Aggarwal CLASSES
A Division of Aggarwal Educare

JEE (MAIN)

TEST PAPER

SUBJECT : PHYSICS,CHEMISTRY, MATHEMATICS ***TEST CODE : TEST PAPER-6***

ANSWER PAPER

TIME : 3 HRS ***MARKS : 300***

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark (-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or “un- darken” a darkened bubble.

Part A - PHYSICS

Q.1 **From the equation $\theta = rg / v^2$, one can obtain the angle of banking θ for a cyclist taking a curve (the symbols have their usual meaning). Then say, it is**
(a) Both dimensionally and numerically correct
(b) Neither numerically nor dimensonally correct
(c) Dimensionally correct only
(d) Numerically correct only

Ans: (c)

Sol: Given equation of dimensionally correct because both sides are dimensionless, but numerically wrong because the correct equation is

$$\tan\theta = v^2 / rg$$

Q.2 **An object moving with a speed of 6.25 m/s is decelerated at a rate given by :**

$$\frac{dv}{dt} = -25\sqrt{v}$$

Where v is instantaneous speed. The time taken by the object to come to rest would be

(a) 1 s **(b) 2 s** **(c) 4 s** **(d) 8 s**

Ans: (b)

Sol: $\int_{6.25}^{0} \frac{dv}{dt} = -25\int_{0}^{t} dt$

$\Rightarrow \quad \left|2\sqrt{v}\right|_{6..25}^{0} = -2.5t$

$\Rightarrow \quad = 2\sqrt{6.25} = 2.5t$

$\Rightarrow \quad t = 2s$

Q.3 **A water fountain on the ground sprinkles water all around it, if the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is**

(a) $\pi\frac{v^2}{g}$ **(b)** $\pi\frac{v^4}{g^2}$ **(c)** $\frac{\pi v^2}{2g^2}$ **(d)** $\pi\frac{v^2}{g^2}$

Ans: (b)

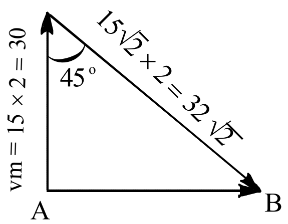
Sol: $R_{max} = \frac{v^2}{g}\sin 2\theta = \frac{v^2}{g}$, area $= \pi R^2 = \pi\frac{v^2}{g^2}$.

Q.4 **A man cycling at the rate of 15 km/h along the north. He is under the influence of wind blowing at the rate of $15\sqrt{2}$ km/h in south-east direction. Find the direction and the distance covered by him in a time of 2 h, from his starting position.**

(a) 45 km along the east **(b) 45 km along the west**
(c) 30 km along the east **(d) 30 km along the west**

Ans: (c)

Sol:



Final distance covered, AB = 30 km due east

Q.5 **A light string passing over a smooth light pulley connects two blocks of masses, m_1 and m_2 (vertically). If the acceleration of the system is g/8 then the ratio of the masses is :**

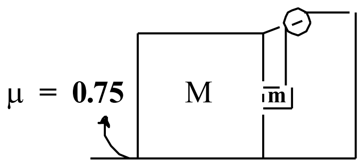
(a) 8 : 1 **(b) 9 : 7** **(c) 1 : 8** **(d) 7 : 9**

Ans: (b)

Sol: $a = \frac{m_1 - m_2}{m_1 + m_2} g$. if $m_1 > m_2$. But $a = g/8$.

$\therefore \quad \frac{g}{8} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g \; or \; m_1 : m_2 = 9 : 7$

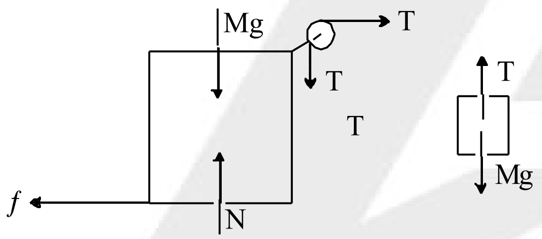
Q.6 **Two blocks (m and M) are arrenged as shown in fig. If there is friction between ground and M only and other surfaces are frictionless. The coefficient of friction between ground and M is $\mu = 0.75$. The maximum ratio of m and M (m/M) so that the system remains at rest is**



(a) 1/4 **(b) 3** **(c) 1/3** **(d) None of these**

Ans: (b)

Sol:



$T = mg$ ....(i)

$f = T$ ....(ii)

and $f = \leq \mu N$

Here $N = Mg + T = (M = m)g$

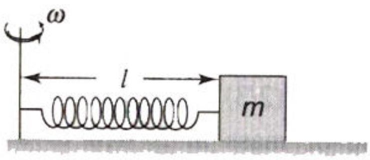
$\therefore \quad T \leq m(M + m)g$

$mg \leq \mu(M + m)g$

$\frac{m}{M} \leq \frac{\mu}{(1 - \mu)}$

$\frac{m}{M} \leq \frac{0.75}{(1 - 0.75)} \Rightarrow \frac{m}{M} \leq 3$

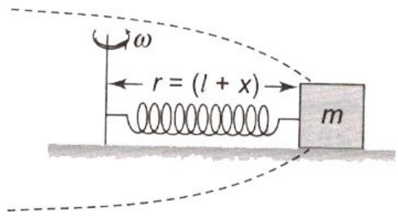
Q.7 **A particle of mass m is fixed to one end of a light spring of force constant k and unstretched lenth *l*. The system is roteded about the other end of the spring with an angular velocity ω, in gravity -free space. The increase in length of the spring will be**



(a) $\frac{m\omega^2 l}{k}$ **(b)** $\frac{m\omega^2 l}{k - m\omega^2}$ **(c)** $\frac{m\omega^2 l}{k + m\omega^2}$ **(d) None of these**

Ans: (b)

Sol: In the given condition, elastic force will provide the required centripetal force.



$$kx = m\omega^2 r = m\omega^2(l+x)$$

$$\Rightarrow \quad kx = m\omega^2 l + m\omega^2 x$$

$$\Rightarrow \quad x(k - m\omega^2) = m\omega^2 l$$

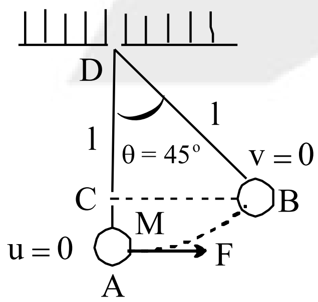
$$\therefore \quad x = \frac{m\omega^2 l}{k - m\omega^2}$$

Q.8 A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with the initial vertical direction is

(a) $\frac{mg}{\sqrt{2}}$ **(b)** $mg(\sqrt{2}-1)$ **(c)** $mg(\sqrt{2}+1)$ **(d)** $mg\sqrt{2}$

Ans: (b)

Sol: Work done by force = Increase in potential energy of mass



$$\Rightarrow \quad F(CB) = Mg(AC)$$

$$\Rightarrow \quad F\left(\frac{l}{\sqrt{2}}\right) = Mg\left(l - \frac{l}{\sqrt{2}}\right)$$

$$\Rightarrow \quad F = Mg(\sqrt{2}-1)$$

Q.9 A neutron travelin with a velocity v and $K \cdot E$, E collides perfectly elastically head on with the nucleus of an atom of mass number A at rest. The fraction of total energy

retained by neutron is

(a) $\left(\frac{A-1}{A+1}\right)^2$ (b) $\left(\frac{A+1}{A-1}\right)^2$ (c) $\left(\frac{A-1}{A}\right)^2$ (d) $\left(\frac{A+1}{A}\right)^2$

Ans: (a)

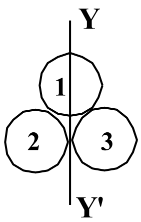
Sol: Friction of kinetic energy retained by projectile,

$$\frac{\Delta K}{K} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

Mass of neutron $(m_1) = 1$ and mass of atom $(m_2) = A$

$$\therefore \qquad \frac{\Delta K}{K} = \left(\frac{1-A}{1+A}\right)^2 \text{ or } \left(\frac{A-A}{A+1}\right)^2.$$

Q.10 Three rings each of mass M and radius R are arrenged as shown in the fig. The moment of inertia of the system about YY' will be



(a) $3MR^2$ **(b) $\frac{3}{2}MR^2$** **(c) $5MR^2$** **(d) $\frac{7}{2}MR^2$**

Ans: (d)

Sol: Moment of inertia of system about YY' $I = I_1 + I_2 + I_3$ where I_1 = moment of inertia of ring about diameter, $I_2 = I_3 =$ moment of inertia of ring about a tangent in a plane

$$\therefore \qquad I = \frac{1}{2}mR^2 + \frac{3}{2}mR^2 + \frac{3}{2}mR^2 = \frac{7}{2}mR^2$$

Q.11 The escape velocity for a plane is v_e. A tunnel is dug along a diameter of the planet and small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be

(a) v_e **(b) $\frac{v_e}{\sqrt{2}}$** **(c) $\frac{v_e}{2}$** **(d) Zero**

Ans: (b)

Sol: Gravitational potential at the surface of the Earth,

$$V_s = -\frac{GM}{R}$$

Graviatational potential at the center of Earth,

$$V_c = -\frac{3GM}{2R}$$

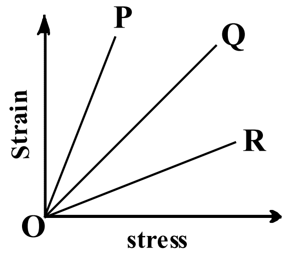
By the coservation of energy.

$$\frac{1}{2}mv^2 = m(V_s - V_c)$$

$$v^2 = 2\frac{GM}{R}\left(\frac{3}{2}-1\right) = \frac{GM}{R}\ gR = \frac{v_e^2}{2} \quad (\text{as } v_e = \sqrt{2gR})$$

$$\therefore \quad v = \frac{v_e^2}{\sqrt{2}}$$

Q.12 The strain-stress curves of three wires different materials are shown in fig. P, Q and R are the elastic limits of the wires. The fig shows that



(a) Elasticity of wire P is maximum **(b) Elasticity of wire Q is maximum**
(c) Tensile strength of R is maximum **(d) None of the above is true.**

Ans: (d)

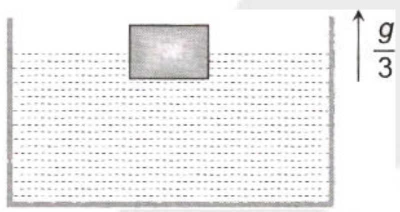
Sol: On the graph, stress is represented on X-axis and strain Y-axis.

So, from the graph, $Y = \cot\theta = \frac{1}{\tan\theta} \propto \frac{1}{\theta}$ [where θ is the angle from stress axis]

$\therefore \quad Y_P < Y_Q < Y_R \quad [\text{as } \theta_P > \theta_Q > \theta_R]$

We can say that elasticity of wire P is minimum and R is maximum.

Q.13 A cubical block is floating in a liquid with half of its volume imersed in the liquid fig when the whole system accelerates upward with acceleration of g/3. the fraction of volume immersed in the liquid will be



(a) $\frac{1}{2}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{2}{3}$ **(d)** $\frac{3}{4}$

Ans: (a)

Sol: Fraction of volume immersed in the liquid, $V_{in} = \left(\frac{\rho}{\sigma}\right)V$, i.e. depends upon the densities of the block and liquid .So there will be no change in it if system moves upward or downward with constant velocity of some acceleration.

Q.14 The coefficienty of apparent expansion of a liquid in a copper vessal is C and in a silver vessel S. The coefficient of volume expansion of copper is λ_C · What is the coefficient of linear expansion of silver ?

(a) $(C + \gamma_C + S)/3$ **(b)** $(C - \gamma_C + S)/3$

(c) $(C + \gamma_C - S)/3$ **(d)** $(C - \gamma_C - S)/3$

Ans: (c)

Sol: Apparent coefficient of volume expansion for liquid $\gamma_{app} = \gamma_L - \gamma_s$

$\therefore \quad \gamma_L = \gamma_{app} + \gamma_s$

Where γ_s is coefficient of volume expansion for solid vessel.

When liquid is placed in copper vessel, then $\gamma_L = C + \gamma_{copper}$ ...(i)

[as γ_{app} for liquid in copper vessel = C]

When liquid is placed in silver vessel, then $\gamma_L = S + \gamma_{silver}$ ...(ii)

[as γ_{app} for liquid in silver vessel = S]

From (i) and (ii) we get

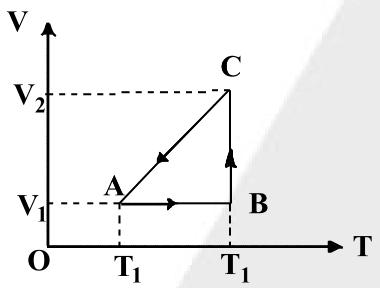
$$C + \gamma_{copper} = S + \gamma_{silver}$$

$\therefore \quad \gamma_{silver} = C + \gamma_{copper} - S$

Coefficient of volume expansion $= 3 \times$ Coefficient of linear expansion

$\Rightarrow \quad \alpha_{silver} \frac{\gamma_{silver}}{3} = \frac{C + \gamma_{copper} - S}{3}$

Q.15 A cyclic process for 1 mol of an ideal gas is shown in fig in the V -T diagram. The work done in AB, BC and CA respectively, is



(a) $0, RT_2 \ In\left(\frac{V_1}{V_2}\right), R(T_1 - T_2)$ **(b)** $R(T_1 - T_2), 0, RT_1 \ In \frac{V_1}{V_2}$

(c) $0, RT_2 \ In\left(\frac{V_1}{V_2}\right), R(T_1 - R_2)$ **(d)** $0, RT_2 \ In\left(\frac{V_2}{V_1}\right), R(T_2 - R_1)$

Ans: (c)

Sol: Process AB is isochoric,

$\therefore \quad W_{AB} = P\Delta V = 0$

Process BC is isothermal,

$\therefore \quad W_{BC} = RT_2 \cdot In\left(\frac{V_2}{V_1}\right)$

Process CA is isobaric

$\therefore \quad W_{CA} = P\,\Delta V = R\,\Delta T = R(T_2 - T_1)$

Q.16 A gas mixture consists of 2 mol of oxygen and 4 mol of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

(a) 4RT (b) 15RT (c) 9RT (d) 11RT

Ans: (d)

Sol: Total internal energy of system $= U_{oxygen} + U_{argon}$

$$= \mu_1 \frac{f_1}{2} RT + \mu_2 \frac{f_2}{2} RT$$

$$= 2\frac{5}{2} RT + 4RT = 5RT + 6RT = 11RT$$

[as $f_1 = 5$ (for oxygen) and $f_2 = 3$ (for argon)]

Q.17 A simple pendulum is executivng SHM with a time period T. If the length of the pendulum is increased by 21 % the percentage increase in the time period of the pendulum is

(a) 10% (b) 21% (c) 30% (d) 50%

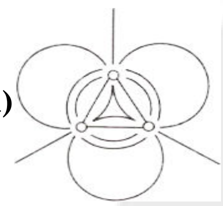
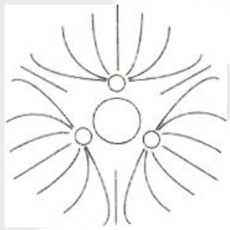
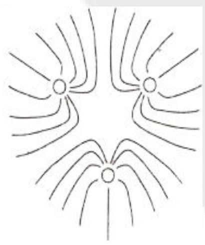
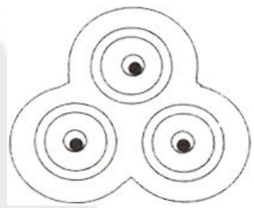
Ans: (a)

Sol: As $T \infty \sqrt{1}$

$\therefore \quad \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_2}} = \sqrt{1.21}$

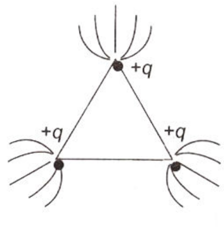
$\Rightarrow \quad T_2 = 1.1\,T = T + 10\%\,(T)$

Q.18 Three positive charges of equal value q are palced at the vertices of an equilateral triangle . The resulting lines of force should be sketched as in

(a)  **(b)**  **(c)**  **(d)** 

Ans: (c)

Sol: Option (a) shows lines of force starting from it, one positive charge and terminating at another. Option (b) has one line of force making closed loop. Option (d) shows all lines making closed loops. All these are not correct. Hence, option (c) is correct.



Q.19 A solid conducting sphere having a charge Q is surrounding by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V.

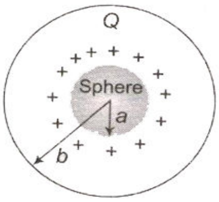
If the shell is now given a charge of -3Q, the new potential differece between the two surfaces is

(a) V **(b) 2V** **(c) 4V** **(d) -2V**

Ans: (a)

Sol: If a and b are radii of spheres and spherical shell, repectively, potential at their surface will be

$$V_{sphere} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{a}\ V_{shell} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{b}$$



and so according to the given problem,

$$V = V_{sphere} - V_{shell} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a},\frac{1}{a}\right] \quad ...(i)$$

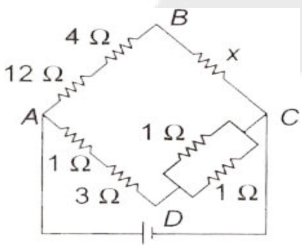
Now when the shell is given a charge -3Q, the potential at its surface and also and inside will change by $v_0 = \frac{1}{4\pi\varepsilon_0}\left[-\frac{3Q}{b}\right]$

So, that now $V_{sphere} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{a}-\frac{3Q}{b}\right]$

and $V_{shell} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{b}-\frac{3Q}{b}\right]$

Hence, $V_{sphere} - V_{shell} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{a}\right] = V$

Q.20 In the combination of resistance shown in fig 32, the potential difference between B and D is zero when unknown resistance (*x*) is



(a) 4Ω **(b)** 2Ω

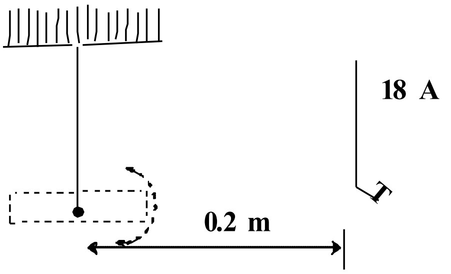
(c) 3Ω **(d) The emf of the cell is reuired**

Ans: (b)

Sol: The potential difference across B, D will be zero when the circuit will act as a balanced wheatstone bridge and

$$\frac{P}{Q}=\frac{R}{S}\Rightarrow\frac{12+4}{x}=\frac{1+3}{1/2}\Rightarrow x = 2Q$$

Q.21 **Fig shows a short magnet executing small oscillations in a vibration magnetometers in Earth's mangnetic field having horizontal component $24\mu T$. The period of oscilation is 0.1 s. When the key K is closed, an upward current of 18 A is established as shown. The new time period is**



Sol: $B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 18}{225} T = 18\ \mu T$

Now $T = 2\pi\sqrt{\frac{I}{MB_H}}$ and $T' = 2\pi\sqrt{\frac{I}{M(B_H - B)}}$

Dividing ,

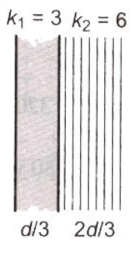
$$\frac{T'}{T} = \sqrt{\frac{B_H}{B_H - B}} \text{ or } \frac{T'}{T} = \sqrt{\frac{24}{24-18}} = 2$$

$T' = 2 \times 0.1 = 0.2s$

Q.22 **A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is *d*. The space between the plates is now filled with two dielectrics. One of the the eielectrics has dielectric constant $k_1 = 3$ and thickness d/3 while the other on has dielectric constant $k_2 = 6$ and thickness 2d/3. Capacitance of the capacitor is now**

Sol: $C = \frac{\varepsilon_0 A}{d} = 9 \times 10^{-12}$ F

with dielectric, $C = \frac{\varepsilon_0 kA}{d}$



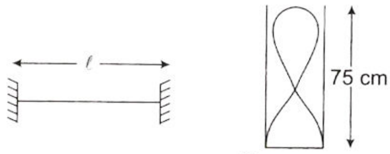
$$C_1 = \frac{\varepsilon_0 A \cdot 3}{d/3} = 9C,\ C_2 = \frac{\varepsilon_0 A \cdot 6}{2d/3} = 9C$$

$C_{total} = \frac{C_1 C_2}{C_1 + C_2}$ [as they are in series]

$$= \frac{9C \times 9C}{18C} = \frac{9}{2} \times C \text{ or } \frac{9}{2} \times 9 \times 10^{-12} \text{ F } C_{total} = 40.5\,\text{pF}$$

Q.23 **A vibrating string of certain length l under the tension T resonates with a mode corresponding to the first overtune (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string alos generates 4 beats/s when excited along with a tuning fork of frequency n, Now when the tension of the string is slightly increased, the number of beats reduces to 2 s. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tunining fork in Hz is**

Sol: Let the frequency of vibrating string be f_1, then vibration frequency of organ pipe (tube) is also f_1.



Let frequency of tunning fork be f_0, then $f_0 - f_{21} = 4 \quad f_1 - f_0 = 4$

As tension in string increases f_1 increases and it is given that beat frequency decreases, so $f_0 - f_1 = 4$ gives correct ralation between f_0 and f_1 so, $f_0 - f_1 = 4$

From tube (organ pipe), $f_1 = 3v/4l$ where v is the velocity of sound wave in air and l_0 is length of tube

So, $f_0 = \dfrac{3 \times 340}{4 \times 3/4} + 4 = 344$ Hz

Q.24 **The minimum intensity of light to be detected by human eye is 10^{10} W/m. The number of photons of wavelength 5.6×10^7 m entering the eye with pupil area 10^{-6} m2 per second for vision is nearly**

Sol: $P = nhv$

$$n = \frac{P}{hv} = \frac{p\lambda}{hc} = \frac{IA\lambda}{hc}$$

$$= \frac{10^{10} \times 10^{-6} \times 5.6 \times 10^{-7}}{6.6 \times 10^{-34} \times 3 \times 10^8}$$

$$= 282.3 \approx 283.$$

Q.25 **A parallel plate capacitor consists of two circular plates each of radius 12 cm and separated by 5.0 mm. The capacitor is being charged by external source. The charging current is constant and is equal to 0.15 A. The rate of change of potential difference between the plates will be**

Sol: $\dfrac{dv}{dt} = \dfrac{I}{C} = \dfrac{I \cdot d}{A \in_0} = 1.87 \times 10^9$ V / s

Part - B - CHEMISTRY

Q.26 **About 0.078 g of hydrocarbon occupy 22.4 g mL of volume at 1 atm and 0^0C. The empirical formula of the hydrocarbon in CH. The molecular formula is :**

(a) C_2H_2 **(b) C_4H_4** **(c) C_6H_6** **(d) C_8H_8**

Ans: (c)

Sol: $M = \frac{0.0821 \times 273 \times 1000}{22.4 \times 1} = 78g$

$n = \frac{78}{13} = 6$

$\therefore$ Molecular formula = $(CH)_6$ or C_6H_6

Q.27 The wave number of electromagnetic radiation emitted during the transition of electron in-between two levels of Li^{2+} ion whose principal quantum numbers sum is 4 and difference is 2 is :

(a) 3.5 R_H **(b) $4R_H$** **(c) $8R_H$** **(d) $\frac{8}{9}R_H$**

Ans: (c)

Sol: $\left.\begin{matrix} n_1 + n_2 = 4 \\ n_1 - n_2 = 4 \end{matrix}\right\}$ so $n_1 = 3$ and $n_2 = 1$

$\bar{v} = R(3)^2 \left\{\frac{1}{(3)^2} - \frac{1}{(1)^2}\right\} = 8R$

Q.28 Strongest intermolecular hydrogen bond is present in the following molecule pairs :

(a) SiH_4 and SiF **(b) $CH_3-\overset{\overset{O}{||}}{C}-CH_3$ and $CHCl_3$**

(c) $H-\overset{\overset{O}{||}}{C}-OH$ and $CH_3-\overset{\overset{O}{||}}{C}-OH$ **(d) H_2O and H_2O_2**

Ans: (c)
Sol: -----------

Q.30 A gaseous mixture containing He, CH_4, and SO_4 was allowed to effuse through a fine hole. Find the molar ratio of the gases coming out initially if the mixture contain He, CH_4 and SO_2 in 1 : 2 : 3 mole ratio :

(a) 2 : 2 : 3 **(b) 6 : 6 : 1** **(c) $\sqrt{2}:\sqrt{2}:3$** **(d) 4 : 4 : 3**

Ans: (d)

Sol: $\frac{n'_{He}}{n_{CH_4}} = \frac{1}{2}\sqrt{\frac{16}{4}} = \frac{1}{1}$

$\frac{n'_{He}}{n_{SO_2}} = \frac{1}{3}\sqrt{\frac{64}{4}} = \frac{4}{3}$

$n'_{He} : n_{CH_4} : n_{SO_2} = 4 : 4 : 3$

Q.31 For which of the following equation will ΔH be equal to ΔU ?

(a) $H_{2(g)} + \frac{1}{2}O_{2(g)} \rightarrow H_2O_{(1)}$ **(b) $H_{2(g)} + I_{2(g)} \rightarrow 2HI_{(g)}$**

(c) $2NO_{2(g)} \rightarrow N_2O_{4(g)}$ **(d) $4NO_{2(g)} + O_{2(g)} \rightarrow 2N_2O_{5(g)}$**

Ans: (b)

Sol: --------

Q.32 **For the system $3A + 2B \rightleftharpoons C$, the concentration of C and D at equilibrium was 0.8 mol /L, then the equilibrium constant K_c will be :**

(a) $\frac{[3A][2B]}{C}$ (b) $\frac{C}{[3A][2B]}$ (c) $\frac{[A]^3[B]^2}{[C]}$ (d) $\frac{[C]}{[A]^3[B]^2}$

Ans: (d)

Sol: Equilibrium constat for the reaction $3A + 2B \rightleftharpoons C$ is $K = \frac{[C]}{[A]^3[B]^2}$

Q.33 **Select the right expression for determining packing fraction (PF) of NaCl unit cell (assume ideal), if ions along an edge diagnol are absent :**

(a) $PF = \frac{\frac{4}{3}\pi(r_+^3 + r_-^3)}{16\sqrt{2}r^3}$ (b) $PF = \frac{\frac{4}{3}\pi(\frac{5}{3}r_+^3 + 4r_-^3)}{16\sqrt{2}r^3}$

(c) $PF = \frac{\frac{4}{3}\pi(\frac{5}{3}r_+^3 + r_-^3)}{16\sqrt{2}r_-^3}$ (d) $PF = \frac{\frac{4}{3}\pi(\frac{7}{2}r_+^3 + 4r_-^3)}{16\sqrt{2}r_-^3}$

Ans: (b)

Sol: Effective number of $Na^+ = 4 - \left(1 + 2\times\frac{1}{4}\right) = \frac{5}{2}$

Effective number of $Cl^- = 4\sqrt{2}\,a = 4r^-$

$$PF = \frac{\text{Volume of effective number of cations and anions}}{\text{Volume of unit cell}}$$

Q.34 **A 0.001 molal solution of $[Pt(NH_3)_4Cl_4]$ in water had a freezing point depression of 0.0054^0C. If K_f for water is 1.80 the correct formulation for the above molecule is :**

(a) $[Pt(NH_3)_4\ Cl_3]Cl$ **(b) $[Pt(NH_3)_4Cl]Cl_2$**
(c) $[Pt(NH_3)_4Cl_2]Cl_3$ **(d) $[Pt(NH_3)_4Cl_4]$**

Ans: (b)

Sol: $\Delta T_f = imK_f$; $0.0054 = i \times 1.8 \times 0.001$

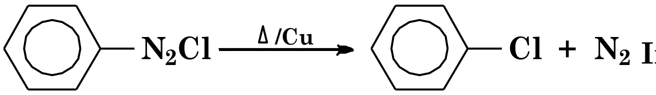
$i = 3$. So it is $[Pt(NH_3)_4Cl]Cl_2$.

Q.35 **In a cell that utilizes the reaction $Zn(s) + 2H^+(aq.) \longrightarrow Zn^{2+}(aq.) + H_2(g)$, addition of H_2SO_4 to cathode compartment will ;**
(a) Increase the E and shift equilibrium to the left
(b) Lower the E and shift equilibrium to the right.
(c) Increase the E and shift equilibrium to the right
(d) Lower the E and shift equilibrium to the left

Ans: (c)

Sol: $E_{cell} = E^o_{cell} + \frac{0.059}{1}\log\frac{[H^+]^2}{P_H \times [Zn^{2+}]} = \frac{\log [H^+]^2}{PH_2, [Zn^{2+}]}$

Addition of H_2SO_4 will increase $[H^+]$ and thus E_{cell} will be more and equilibrium will shift toward right.

Q.36 

In this reaction, half life is independent of concentration of reactant. After 10 min, the volume of N_2 gas is 10 L and after complete reaction, it is 50 L. Hence the rate constant is :

(a) (2.303/10) log 5 min^{-1} **(b) (2.303/10) log 1.25 min^{-1}**

(c) (2.303/10) log 2 min^{-1} **(d) (2.303/10) log 4 min^{-1}**

Ans: (b)

Sol: $K = \frac{2.303}{10}\log\left(\frac{50}{50-10}\right) = \frac{2.303}{10}\log 1.25\,\text{min}$

Q.37 Which of the following methods is used for sol destruction ?

(a) Condensation

(b) Dialysis

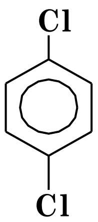
(c) Diffusion through animal membrane

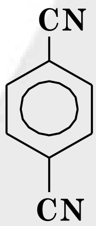
(d) Addition of an electrolyte

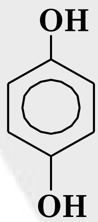
Ans: (d)

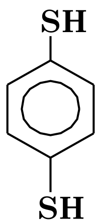
Sol: Traces of electrolytes are essential for stabilizing the sol hence for sol destruction, addition of electrolytes is required.

Q.38 For Which of the following molecules $\mu \neq 0$?

(I) 

(II) 

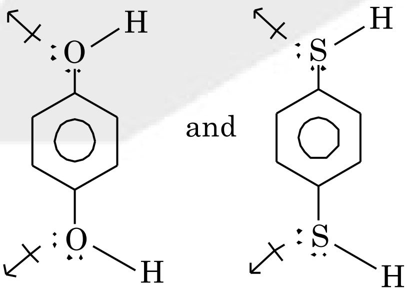
(III) 

(IV) 

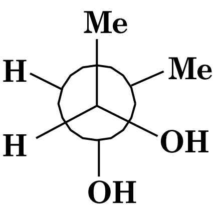
(a) Only I **(b) I and II** **(c) Only III** **(d) III and IV**

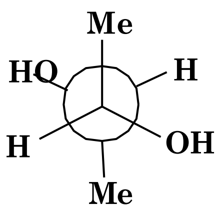
Ans: (d)

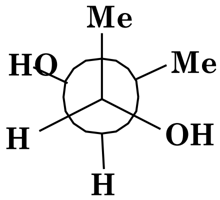
Sol: The net dipole moment is not zero.

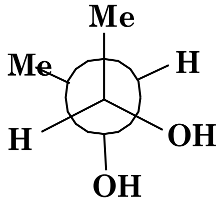


Q.39 Most stable form of meso-2,3-butandiol is :

(a) 

(b) 

(c) 

(d) 

Ans: (a)

Sol: (a) is most stable due to intramolecular H-bonding. (c) and (d) are not meso compounds.

Q.40 Rate of addition of halogen acid (HX) is least in :

(a) $CH_2 = CHCl$ **(b) $CH_2 = CH_2$**

(c) $CH_3 - CH = CH_2$ **(d) $(CH_3)_2\ C = CH_2$**

Ans: (a)

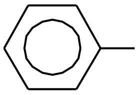
Sol: CH_2 = CH-Cl has got a chlorine atom whose -I effect is so high that is makes the alkene deactivated.

Q.41 Which of the following compound on boiling with $KMnO_4$ (alk,) and subsequent acidification will not give benzoic acid ?

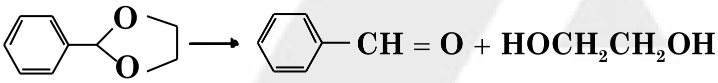
(a) Benzyl alcohol **(b) Acetophenone**

(c) Anisole **(d) Toluene**

Ans: (c)

Sol:  OCH_3 on $KMnO_4$ oxidation does not gives benzoic acid.

Q.42 Which reagent and /or reaction conditions will be best suited to bring about the following conversion ?



(a) $LiAlH_4$, H_2O **(b) H_2O, H_2SO_4**

(c) H_2O, NaOH, heat **(d) PCC, CH_2Cl_2**

Ans: (b)

Sol: --------

Q.43 Nitrosoamines ($R_2N - N = O$) are soluble in water. On heating them with concentrated H_2SO_4, they give secondary amines, The reaction is called :

(a) Perkin's reaction **(b) Fittig's reaction**

(c) Sandmeyer's reaction **(d) Liebermann's nitroso reaction**

Ans: (d)

Sol: (d) Liebermann's nitroso reaction

Q.44 In the estimation of sulphur, organic compound on treating with conc, HNO_3 is converted into :

(a) SO_2 **(b) CH_2**

(c) H_2SO_4 **(d) SO_3**

Ans: (c)

Sol: In Carius method, sulphur of organic compound is converted into H_2SO_4

$$S + H_2O + 30 \xrightarrow[HNO_3]{\Delta} H_2SO_4$$

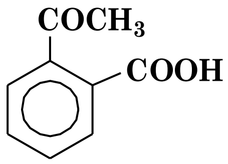
Q.45 In the process of forming mercerized cellulose, the swelling of cellulose is caused by

(a) water **(b) Na_2CO_3**
(c) aqueous NaOH **(d) aqueous HCl**

Ans: (c)

Sol: Cellulose forms a translucent mass on treatment with conc. NaOH which imparts a silky luster to cotton. This process is mercerization and the cotton so produced is known as mercerized cotton.

Q.46 A spirin is a pain reliver with pK_a = 2. Two tablets each containing 0.09 g of asprin are dissolved in 100 mL solution. pH will be :



Sol: Aspirin is a weak acid $= 0.09 \times 2 g / 100$ mL

$= 1.8$ g / 100

$= \frac{1.8}{180}$ mol / L = 0.01 M

pH (weak acid) $= \frac{1}{2}[pK_a - \log C] = \frac{1}{2}[2+2] = 2$

Q.47 Two nuclides X and Y are isotonic to each other with mass number 70 and 72 respectively. If the atomic number of X is 34, then that of Y would be

Sol: Number of neutrons is X = 70 − 34 = 36
= Number of neutrons in Y (being isotonic)
Mass number of Y = 72
$\therefore$ Number of protons in Y = 72 − 36 = 36 = Z

Q.48 A litre of CO_2 gas at 15°C and 1.00 atm dissolves in 1.00 L of water at the same temperature when the pressure of CO_2 is 1.00 atm. Compute the molar concentration of CO_2 in a solution over which the partial pressure of CO_2 is 150 Torr at this temperature

Sol: $n = \frac{pV}{RT} = \frac{(1.00 \text{ atm})(1.00 \text{ L})}{(0.0821 \text{ L atm / mol K})(288 \text{ K})}$

$= 0.0423$ mol

The concentration at 1.00 atm partial pressure is 0.0423 M At 150/760 atm partial pressure, the concentration is

$$0.0423 \times \frac{150}{760} = 8.35 \times 10^{-3} \text{ M}$$

$= 8.35$ mM

Q.49 $\mathbf{MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O \quad E^0 = 1.51V}$

$\mathbf{MnO_2 + 4H^+ + 2e^- \longrightarrow Mn^{2+} + 2H_2O \quad E^0 = 1.23V}$

$\mathbf{E^0_{MnO_4^-} \mid MnO_2}$ **is**

Sol: $MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O \quad E^0 = 1.51V$

$$\Delta G_1^0 = -5(1.51)\ F = -7.55\,F$$

$$MnO_2 + 4H^+ + 2e^- \rightarrow Mn^{2+} + 2H_2O \quad E^0 = 1.23\,V$$

$$\Delta G_2^0 = -2(1.23)\ F = -2.46\,V$$

On substracting

$$MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$$

$$\Delta G_3^0 = -\ 5.09\ \ F$$

$$E^0_{MnO_4^- | MnO_2} = \Delta G_3^0\ / -n\ F$$

$$= -5.09\ F/-3F = 1.70\,V$$

Q.50 Molar conductance of a 1.5 M solution of an electrolyte is found to be 138.9 S cm². The specific conductance of this solution is

Sol: Molar conductance $= \dfrac{k \times 1000}{M}$

$$138.9 = \frac{k \times 1000}{1.5},\ k = 0.208\ S\ cm^{-1}$$

Part - C - MATHEMATICS

Q.51 The value of $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\dfrac{n}{5^n}\right)$ equals

(a) $\dfrac{5}{12}$ (b) $\dfrac{5}{24}$ (c) $\dfrac{5}{36}$ (d) $\dfrac{5}{16}$

Ans: (c)

Sol: We have $S = \dfrac{1}{5} - \dfrac{2}{5^2} + \dfrac{3}{5^3} - \dfrac{4}{5^4} + \dfrac{5}{5^5} ...\infty$ ...(1) multiplying by (common ratio $= \dfrac{1}{5}$) and shift the term we get

$$\frac{S}{5} = \frac{1}{5^2} - \frac{2}{5^3} + \frac{3}{5^4} - \frac{4}{5^5} + ...\infty$$

$$\frac{6S}{5} = \frac{1}{5} - \frac{1}{5^2} + \frac{1}{5^3} - \frac{1}{5^4} + \frac{1}{5^5} .....\infty$$

$$\frac{6S}{5} = \frac{\frac{1}{5}}{1+\frac{1}{5}} = \left(\frac{1}{5}\right)\left(\frac{5}{6}\right) \Rightarrow S = \frac{5}{36} \qquad [\because\ S_\infty = \frac{q}{i-r}]$$

Q.52 If c and d are the roots of the equation $(x-a)\ (x-b) - k = 0$, then a and b are the roots of the equation

(a) $(x+c)\ (x-d)+k=0$ **(b) $(x-c)\ (x-d)-k=0$**

(c) $(x-c)\ (x-d)+k=0$ **(d) $(x+c)\ (x+d)-k=0$**

Ans: (c)

Sol: Since c and d are the roots of the equation $(x-a)(x-b)-\text{k}=0$. Therefore

$(x-a)(x-b)-\text{k} = (x-c)(x-d)$.

$\Rightarrow \quad (x-a)(x-b) = (x-c)(x-d)+\text{k}$

$\Rightarrow \quad (x-c)(x-d)+\text{k} = (x-a)(x-b)$

Clearly, a and b are roots of the equation $(x-a)(x-b)=0$.

$\Rightarrow \quad a,\ b,$ are roots of $(x-c)(x-d)+\text{k}=0$

Q.53 The value of $\sum_{k=1}^{6} \sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}$ is

(a) -1 **(b) 0** **(c) -*i*** **(d) *i***

Ans: (d)

Sol: Let $z = \cos\frac{2\pi k}{7} + i\sin\frac{2\pi k}{7}$

Then by De Moiver's theorem, we have

$$z^{k} = \cos\frac{2\pi k}{7} + i\sin\frac{2\pi k}{7}$$

Now, $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right)$

$$= \sum_{k=1}^{6}(-i)\left(\cos\frac{2\pi k}{7} + i\sin\frac{2k\pi}{7}\right) \qquad \left[\because\ \frac{1}{i} = -i\right]$$

$$= (-i)\sum_{k=1}^{6} z^{k} = -i\frac{z(1-z^{6})}{1-z} = -i\left(\frac{z-z^{7}}{1-z}\right) \qquad \left[\text{By the formula of G.P } S_n = \frac{q(i-r^{n})}{i-r}\right]$$

$$= (-i)\left(\frac{z-1}{1-z}\right) \qquad [\text{using } z^{7} = \cos 2\pi + i\sin 2\pi = 1]$$

$$= (-i)\left(\frac{z-1}{1-z}\right) = i$$

Q.54 Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in N$. Then

(a) There cannot exist any B such that AB = BA
(b) There exists more thant one but finite number B's such that AB = BA
(c) There exists exactly one B such that AB = BA
(d) There exist infinitely many B's such that AB = AB

Ans: (d)

Sol: $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$

$BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$

Hence $AB = BA$ only when $a = b$.

Q.55 **If f $a \neq p,\ b \neq q,\ c \neq r,$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$ then the value of $\frac{p}{p-q}+\frac{q}{q-b}+\frac{r}{r-c}$ is equal to**

(a) -1 **(b) 1** **(c) -2** **(d) 2**

Ans: (d)

Sol: Given $\begin{bmatrix} p & b & c \\ a & q & c \\ a & b & r \end{bmatrix} = 0$

$R_1 \rightarrow R_1 - R_2,\ R_2 \rightarrow R_2 - R_3$ reduces the determinant into

$$\begin{bmatrix} p-a & b-q & 0 \\ a & q-b & c-r \\ a & b & r \end{bmatrix} = 0$$

$\Rightarrow$ $(p-q)(q-b)r + a(b-q)(c-r) - b(p-a)(c-r) = 0$

$\Rightarrow$ Dividing throughout by $(p-q)(q-b)(c-r)$ we get,

$$\frac{r}{r-c}+\frac{a}{p-a}+\frac{b}{q-b}=0$$

$\Rightarrow$ $$\frac{r}{r-c}+1\frac{a}{p-a}+1\frac{b}{q-b}=2$$

$\Rightarrow$ $$\frac{r}{r-c}+\frac{p}{p-a}+\frac{q}{q-b}=2$$

Q.56 **For $2 \leq r \leq n,$ $\binom{n}{r}+2\binom{n}{r-1}+\binom{n}{r-2}=$**

(a) $\binom{n+1}{r-1}$ **(b) $2\binom{n+1}{r+1}$** **(c) $\binom{n+2}{r}$** **(d) $\binom{n+2}{r}$**

Ans: (d)

Sol: $\binom{n}{r}+2\binom{n}{r-1}+\binom{n}{r-2}$

$$=\left[\binom{n}{r}+\binom{n}{r-1}\right]+\left[\binom{n}{r-1}+\binom{n}{r-2}\right]$$

$$=\binom{n+1}{r}+\binom{n+1}{r-1}=\binom{n+2}{r}$$ $[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$

Q.57 **The number of ways in which 10 persons can go in two boats, so that there may be 5 on each boat, supposing that two particular persons will not go in the same boat is**

(a) $\frac{1}{2}({}^{10}C_5)$ **(b) $\frac{1}{2}({}^{8}C_5)$** **(c) $2 \times {}^8C_4$** **(d) ${}^8C_4$**

Ans: (c)

Sol: First omit two particular persons. Remaining eight persons may be four in each boat. This can be done in ${}^{8}C_{4}$ ways. The two paricular persons may be placed in two ways one in each boat

$\therefore$ Total number of ways $= 2 \times {}^{8}C_{4}$

Q.58 The sum of the series $\sum_{n=1}^{\infty} \frac{2n}{(2n+1)!}$ is

(a) e **(b) e^{-1}** **(c) $2e$** **(d) None of these**

Ans: (b)

Sol: We have $= \sum_{n=1}^{\infty} \frac{2n}{(2n+1)!} = \sum_{n=1}^{\infty} \frac{2n+1-1}{(2n+1)!}$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{(2n)!} - \frac{1}{(2n+1)!} \right)$$

$$= \sum_{n=1}^{\infty} \frac{1}{(2n)!} - \sum_{n=1}^{\infty} \frac{1}{(2n+1)!}$$

$$= \left(\frac{e+e}{2} - 1 \right) - \left(\frac{e-e^{-1}}{2} - 1 \right) = e^{-1}$$

Q.59 The points $(-a, -b)$, (a, b), and (a^2, ab) are

(a) Vertices of an equilateral triangle **(b) Vertices of a right-angled triangle**
(c) Vertices of an isosceles triangle **(d) Collinear**

Ans: (d)

Sol: $l_1 = \sqrt{(2a)^2 + (2b)^2} = 2\sqrt{a^2 + b^2}$

$l_2 = \sqrt{(a^2 - a)^2 + b^2(a-1)^2} = (a-1)\sqrt{a^2 + b^2}$

$l_3 = \sqrt{(a^2 - a)^2 + b^2(a+1)^2} = (a+1)\sqrt{a^2 + b^2}$

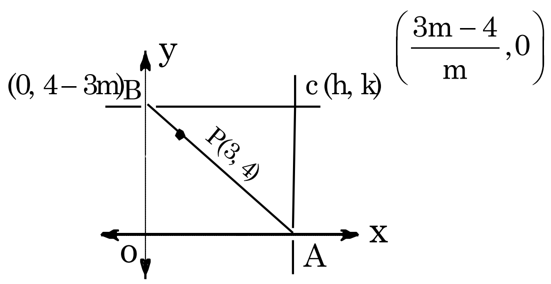
Now $l_1 + l_2 = l_3$. Hence points are collinear.

Q.60 A straight line is drawn through $P(3, 4)$ to meet the axis of x and y at A and B respectively. If the rectangle $OACB$ is completed, then locus of C, is

(a) $\frac{x}{3} + \frac{y}{4} = 1$ **(b) $\frac{4}{x} + \frac{3}{y} = 1$** **(c) $\frac{3}{x} + \frac{4}{y} = 1$** **(d) $\frac{x}{4} + \frac{y}{3} = 1$**

Ans: (c)

Sol:



Equation of any line throught P (3 , 4) with slope m will be

$y-4=m(x-3) \Rightarrow mx-y=3m-4$

Clearly A $\left(\frac{3m-4}{m}, 0\right)$ and B $(0, 4-3m)$

For rectangle OACB coordinates of C

$\left(\frac{3m-4}{m}, 4-3m\right) \equiv C(h, k)$

Now $\frac{k}{h}=-m \Rightarrow m=\frac{-k}{h}$

$\Rightarrow$ $k=4-3m=4-3\left(\frac{-k}{h}\right)$

Therefore, the locus of C(h, k) $y=4+\frac{3y}{x} \Rightarrow \frac{3}{x}+\frac{4}{y}=1.$

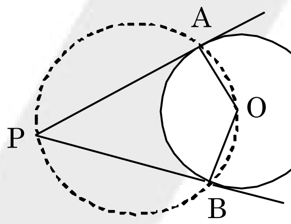
Q.61 Tangent PA and PB are drawn to $x^2+y^2=a^2$ From the point $P\ (x_1, y_1)$. Equation of the circumcircle of triangle PAB is

(a) $x^2+y^2-xx_1-yy_1=0$ **(b) $x^2+y^2+xx_1-yy_1=0$**

(c) $x^2+y^2+xx_1-yy_1=0$ **(d) $x^2+y^2+xx_1+yy_1=0$**

Ans: (a)

Sol:



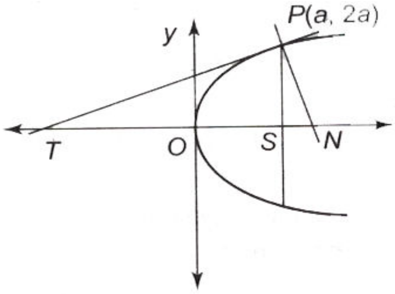
Clearly the point O, A, P and B are concyclic and midpoint of OP is the center of the circle. Thus equation of circumcircle of triangle PAB is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

Q.62 The area of the triangle formed by the tangent and the normal to the parabola $y^2=4ax$, both drawn at the end of the latus rectum, and the axis of the parabola is

(a) $2\sqrt{2}\ a^2$ **(b) $2a^2$** **(c) $4a^2$** **(d) None of these**

Ans: (c)
Sol:



One end of the latus rectum, $P(a, 2a)$. The equation of the tangent PT at $P(a, 2a)$. is $y. = 2a(x+a)$, i.e., $= x+a$. The equation of normal PN at $(a, 2a)$ is $y + x = 2a + a$, i.e. $y + x = 3a$. Solving $y = 0$ and $y = x + a$ we get $x = -a, y = 0$. Solving $y = 0.$ $y + x = 3a, y = 0$. The area of the triangle with vertices $P(a, 2a)$, $T(-a, 0)N(3a, 0)$ $4a^2$.

Q.63 Which of the following equations in parametiric form does not represent a hyperbola, where "t" is a parameter.

(a) $x = \frac{a}{2}\left(t+\frac{1}{t}\right)$ **and** $x = \frac{b}{2}\left(t-\frac{1}{t}\right)$

(b) $\frac{tx}{a} - \frac{y}{b} + t = 0$ **and** $\frac{x}{a} - \frac{ty}{b} - 1 = 0$

(c) $x = e^t + e^{-1}$ **and** $y = e^t - e^{-t}$

(d) $x^2 - 6 = 2 \cos t$ $y^2 + 2 = 4\cos^2 \frac{t}{2}$

Ans: (b)

Sol: ---------

Q.64 If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$ then $f^{-1}(x)$ is

(a) $\left(\frac{1}{2}\right)^{x(x-1)}$

(b) $\frac{1}{2}(1+\sqrt{1+4\log_2 x})$

(c) $\frac{1}{2}(1-\sqrt{1+4\log_2 x})$

(d) not defined

Ans: (b)

Sol: $y = 2^{x(x-1)}$

$\Rightarrow \quad x^2 - x - \log_2 y = 0.$

$x = \frac{1}{2}(1+\sqrt{1+4\log_2 y})$

Since x is +ve we choose only + out of $\pm(\because$ for $y \geq 1$ $\log_2 y \geq 0)$.

$\Rightarrow \quad x = \frac{1}{2}(1+\sqrt{1+4\log_2 y})$

$\Rightarrow \quad f^{-1}(x) = \frac{1}{2}(1+\sqrt{1+4\log_2 x})$

Q.65 The value of $\lim_{x\to\infty}\left(\frac{x^2\sin\left(\frac{1}{x}\right)-x}{1-|x|}\right)$ **is**

(a) 0 **(b) 1** **(c) -1** **(d) None of these**

Ans: (a)

Sol: $\lim_{x\to\infty}\left(\frac{x^2\sin\left(\frac{1}{x}\right)-x}{1-|x|}\right)$

$$= \lim_{x\to\infty}\frac{x^2\sin(x^{-1})-x}{1-x}$$

$$= \lim_{x\to\infty} \frac{\left(\frac{\sin(x^{-1})}{x^{-1}}\right)-1}{x^{-1}-1} = \frac{1-1}{0-1} = 0$$

Q.66 If $f(x) = 2\ \sin^{-1}\sqrt{1-x} + \sin^{-1}\left(2\sqrt{x(1-x)}\right)$ where $x \in \left(0\,\frac{1}{2}\right)$ then $f'(x)$ is equal to

(a) $\frac{2}{\sqrt{x(1-x)}}$ (b) zero (c) $-\frac{2}{\sqrt{x(1-x)}}$ (d) π

Ans: (b)

Sol: $\sqrt{x} = \cos\theta$

$$x \in \left(0,\frac{1}{2}\right) \Rightarrow \sqrt{x} = \cos\theta \in \left(0,\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad \theta \in \left(\frac{\pi}{4},\frac{\pi}{2}\right) \Rightarrow 2\theta \in \left(\frac{\pi}{2},\pi\right)$$

$$\Rightarrow \quad f(x) = 2\sin^{-1}\sqrt{1-\cos^2\theta} + \sin^{-1}\left(2\sqrt{\cos^2\theta\sin^2\theta}\right)$$

$$= 2\sin^{-1}(\sin\theta) + \sin^{-1}(2\sin\theta\cos\theta)$$

$$= 2\theta + \sin^{-1}(\sin 2\theta)$$

$$= 2\theta + \pi - 2\theta$$

$$= \pi$$

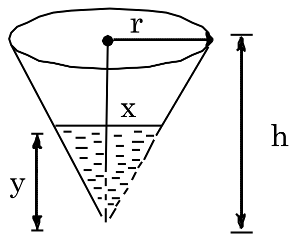
$$\Rightarrow \quad f(x) = 0$$

Q.67 Water runs into an inverted conical tent at the rate of 20 cubic feet per minute and leaks out at the rate of 5 cubic feet per minute. The height of the water in three times the radius of the water's surface. The radius of the water surface in increasing when radius is 5 feet, is

(a) $\frac{1}{5\pi}$ ft / min (b) $\frac{1}{10\pi}$ ft / min (c) $\frac{1}{15\pi}$ ft / min (d) None

Ans: (a)

Sol:



$$\frac{dV}{dt} = 1.5;\ h = 3r$$

$$V = \frac{1}{3}\pi x^2 y;\ \frac{dx}{dt} = ? \quad \text{where } x = 5$$

$$\frac{x}{y} = \frac{r}{h} = \frac{1}{3} \qquad V = \frac{2}{3}\pi r^2 3x = \pi r^3$$

$$\frac{dV}{dt} = 3x r^2 \frac{dx}{dt}$$

$\Rightarrow$ $$15 = 3\pi \cdot 25 \frac{dx}{dt}$$

$\Rightarrow$ $$\frac{dx}{dt} = \frac{1}{5\pi}$$

Q.68 The function $f(x) = \cot^{-1} x + x$ increase in the interval

(a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $(-\infty, \infty)$ (d) $(0, \infty)$

Ans: (c)

Sol: We have $f(x) = \cot^{-1} x + x$

$\therefore$ $$f'(x) = -\frac{1}{1+x^2} + 1 = \frac{x^2}{1+x^2}$$

Clearly $f'(x)$ is > 0 for all x

Hence $f(x)$ is increasing in $(-\infty, \infty)$.

Q.69 $\int \frac{\sin^6 x}{\cos^8 x} dx =$

(a) $\tan^7 x + c$ (b) $\frac{\tan^7 x}{7} + c$ (c) $\frac{\tan 7x}{7} + c$ (d) $\sec^7 x + c$

Ans: (b)

Sol: $\int \frac{\sin^6 x}{\cos^6 x} dx$

$$= \int \frac{\sin^6 x}{\cos^6 x} \cdot \frac{1}{\cos^2 x} dx$$

$$= \int \tan^6 x . \sec^2 x \, dx$$

$$= \frac{\tan^7 x}{7} + c$$

Q.70 If $\int_0^x f(t)\, dt = x + \int_x^1 t\, f(t)\, dt$, then the value of $f(1)$ is

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) $-\frac{1}{2}$

Ans: (a)

Sol: Given $\int_0^x f(t)\, dt = x + \int_x^1 t\, f(t)\, dt$

Differentiate both sides w.r.t. "x" we get

$$f(x) = 1 + (0 + xf(x) = 1 - x\, f(x)$$

At $x = 1,\ f(1) = 1 - f(1)$

$\Rightarrow$ $2f(1) = 1 \ \therefore f(1) = \frac{1}{2}$

Q.71 In a class of 55 students, the number of students studying different subjects are 23 in mathematics, 24 in physics, 19 in chemistry, 12 in mathematics and physics, 9 in mathematics and chemistry, 7 in physics and chemistry, and 4 in all the three subject. The number of students who have taken exactly one subject is

Sol: $n(\text{M}) = 23,\ n(\text{P}) = 24,\ n(\text{C}) = 19$

$n(\text{P} \cap \text{C}) = 12,\ n(\text{M} \cap \text{C}) = 9$

$n(\text{P} \cap \text{C}) = 7$

$n(\text{M} \cap \text{P} \cap \text{C}) = 4$

We have to find $n(\text{M} \cap \text{P}' \cap \text{C}'), n(\text{P} \cap \text{M}' \cap \text{C}'), n(\text{C} \cap \text{M}' \cap \text{P}')$

Now $n(\text{M} \cap \text{P}' \cap \text{C}') = n[\text{M} \cap (\text{P}' \cap \text{C}')]$

$= n(\text{M}) - n(\text{M} \cap (\text{P}' \cap \text{C}'))$

$= n(\text{M}) - n[(\text{M} \cap \text{P}) \cup (\text{M} \cap \text{C})]$

$= n(\text{M}) - n(\text{M} \cap \text{P}) - n(\text{M} \cap \text{C}) + n(\text{M} \cap \text{P} \cap \text{C})$

$= 23 - 12 - 9 + 4 = 27 - 21 = 6$

$n(\text{P}' \cap \text{M}' \cap \text{C}') = n[\text{P} \cap (\text{M}' \cap \text{C})']$

$= n(\text{P}) - n[\text{P} \cap (\text{M} \cap \text{C})]$

$= n(\text{P}) - n[(\text{P} \cap \text{M}) \cup (\text{P} \cap \text{C})]$

$= n(\text{P}) - n(\text{P} \cap \text{M}) - n(\text{P} \cap \text{C}) + n(\text{P} \cap (\text{M} \cap \text{C})]$

$= 24 - 12 - 7 + 4 = 9$

$n(\text{C}' \cap \text{M}' \cap \text{P}')$

$= n(\text{C}) - n(\text{C} \cap \text{P}) - n(\text{C} \cap \text{M}) + n(\text{C} \cap \text{P} \cap \text{M})$

$= 19 - 7 - 9 + 4 = 23 - 16 = 7$

Q.72 If $f(x) = \begin{cases} x, & x < 0 \\ 1, & 0 \le x < 2 \\ -1, & x \ge 2 \end{cases}$ **and** $g(x) = \begin{cases} 3x; & x \le 1 \\ 1; & x > 1. \end{cases}$ **Then the sum of all values of x where $f(x) + g(x)$ is discontinuous is**

Sol: Since $f(x)$ is continuous at $x = 1$ but $g(x)$ is discontinous at $x = 1$, Hence $f(x) + g(x)$ is discontinuous at $x = 0$ and 2.

Thus $f(x) + g(x)$ is discontinuous at $x = 0, 1, 2$.

Q.73 A particle acted by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ displaced from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The total work done by the forces is

Sol: Here, $\vec{F} = \vec{F_1} + \vec{F_2}$

$= (4\hat{i} + \hat{j} - 3\hat{k}) + (3\hat{i} + \hat{j} - \hat{k})$

$= 7\hat{i} + 2\hat{j} - 4\hat{k}$

and $\vec{d} = \vec{d}_2 - \vec{d}_1 = (5\hat{i} + 4\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})$

$$= 4\hat{i} + 2\hat{j} - 2\hat{k}$$

$\therefore$ Work done $= \vec{F} \cdot \vec{d}$

$$= (7\hat{i} + 2\hat{j} - 4\hat{k}) \cdot (4\hat{i} + 2\hat{j} - 2\hat{k})$$

$$= (7) + (4) + (2)(2) + (-4)(-2)$$

$$28 + 4 + 8 = 40 \text{ units}$$

Q.74 $\lim_{x \to \frac{\pi}{4}} \left(\dfrac{1 - \tan x}{1 - \sqrt{2} \sin x} \right)$ **is equal to**

Sol: $\lim_{x \to \frac{\pi}{4}} \left(\dfrac{1 - \tan x}{1 - \sqrt{2} \sin x} \right) = \lim_{x \to \frac{\pi}{4}} \dfrac{-\sec^2 x}{-\sqrt{2} \cos x}$ [applying L' Hospital's rule]

$$= \frac{\sec^2 \frac{\pi}{4}}{\sqrt{2} \cos \frac{\pi}{4}} = \frac{(\sqrt{2})^2}{\sqrt{2} \cdot \frac{1}{\sqrt{2}}} = 2.$$

Q.75 **The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently, is**

Sol: Given word is BANANA.
Here, presence of alphabet A = 3 times and N = 2 times.
Required number of arrangements

$$= \frac{6!}{2!3!} - \frac{5!}{3!} = 60 - 20 = 40$$

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