.

(b) There is an isolated integer that does not satisfy the inequation.

(c) Not all positive integers satisfy the inequation.

(d) The greatest negative integer satisfying

Ans: (B)

Sol: $\frac{(2x+1)(x+4)}{x^2} < 0$

$\Leftrightarrow (2x+1)(x+4) \geq 0, \; x \neq 0$

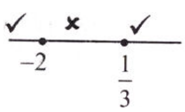
$\Leftrightarrow x \leq -4 \text{ or } x \geq -\frac{1}{2} \text{ and } x \neq 0$

Solution set $= (-\infty, -4)] \cup \left[-\frac{1}{2}, \infty\right) - \{0\}$

Q.52 The greatest negative integer satisfying the inequation $\log_{\frac{1}{3}}\left(\frac{3x - 1}{x + 2}\right) < 1$ is

(a) -1 **(b) -2** **(c) -3** **(d) -7**

Ans: (C)

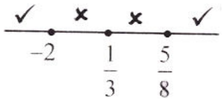
Sol: We first find the feasible region 

$\frac{3x-1}{x+2} > 0 \Leftrightarrow x < -2$ or $x > \frac{1}{3}$

Feasible region $= (-\infty, -2) \cup \left(\frac{1}{3}, \infty\right)$

Given inequation $\Leftrightarrow \frac{3x-1}{x+2} > \frac{1}{3}$

$$\Leftrightarrow \frac{9x-3-x-2}{3(x+2)} > 0$$



$$\Leftrightarrow \frac{8x-5}{3(x+2)} > 0$$

$$\Leftrightarrow x < -2 \text{ or } x > \frac{5}{8}$$

The solution set is $(-\infty, -2) \cup \left(\frac{5}{8}, \infty\right)$

The greatest negative integer is -3

Q.53 The domain of the function

$f(x) = \sin^{-1}\left(\frac{8.3^{x-2}}{1-3^{2(x-1)}}\right)$ is

(a) $(-\infty, 0]$ **(b) $[2, \infty)$** **(c) $(-\infty, 0] \cup [2, \infty)$** **(d) $(-\infty, -1] \cup [1, \infty)$**

Ans: (C)

Sol: f(x) is defined, if $-1 \le \frac{8.3^{x-2}}{1-3^{2(x-1)}} \le 1$

$$\Rightarrow -1 \le \frac{3^x - 3^{x-2}}{1-3^{2x-2}} \le 1$$

$$\Rightarrow \frac{3^x - 3^{x-2}}{1-3^{2x-2}} + 1 \ge 0 \text{ and } \frac{3^x - 3^{x-2}}{1-3^{2x-2}} - 1 \le 0$$

$$\Rightarrow \frac{1+3^x-3^{x-2}-3^{2x-2}}{1-3^{2x-2}} \ge 0 \text{ and } \frac{3^x-3^{x-2}-1+3^{2x-2}}{1-3^{2x-2}} \le 0$$

$$\Rightarrow \frac{(1+3^x)(1-3^{x-2})}{(1-3^x.3^{x-2})} \ge 0 \text{ and } \frac{(3^x-1)(3^{x-2}+1)}{(3^{2x-2}-1)} \ge 0$$

$$\Rightarrow \frac{(3^x+1)(3^{x-2}-1)}{(3^x.3^{x-2}-1)} \ge 0 \text{ and } \frac{(3^x-1)}{(3^{2x-2}-1)} \ge 0$$

$$\Rightarrow \frac{(3^x-3^2)}{(3^{2x}-3^2)} \ge 0 \text{ and } \frac{(3^x-1)}{(3^{2x}-3^2)} \ge 0$$

Let $3^x = t, t > 0$

$\frac{t-9}{t-3} \ge 0$ and $\frac{t-1}{t-3} \ge 0$

$t-3 > 0 \Rightarrow t-9 \ge 0$ and $t-1 \ge 0$

$\Rightarrow t \ge 9$

$t-3<0 \Rightarrow t-9 \le 0$ and $t-1 \le 0$

$\Rightarrow t-1 \le 0$

$\Rightarrow t \le 1$

$\therefore \quad t \le 1 \quad$ or $\quad t \ge 9$

$\Leftrightarrow 3^x \le 1 \quad$ or $\quad 3^x \ge 9$

$\Leftrightarrow x \le 0 \quad$ or $\quad x \ge 2$

$\Rightarrow x \in (-\infty, 0] \cup [2, \infty)$

Q.54 The points on the line x+y=4 which lie at a unit distance from the line 4x+3y=10 are
(a) (3, 1), (-7, 11) (b) (3, 1)(7, 11) (c) (-3, 1), (-7, 11) (d) (1, 3), (-7, 11)

Ans: (A)

Sol: Let the point (h, k) lies on the line x+y=4

$\Rightarrow h+k=4 \quad ...(i)$

According to the given condition,

$$\left|\frac{4h+3k-10}{\sqrt{4^2+3^2}}\right|=1$$

$$\Rightarrow \frac{4h+3k-10}{5}=\pm 1$$

$\Rightarrow 4h+3k=15 \quad ...(ii)$

and $4h+3k=5 \quad ...(iii)$

From equation (i) & (ii), we get h = 3, k = 1
From equation (i) & (iii), we get h = -7, k = 11

Q.55 L is a line whose complex slope is m. The complex slope of the reflection of line L in imaginary axis, is

(a) -m (b) $\frac{1}{m}$ (c) $\frac{1}{m^2}$ (d) $-\frac{1}{m}$

Ans: (B)

Sol: Let L_1 be the reflection of line L, in the imaginary axis. Let P and Q be the point , on line L, representing z_1 and z_2 respectively. Then points, R and S, say, on Line L_1, representing $-\bar{z}_1, -\bar{z}_2$ are images of P and Q respectively.

Complex slope of line $L_1 = \dfrac{-\bar{z}_1-(-\bar{z}_2)}{-z_1-(-z_2)} = \dfrac{1}{m}$

Q.56 The equation of the circle passing through the origin and cutting intercepts of length 3 and 4 units from the positive axes, is

(a) $x^2+y^2+6x+8y+1=0$

(b) $x^2+y^2-6x-8y=0$

(c) $x^2+y^2+3x+4y=0$

(d) $x^2+y^2-3x-4y=0$

Ans: (D)

Sol: Given, $|OA|=3$ and
$|OB|=4$

$\therefore \quad \Rightarrow |OL| = \frac{3}{2}$ and $|CL| = 2$

$$OC^2 = \left(\frac{3}{2}\right)^2 + 2^2$$

$$= \frac{25}{4}$$

$\Rightarrow |OC| = \frac{5}{2}$

The centre of the circle is $\left(\frac{3}{2}, 2\right)$ and radius $= \frac{5}{2}$

The equation of the circle is

$$\left(x - \frac{3}{2}\right)^2 + (y-2)^2 = \left(\frac{5}{2}\right)^2$$

$\Rightarrow x^2 + y^2 - 3x - 4y = 0$

Q.57 The line 2x+y=1 is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If this lines passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

(a) $\sqrt{2}$ **(b) 2** **(c) $\sqrt{3}$** **(d) 3**

Ans: (B)

Sol: According to the given condition, the line 2x+y=1 passes through $\left(\frac{a}{e}, 0\right)$,

$\Rightarrow \frac{2a}{e} + 0 = 1 \Rightarrow a = \frac{e}{2}$ ...(i)

Since, $2x + y = 1$ i.e., $y = -2x + 1$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

$\Rightarrow 1^2 = a^2(-2)^2 - b^2$

$\Rightarrow 4a^2 - b^2 = 1$

$\Rightarrow 4a^2 - a^2(e^2 - 1) = 1$

$\Rightarrow e^2 - \frac{e^2}{4}(e^2 - 1) = 1$ ....[From (i)]

$\Rightarrow 5e^2 - e^4 = 4$

$\Rightarrow e^4 - 5e^2 + 4 = 0$

$\Rightarrow (e^2 - 1)(e^2 - 4) = 0$

$\Rightarrow e = 2$ ....[$\because e > 1$]

Q.58 Between two junction stations A and B there are 12 intermediate stations. The number of ways in which a train can be made to stop at 4 of these stations so that no two of these halting stations are consecutive is

(a) $^6C_4$ **(b) $^5C_4$** **(c) $^8C_4$** **(d) $^9C_4$**

Ans: (D)

Sol: Let

x_1= number of stations before 1^{st} and halting station
x_2= between 1^{st} and 2^{nd}
x_3= between 2^{nd} and 3^{rd}
x_4= between 3^{rd} and 4^{th}
and x_5 on the right of the 4^{th} stations.

Then, $x_1 \geq 0, x_5 \geq 0, x_2, x_3, x_4 \geq 1$

such that $x_1 + x_2 + x_3 + x_4 + x_5 = 8$ ...(i)

The total number of ways is the number of solutions of the above equation.

Let $y_2 = x_2 - 1, y_3 = x_3 - 1, y_4 = x_4 - 1$.

Then equation (i) reduces to

$x_1 + y_2 + y_3 + y_4 + x_5 = 5, x_1, y_2, y_3, y_4 \geq 0$ ...(ii)

The number of solutions to this equation is

$^{5+5-1}C_{5-1} = {}^9C_4$.

There is bijection between two sets containig solutions of (i) and (ii)

$\Rightarrow$ Required number of solutions to (i) $= {}^9C_4$

Q.59 The coefficient if x^{99} in (x+1)(x+3)(x+5)...(x+199) is

(a) 40000 **(b) $\frac{200!}{2^{100}(100!)}$** **(c) 10000** **(d) 39601**

Ans: (C)

Sol: There are 100 linear factors. To get x^{99} we take factor for a constant term and the remaining factor for the term containing x.

$\Rightarrow$ Required coefficient $=1+3+5+....+199$
$=(100)^2=10000$

Q.60 The probability of India winning a test match against West Indies is $\frac{1}{2}$. Assuming independence from match to match, the probability that in a 5-atch series India's second win occurs at the third test, is

(a) $\frac{2}{3}$ **(b) $\frac{1}{2}$** **(c) $\frac{1}{4}$** **(d) $\frac{1}{8}$**

Ans: (C)

Sol: The sample space is [LWW, WLW]

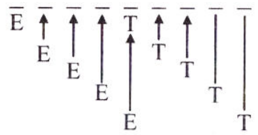
$\therefore$ P(LWW)+P(WLW)
= Probability that in 5-match series, it is India's second win
= P(L)P(W)P(W)+P(W)P(L)P(W)
$= \frac{1}{8} + \frac{1}{8} = \frac{2}{8}$
$= \frac{1}{4}$

Q.61 If the letters of the word 'BEAUTIFUL' be arranged at random, the probability that there will be exactly 3 letters between E and T is

(a) $\mathbf{\frac{5}{72}}$ **(b)** $\mathbf{\frac{5}{36}}$ **(c)** $\mathbf{\frac{5}{19}}$ **(d)** $\mathbf{\frac{4}{9}}$

Ans: (B)

Sol: The word BEAUTIFUL has 9 letters.
Total number of words formed from the letters of the word BEAUTIFUL =9!=n(S)



There are five situtaions in which E and T differ by 3 places.
E and T can interchange their positions.

Number of favourable cases $= \frac{(5\times 2)\times 7!}{9!}$

$= \frac{5}{36}$

Q.62 Let *n* be an odd number and S(n) denote the sum of the unordered products of all the pairs of positive integers whose sum $=$ n. Then $\lim_{n\to 0}\frac{S(n)}{n^3}$ is

(a) $\mathbf{\frac{1}{4}}$ **(b)** $\mathbf{\frac{1}{3}}$ **(c)** $\mathbf{\frac{1}{12}}$ **(d)** $\mathbf{\frac{1}{24}}$

Ans: (C)

Sol: $S(n) = \frac{1}{2}[(n-1)\cdot 1 + ... + 2\cdot(n-2) + 1\cdot(n-1)]$ ...[division by 2 as S(n) denotes the product of unordered pairs]

$$= \frac{1}{2}\sum_{r=1}^{n-1}(n-r)\cdot r$$

$$= \frac{1}{2}n\left(\sum_{r=1}^{n-1} r\right) - \frac{1}{2}\left(\sum_{r=1}^{n-1} r^2\right)$$

$$= \frac{1}{4}n^2(n-1) - \frac{1}{12}(n-1)n(2n-1)$$

$$= \frac{1}{4}n(n-1)\left(n - \frac{1}{3}(2n-1)\right) = \frac{n(n-1)(n+1)}{12}$$

$$\lim_{n\to\infty}\frac{S(n)}{n^3} = \frac{1}{12}\lim_{n\to\infty}\frac{n(n-1)(n+1)}{n^3}$$

$$= \frac{1}{12}(1) = \frac{1}{12}$$

Q.63 Let f(x)=|x²-3|x|+2|, then which of the following is NOT correct?

(a) $\mathbf{f'(x) = 2x - 3,\ x \in (0,1)\cup(2,\infty)}$

(b) $\mathbf{f'(x) = 2x + 3,\ x \in (-\infty,-2)\cup(-1,0)}$

(c) $\mathbf{f'(x) = -2x - 3, x \in (-2,-1)}$

(d) $\mathbf{f'(x) = 2x + 3,\ x \in (1,2)}$

Ans: (D)

Sol: $f(x) = |x^2 - 3|x| + 2|$

$= |x^2 + 3x + 2|, \qquad x < 0$

$= |x^2 - 3x + 2|, \qquad x \geq 0$

$\Leftrightarrow f(x) = |(x+1)(x+2)|, \qquad x < 0$

$= |(x-1)(x-2)|, \qquad x \geq 0$

$\Leftrightarrow f(x) = (x+1)(x+2), \qquad x < -2 \text{ or } -1 \leq x < 0$

$= -(x+1)(x+2), \qquad -2 \leq x \leq -1$

$= (x-1)(x-2), \qquad 0 \leq x \leq 1 \text{ or } x \geq 2$

$= -(x-1)(x-2) \qquad 1 \leq x < 2$

$\Rightarrow f$ may not be differentiable at $x = -2, -1, 0, 1, 2$

$\Rightarrow f'(x) = 2x + 3, \qquad x < -2$

$= -(2x+3), \qquad -2 < x < -1$

$= 2x + 3, \qquad -1 < x < 0$

$= 2x - 3, \qquad 0 < x < 1$

$= -(2x-3), \qquad 1 < x < 2$

$= 2x - 3, \qquad x > 2$

Q.64 $\lim\limits_{x \to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$ **equals**

(a) $-\frac{1}{12}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{6}$ **(d)** $-\frac{1}{3}$

Ans: (D)

Sol: Let $L = \lim\limits_{x \to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$

$= \lim\limits_{x \to 0} \frac{\sin^2 x - x^2}{x^2 \sin^2 x}$

$= \lim\limits_{x \to 0} \frac{\sin^2 x - x^2}{x^4} \qquad \ldots\left[\lim\limits_{x \to 0} \frac{\sin x}{x} = 1\right]$

Applying L' Hopital's rule repeatedly,

$L = \lim\limits_{x \to 0} \frac{2\sin x \cos x - 2x}{4x^3}$

$= \lim\limits_{x \to 0} \frac{2\cos 2x - 2}{12x^2}$

$= \lim\limits_{x \to 0} \frac{-4\sin 2x}{24x}$

$= -\frac{1}{6} \times 2 = -\frac{1}{3}$

Q.65 $\int \frac{dx}{(x+1)\sqrt{4x+3}}$ **is equal to**

(a) $\tan^{-1}\sqrt{4x+3}+c$ **(b)** $3\tan^{-1}\sqrt{4x+3}+c$

(c) $2\tan^{-1}\sqrt{4x+3}+c$ **(d)** $4\tan^{-1}\sqrt{4x+3}+c$

Ans: (C)

Sol: Let $4x+3=t^2 \Rightarrow 4dx=2t\ dt$

$$\int \frac{dx}{(x+1)\sqrt{4x+3}}=\frac{1}{2}\int \frac{tdt}{\left(\frac{t^2-3}{4}+1\right)t}=2\int \frac{dt}{1+t^2}$$

$$=2\tan^{-1}t+c$$

$$=2\tan^{-1}\sqrt{4x+3}+c$$

Q.66 $\lim\limits_{n\to\infty}\left(\frac{(n+1)(n+2)...3n}{n^{2n}}\right)^{\frac{1}{n}}$ **is equal to**

(a) $\frac{27}{e^2}$ **(b)** $\frac{9}{e^2}$ **(c)** $3\log 3-2$ **(d)** $\frac{18}{e^4}$

Ans: (A)

Sol: Let $P=\lim\limits_{n\to\infty}\left(\frac{(n+1)(n+2)...3n}{n^{2n}}\right)^{\frac{1}{n}}$

$$=\lim_{n\to\infty}\left(\frac{n+1}{n}\cdot\frac{n+2}{n}...\frac{(n+2n)}{n}\right)^{\frac{1}{n}}$$

$$\Rightarrow \log P=\frac{1}{n}\lim_{n\to\infty}\left[\log\left(\frac{n+1}{n}\right)+\log\left(\frac{n+2}{n}\right)+...\log\left(\frac{n+2n}{n}\right)\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\log\left(1+\frac{r}{n}\right)$$

$$=\int_0^2 \log(1+x)dx$$

$$=\int_1^3 \log t\,dt \qquad ...[\text{Let } 1+x=t]$$

$$=[t(\log-1]_1^3$$

$$=3(\log 3-1)-1(\log 1-1)$$

$$=3\log 3-2$$

$$=\log 3^3-\log e^2$$

$$\Rightarrow \log\ P=\log\frac{3^3}{e^2}=\log\frac{27}{e^2}\Rightarrow P=\frac{27}{e^2}$$

Q.67 **The value of determinant** $\begin{vmatrix} yz & zx & xy \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ **where x, y, z are** p^{th}, $(2q)^{th}$, $(3r)^{th}$ **terms,**

repectively of an H.P., is

(a) -1 **(b) 0** **(c) 1** **(d) none**

Ans: (B)

Sol:

$\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are p^{th}, $(2q)^{th}$, $(3r)^{th}$ of an A.P > with common difference d.

Given determinant

$$= xyz\begin{vmatrix} \frac{1}{x} & \frac{1}{y} & \frac{1}{z} \\ p & 2q & 3r \\ 1 & 1 & 1 \end{vmatrix}$$

$$= xyz\begin{vmatrix} a+(p-1)d & a+(2q-1)d & a+(3r-1)d \\ p & 2q & 3r \\ 1 & 1 & 1 \end{vmatrix}$$

$R_1 \to R_1 - aR_3$, $R_2 \to R_2 - R_3$ and taking d common from R_1

$$= xyzd\begin{vmatrix} p-1 & 2q-1 & 3r-1 \\ p-1 & 2q-1 & 3r-1 \\ 1 & 1 & 1 \end{vmatrix}$$

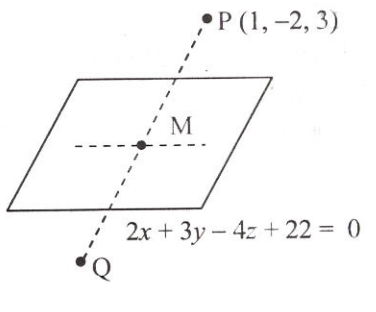
$= 0$

Q.68 If the image of the point P(1, -2, 3) in the plane 2x+3y-4z+22=0 measured parallel to the line, $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q, then PQ is equal to

(a) $6\sqrt{5}$ **(b)** $3\sqrt{5}$ **(c)** $2\sqrt{42}$ **(d)** $\sqrt{42}$

Ans: (C)

Sol: Since, distance PQ measured parallel to the line $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$.



$\Rightarrow$ direction ratios of line PQ are 1,4,5.

Equation of PQ is given by,

$\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}=\lambda$, say

$\Rightarrow$ A point on the line is $(\lambda+1, 4\lambda-2, 5\lambda+3)$

$= M$, say

Point M lies on $2x+3y-4z+22=0$

$\Rightarrow 2(\lambda+1)+3(4\lambda-2)-4(5\lambda+3)+22=0$

$\Leftrightarrow -6\lambda+6=0 \Rightarrow \lambda=1$

$\Rightarrow M=(2,2,8)$

$|PQ|=2|PM|$ ... [M is the midpoint of PQ]

$\Rightarrow |PQ|=2\sqrt{1^2+4^2+5^2}=2\sqrt{42}$

Q.69 If the points (-2, 3, 5), (1, 2, 3) and $(\lambda, 0, -1)$ are collinear, then the value of λ is

(a) 5 (b) -5 (c) 7 (d) -7

Ans: (C)

Sol: The given points are collinear.

$\Rightarrow \frac{-2-1}{1-\lambda}=\frac{3-2}{2-0}=\frac{5-3}{3+1}$

$\Leftrightarrow \frac{-3}{1-\lambda}=\frac{1}{2}=\frac{2}{4}$

$\Leftrightarrow \lambda=7$

Q.70 Which of the following quantified statement is true?

(a) The square of every real number is positive

(b) There exists a real number whose square is negative

(c) There exists a real number whose square is not positive

(d) Every real number is rational

Ans: (C)

Sol: Since there exists a real number x=0, such that $x^2=0$. Zero is neither positive nor negative. Option (C) is the correct answer.

Q.71 The area (in sq. units) of the parallelogram whose diagonals are along the vectors $8\hat{i}-6\hat{j}$ and $3\hat{i}+4\hat{j}-12\hat{k}$, is_____?

Sol: Let $\vec{d}_1=8\hat{i}-6\hat{j}$ and $\vec{d}_2=3\hat{i}+4\hat{j}-12\hat{k}$

$$\vec{d}_1\times\vec{d}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 8 & -6 & 0\\ 3 & 4 & -12\end{vmatrix}$$

$$=\hat{i}(72-0)-\hat{j}(-96-0)+\hat{k}(32+18)$$

$\Rightarrow \vec{d}_1\times\vec{d}_2=72\hat{i}+96\hat{j}+50\hat{k}$

$\Rightarrow |\vec{d}_1\times\vec{d}_2|=\sqrt{72^2+96^2+50^2}=130$

$$\text{Area of parallelogram}=\frac{1}{2}|\vec{d}_1\times\vec{d}_2|$$

$$=\frac{1}{2}\times 130$$

$$=65 \text{ sq. units}$$

Q.72 The order of the differential equation whose general solution is given by $y=(c_1+c_2)\cos(x+c_3)-c_4\sin x+c_5\,e^{-x+c_6}$ is_____?

Sol: $y=(c_1+c_2)\cos(x+c_3)-c_4\sin x+cs\ e^{-x+c_6}$

Let $c_1+c_2=c_7$

$\Rightarrow y=c_7\cos(x+c_3)-c_4\sin x+c_5\cdot e^{C_6}\cdot e^{-x}$

Let $c_5\,e^{C_6}=c_8$

$\Rightarrow y=c_7\cos(x+c_3)-c_4\sin x+c_8e^{-x}$

Clearly, the relation contains three arbitary constants. So, the order of differential equation, satisfying it, is 3.

Q.73 S is a circle (radius=13) and P is a point at a distance of 12 from the centre os S. Using continuity considerations, the numbers of chords of the circle through P, which have integral lengths, is_____?

Sol: The longest chord through P is the diameter
AB of length 26. The shortest chord through P is $XY\perp AB$, of length 10.
Draw a variable chord through P, as shown. Its length varies continiously from 10 to 26.
Now, there are 15 integers in (10, 26), hence the number of chords through P (having integral lengths) is $2\times15+2=32$

Q.74 The area bounded by the curve y=x(x-3)² and y=x, in square units, is _____?

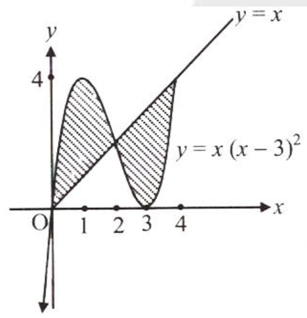
Sol: $y=(x-3)^2$ is a cubic polynomial.

$\Rightarrow y'=(x-3)^2+2x(x-3)$

$=3(x-3)(x-1)$

$y'=0\Leftrightarrow x=3,\ x=1$

x=1 is a point of relative maxima and x=3 is a point of relative minima.
For x>0, y>0
For x<0, y<0
y(1)=4, y(3)=0
We sketch the curve using this information,



Solving y=x and $y=x(x-3)^2$, we get
$(x-3)^2=1\Rightarrow x-3=\pm1$
$\Rightarrow x=2,\ 4$
Due to symmetry of the figure, the required

area$=2\int_0^2 x(x-3)^2-x)dx$

$=2\int_0^2(x^3-6x^2+8x)dx$

$$= 2\left(\frac{16}{4} - \frac{6\times 8}{3} + \frac{8\times 4}{2}\right)$$

$$= 8$$

Q.75 The nuber of common terms to the two sequences 11, 15, 19, 23, ..., 411 and 6, 11, 16, 21, ..., 431 is _____?

Sol: In the first sequence, 11, 15, 19, 23, ..., 411,

$a = 11, d = 4, t_p = 411$

$\Rightarrow 411 = 11 + (p-1)4$

$\Rightarrow p = 101$

1st sequence is 11, 15, 19, 23............411

$d_1 = 4$

no. of terms $= 101$

2nd sequence is 6, 11, 16.................431

$d_2 = 5$

$431 = 6 + (4-1)5 \qquad = \frac{425}{5} = n-1$

$= n = 85+1 = 86$

L.C.M of d_1 and d_2 is $= 20$

Then,

Resultant sequence 11, 31, 51................No of common terms is 20.

There are 101 terms in the first sequence and 86 terms in the second.

$\Rightarrow 0 \le k \le 20$ and $0 \le k \le 21$

$\Rightarrow 0 \le k \le 20$

$\therefore$ Number of common terms = 20

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