Aggarwal CLASSES ™
A Division of Aggarwal Educare

JEE (MAIN)

TEST PAPER

SUBJECT : PHYSICS,CHEMISTRY, MATHEMATICS ***TEST CODE : TEST PAPER-5***

ANSWER PAPER

TIME : 3 HRS ***MARKS : 300***

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark(-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or "un- darken" a darkened bubble.

Part A - PHYSICS

Q.1 A vector $\vec{a}$ makes an angle 30^0 and $\vec{b}$ makes an angle 120^0 with X-axis. The magnitude of these vectors are 9 unit and 12 unit respectively. The magnitude of resultant vector is

(a) 23 unit (b) 4 unit (c) 15 unit (d) 3 unit

Ans: (c)

Sol: $\theta = 120^0 - 30^0 = 90^0$

$\therefore |R| = \sqrt{9^2 + 12^2 + 2\times 9\times 12\cos 90^0} = 15$ units

Q.2 A ball of mass m is moving towards a batsman at a speed v. The batsman strikes the ball and deflects it by an angle θ without changing its speed. The impulse imparted to the ball is given by

(a) $mv\cos\theta$ (b) $mv\sin\theta$ (c) $2mv\cos\left(\frac{\theta}{2}\right)$ (d) $2mv\sin\left(\frac{\theta}{2}\right)$

Ans: (d)

Sol: Impulse=change in momentum $= P_f - P_i$

Resultant of two vectors having same magnitude and angle separated θ is,

$R = 2A\cos\frac{\theta}{2}$

$\therefore$ Here, I=$2mv\cos\frac{(180-\theta)}{2} = 2mv\sin\frac{\theta}{2}$

Q.3 At the highest point of the path of a projectile,

(a) kinetic energy is maximum

(b) potential energy is minimum

(c) kinetic energy is minimum

(d) total energy is maximum.

Ans: (c)

Sol: At the highest point of the path, potenial energy is maximum, so the kinetic energy will be minimum.

Q.4 A sphere 'P' of mass 'm' moving with velocity 10 m/s collides head-on with another sphere 'Q' of mass 'm' which is moving with 8 m/s. The ratio of final velocity of 'Q' to initial velocity of 'P' is (e= coefficient of restitution)

(a) $\frac{e-9}{10}$ (b) $\left[\frac{e-9}{10}\right]^{1/2}$ (c) $\frac{e+9}{10}$ (d) $\left[\frac{e+9}{10}\right]^2$

Ans: (c)

Sol: Initial momentum= $mu_1 + mu_2$

Final momentum= $mv_1 + mv_2$

$\therefore mu_1 + mu_2 = mv_1 + mv_2$

$\therefore v_1 = 18 - v_2$

$\because e = \dfrac{v_2 - v_1}{u_1 - u_2}$

$\therefore e = \dfrac{v_2 - (18 - v_2)}{u_1 - u_2} = \dfrac{2v_2 - 18}{2} = v_2 - 9$

$\therefore \dfrac{v_2}{u_1} = \dfrac{e+9}{10}$

Q.5 Assertion: Speed of innerlayers of a whirlwind in a tornadois alarmingly high.
Reason: Moment of inertia is low about tornado's axis of rotation.
(a) Assertionis true, Reason is True
Reason is a correct explanation for Assertion
(b) Assertion is True, Reason is True;
Reason is not a correct explanation for Assertion
(c) Assertion is True, Reason is False.
(d) Assertion is False, Reason is True.

Ans: (a)

Sol: In a whirl wind, air from surrounding regions concentrate in small space. Hence, M.I. decreases. As angular momentum is conserved, decrease in M.I. leads to increase in ω.

Q.6 The pans of a physical balance are in equilibrium. Air is blown under the right hand pan; then the right hand pan will
(a) move up **(b) move down**
(c) move erratically **(d)remain at the same level**

Ans: (b)

Sol: The blowing of air under the pan causes low pressure under it. As the pressure at the upper side of pan is more, this difference in pressure displacesthe pan vertically down.

Q.7 The net force acting on a water molecule when it is situated at surface, middle and bottom layer of water placed in an open glass container in air is repectively (neglecting effect of gravity)
(a) zero, non-zero, zero.
(b) non-zero, zero, zero
(c) non-zero, zero , non-zero
(d) zero, zero, non-zero

Ans: (c)

Sol: As, force of adhesion due to air molecules above the water surface is negligible, for a water molecule situated at this surface, there exists net non-zero force acting in downward direction. For the molecule in middle layer of water, equal forces of cohension act along all directions onit giving rise to zero net force. For the bottom layer molecule, the adhesive force exerted by glass molecles is greater than the cohesive force exerted by water molecules. Hence, anet non-zero downward force acts on the bottom layer molecule.

Q.8 60 g of ice at 0^0 is mixed with 60 g of steam at 100^0 C. At thermal equilibrium, the mixture contains (Latent heat of steam and ice are 540 $cal\ g^{-1}$ and 80 $cal\ g^{-1}$ respectively, specific heat of water=1 $cal\ g^{-1}\ {}^0C^{-1}$)
(a) 80 g of water and 40 g of steam at 100^0 C
(b) 120 g of water at 90^0C
(c) 120 g of water at 100^0C
(d) 40 g of steam and 80 g of water at 0^0C

Ans: (a)

Sol: Heat required to melt ice$=m_iL_i$

$=60\times 80=4800$ cal

Heat required to change the temperature of water at 100^0C (steam)

$=m_s c_w \Delta\theta$

$=601\times(100-0)$

$=6000$ cal

$\therefore$ Total heat $Q_1=10800$ cal

Now, heat required to condense 60 g of steam

$Q_2=60\times 540=32400$ cal

as $Q_2>Q_1$

whole 60 g of steam does not get condensed.

$\therefore$ temperature of mixture remains 100^0C

But Q_1 amount of heat condenses m g of steam

$\therefore M=\frac{Q_1}{L_s}=\frac{10800}{540}=20g$

Hence out of 60 g, 20 g of steam is converted into water

$\therefore$ mixture contains 40 g of steam and 120-40=80 g of water.

Q.9 One mole of an ideal monoatomic gas undergoes a process described by the equation $PV^3=$ constant. The heat apacity of the gas during this process is

(a) R **(b) $\frac{3}{2}R$** **(c) $\frac{5}{2}R$** **(d) 2R**

Ans: (a)

Sol: Given, $PV^3=$ constant

This is a polytropic process with n=3.

$\therefore$ specific heat capacity,

$C=C_v+\frac{R}{1-n}$

$=\frac{R}{\gamma-1}+\frac{R}{1-n}$

$=\frac{R}{\frac{5}{3}-1}+\frac{R}{1-3}$ ....$\left(\because \gamma_{mono}=\frac{5}{3}\right)$

$\therefore$ C=R

Q.10 An ideal gas having f degrees of freedom is isobarically heated. the ratio of the work done by it to the change in its internal energy will be

(a) $\frac{2}{f-2}$ **(b) $\frac{f-2}{2}$** **(c) $\frac{2}{f}$** **(d) $\frac{f}{2}$**

Ans: (c)

Sol: $\frac{\Delta W}{\Delta U}=\frac{(C_p-C_v)\Delta T}{C_v\Delta T}$

$=\frac{C_p}{C_v}-1=\gamma-1$

$=\left(1+\frac{2}{f}\right)-1=\frac{2}{f}$

Q.11 Two particles each of mass 10 g and charge $10\mu C$ are at some distance from each other on a horizontal surface . If the coefficient of static friction between each particle and horizontal surface is 0.5, then what is the distance between them if they are in limitting equilibrium?(g=10 m/s^2)

(a) 3.21 m **(b)1.5 m** **(c)10.32 m** **(d) 4.24 m**

Ans: (d)

Sol: The electric forceon one particledue to other

$=\frac{1}{4\pi\varepsilon_0}\frac{qq}{r^2}=\frac{9\times10^9\times(10\times10^{-6})^2}{(x)^2}N$

The frictional force in limiting case= $\mu_s mg$

$=0.5\times10\times10^{-3}\times10$

=0.05N

$\therefore\frac{9\times10^9\times(10\times10^{-6})^2}{(x)^2}=0.05$

$x^2=\frac{9\times10^9\times10^{-10}}{0.05}$

$=\frac{90}{5}$

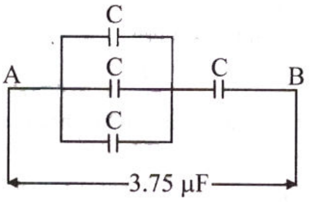
$x=\sqrt{18}=4.24$

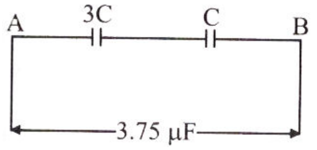
Q.12 When three capacitors of equal capacities are connected in parallel and one capacitor of the same capacity is connected in series with its combination . The resultant capacity is $3.75\mu F$ .The capacity of each capacitor is

(a) 5 μF **(b)6 μF** **(c)7 μF** **(d) 8 μF**

Ans: (a)

Sol:





$$C_{eq} = \frac{3C \times C}{3C + C}$$

$$\therefore \quad 3.75 = \frac{3C^2}{4C}$$

$$\therefore \quad 3.75 = \frac{3C}{4}$$

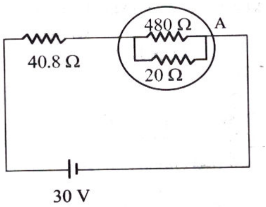
$$\therefore \quad C = \frac{3.75 \times 4}{3} = 5\mu F$$

Q.13 A circuit contains an ammeter, a battery of 30 V and resisitance 40.8 ohm all connected in series. If the ammeter has a coil resisitance 480 ohm and a shunt of 20 ohm, the reading in the ammeter will be:

(a) 1A **(b) 0.5A** **(c) 0.25A** **(d) 2A**

Ans: (b)

Sol:



Here combination of ($480\,\Omega \,||\, 20\,\Omega$) is n series with $40.8\,\Omega$

$$R_{eff} = 40.8 + \frac{480 \times 20}{480 + 20} = 40.8 + 19.2 = 60\Omega$$

$$I = \frac{V_{eff}}{R_{eff}} = \frac{30}{60} = 0.5A$$

Q.14 The deflection of galvanometerfalls from 60 to 20, when 15 Ω shunt is connected across it.The galvanometer resistance is

(a) 15 Ω **(b) 24 Ω** **(c) 30 Ω** **(d) 48 Ω**

Ans: (c)

Sol: $n = \frac{60}{20} = 3,\ S = \frac{G}{n-1} = \frac{G}{3-1} \Rightarrow S = \frac{G}{2}$

$\therefore 2S = G = 2 \times 15 = 30\Omega$

Q.15 The orbital speed of an electron orbiting around the nucleus in a circular orbit of radius r is v. Then the magnetic dipole moment of the electron will be

(a) evr **(b) $\frac{evr}{2}$** **(c) $\frac{ev}{2r}$** **(d) $\frac{vr}{2e}$**

Ans: (b)

Sol: Magnetic dipole moment,

M = iA

$$= \frac{e}{T} \times \pi r^2$$

$$= \frac{e}{\left(\frac{2\pi r}{v}\right)} \times \pi r^2 \qquad \left[\because T = \frac{2\pi r}{v}\right]$$

$$\therefore M = \frac{evr}{2}$$

Q.16 One require 11eV of energy to dissociate a carbon monoxide molecule into carbon and oxygen atoms. The minimum frequency of the appropriate electromagnetic radiation to achieve the dissociation lies in

(a) visible region **(b) infrared region**

(c) ultraviolet region **(d) microwave region**

Ans: (c)

Sol: E=hv, E=11eV=$11 \times 1.6 \times 10^{-19}$ J

$$\therefore v = \frac{E}{h} = \frac{11 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = 2.65 \times 10^{15}\,\text{Hz}$$

This frequency value belongs to UV region.

Q.17 A concave lens of glass refractive index 1.5 has both surfaces of the same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a

(a) convergent lens of focal length 3.5 R

(b) convergent lens of focal length 3.0R.

(c) divergent lens of focal length 3.5 R.

(d) divergent lens of focal length 3.0R.

Ans: (a)

Sol: $\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

For concave lens, R_1=-R,

R_2=-R_1=R

μ_2=1.5, μ_1=1.75

$$\therefore \frac{1}{f} = \left(\frac{1.5 - 1.75}{1.75}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= \left(\frac{0.25}{1.75}\right)\left(\frac{2}{R}\right)$$

$\therefore$ f=+3.5R

Q.18 In Young's double slit experiment, bichromatic light of wavelengths 600 nm and 840 nm are used. The distance between the slits is 0.12 nm and the distance between the plane of the slits and the screen is 120 cm. The minimum distance between two successive region of complete darkness is

(a) 42 mm **(b) 5.6 mm** **(c) 14 mm** **(d) 28 mm**

Ans: (a)

Sol: If n^{th} minima of 600 nm coincides with m^{th} minima of 840 nm then,

$$\frac{(2n+1)D(600)}{2d} = \frac{(2m+1)D(840)}{2d}$$

$$\therefore \frac{2n+1}{2m+1} = \frac{7}{5} \qquad ...(i)$$

For the values of n=3 and m=2, the above condition is satisfied.

For y_1 as distance of 1^{st} minimum,

$$y_1 = \frac{(2n+1)\lambda D}{2D}$$

$$\therefore y_1 = \frac{[2(3)+1]\times(1.2)(600\times10^{-9})}{2\times(0.12\times10^{-3})} = 21 \text{ mm}$$

The next values that satisfies equation (i) are n=10 and m=7.

$$\therefore y_2 = \frac{[2(10)+1]\times(1.2)(600\times10^{-9})}{2\times(0.12\times10^{-3})} = 63 \text{ mm}$$

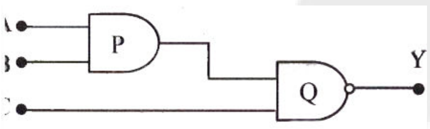
Required distance $= \therefore y_2 - y_1 = (63-21) = 42\text{mm}$

Q.19 In β-decay, the subatomic particles that are emitted can be
(a) positively charged
(b) negatively charged
(c) neutral
(d) any of these

Ans: (d)

Sol: In β-decay, radioactive nucleus emits an electron or positron and a neutrino. Hence, emitted particles can be negatively or positively charged or neutral.

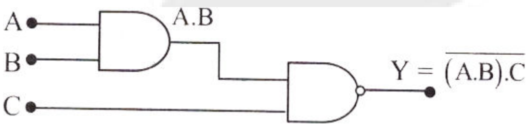
Q.20 What is the output Y in the following circuit, when all the three inputs A,B,C are first 0 and then 1 ?



(a) 1, 1 **(b) 0,1** **(c) 0,0** **(d) 1,0**

Ans: (d)

Sol:

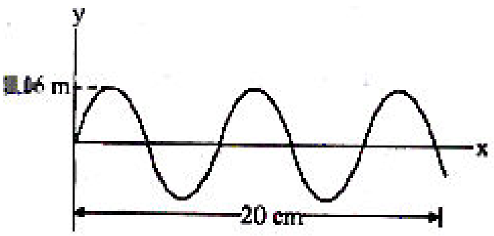


A	B	(A,B)	c	$Y = \overline{(A.B).C}$
0	0	0	0	1
1	1	1	1	0

Q.21 Figure given shows a sinusodial wave on a string. If the frequency of the wave is 200 Hz and the mass per unit length of the string is 0.2 g/m, the power transmitted by the wave is_____?

Sol: Given:
Mass per unit length of the string,

$$\mu = 0.2 \text{ g/m} = 0.2\times10^{-3} \text{ kg/m}$$



Frequency of the wave, A=0.06 m and

$\frac{5}{2}\lambda = 20\,\text{cm}$

$\therefore$ Wavelength of the wave, $\lambda = \frac{40}{5}$ cm

=8 cm

$= 8 \times 10^{-2}$ m

Velocity of the wave, v= $v\lambda$ =200$\times$ 8×10^{-2}

=16 m/s

The power transmitted by the wave is

$P = 2\pi^2 v^2 A^2 \mu v$

Substituing the given values, we get

P= $2 \times (3.14)^2 \times (200)^2 \times (0.06)^2 \times (0.2 \times 10^{-3}) \times 16$

=9.09 W.

Q.22 The percentage decrease in the accleration due to the gravity at a depth of 160 km below the surface of the earth is (Given that radius of the earth is R = 6400 km)

Sol: $g' = g\left(1 - \frac{d}{R}\right)$

$= g\left(1 - \frac{160}{6400}\right)$

$= g\left(1 - \frac{1}{40}\right) = \frac{39g}{40}$

$\therefore g - g' = g - \frac{39g}{40} = \frac{g}{40}$

$\therefore \frac{g - g'}{g} \times 100 = \frac{g}{40} \times \frac{100}{g} = 2.5\%$

Q.23 1 cc of water is taken from the surface to the bottom of the ocean 8 km deep. If volume elasticity of water is 24,000 atmosphere, then the change in volume of water will be?

Sol: $K = \frac{P}{dV/V} = \frac{h\rho gV}{dV}$

$\therefore dV = \frac{h\rho gV}{K} = \frac{8 \times 10^5 \times 1 \times 980 \times 1}{24000 \times 10^5 \times 10} = 0.033$ cc

Q.24 A particle performs S.H.M with amplitude 25 cm and period 3 s. The minimum time required for it to move between two points 12.5 cm on either side of the mean position is_____?

Sol: OP = A = 25 cm and OQ = $\frac{A}{2}$ = 12.5 cm

$\Rightarrow \angle OPQ = 30^0$

Similarly $\angle MNQ = 30^0$

$\therefore \quad \angle PON = 60^0 = \frac{\pi}{3}$

$\therefore \quad \omega t = \frac{\pi}{3}$

$\therefore \quad \frac{2\pi}{T} \times t = \frac{\pi}{3}$

$\therefore \quad t = \frac{T}{6} = \frac{3}{6} = 0.5s$

Q.25 The radius of a uniform wire is r = 0.021 cm.the value of π is given to be 3.142. What is the area of cross section of the wire upto approximate significant figures?

Sol: Area, A= $\pi r^2 = 3.142 \times (0.021)^2$

$= 0.00138562\ cm^2$

There are only two significant figures in 0.021 cm. Hence the result must be rounded off to two significant figures i.e., A = 0.0014 cm^2

Part - B - CHEMISTRY

Q.26 _______________ are the smallest particles of matter was one of the assumptions of Dalton's atomic theory.

(a)Atoms **(b) Molecules** **(c) Ions** **(d) Elements**

Ans: (a)

Q.27 The energy of a photon is calculated by

(a) $E = h\upsilon$ **(b)** $E = \frac{h}{\upsilon}$ **(c)** $E = \upsilon\lambda$ **(d)** $E = \frac{h\lambda}{c}$

Ans: (a)

Q.28 The group having isoelectronic species is _______________.

(a) $O^{2-}, F^-, Na^+, Mg^{2+}$ **(b)** O^-, F^-, Na, Mg^+

(c) O^{2-}, F^-, Na, Mg^{2+} **(d)** O^-, F^-, Na^+, Mg^{2+}

Ans: (a)

Sol: Isoeletronic species have same number of electrons.

Species	O^{2-}	F^{-}	Na^{+}	Mg^{2+}
Z	8	9	11	12
Charge	–2	–1	+1	+2
Total no. of electrons	10	10	10	10

Q.29 Predict the CORRECT order among the following ____________,
(a) bond pair-bond pair> lone pair-bond pair> lone pair-lone pair
(b) lone pair- bond pair> bond pair-bond pair> lone pair -lone pair
(c) lone pair-lone pair > lone pair-bond pair> bond pair-bond pair
(d) lone pair- lone pair bond pair-bond pair> lone pair - bond pair

Ans: (c)

Sol: The forces of repulsion between electron pairs decrease in the following order:
lone pair- lone pair> lone pair- bond pair> bond pair- bond pair

Q.30 For a sample of perfect gas when its pressure is changed isothermally from P_i to P_f, the entropy change is given by

(a) $\Delta S = RT\, In\left(\frac{P_i}{P_f}\right)$ **(b)** $\Delta S = nR\, In\left(\frac{P_f}{P_i}\right)$

(c) $\Delta S = nR\, In\left(\frac{P_i}{P_f}\right)$ **(d)** $\Delta S = nRT\, In\left(\frac{P_f}{P_i}\right)$

Ans: (c)

Sol: The entropy change of a system in a process is equal to the amount of heat transfferred to it in a reversible manner divided by the temperature at which the process occurs.

$$\therefore \quad \Delta S = \frac{q_{rev}}{T}$$

Now, q_{rev} for an isothermal changes given as:

$$q_{rev} = -w = nRT\, In\left(\frac{P_i}{P_f}\right)$$

$$\therefore \quad \Delta S = \frac{nRT\, In\left(\frac{P_i}{P_f}\right)}{T}$$

$$= nR\, In\left(\frac{P_i}{P_f}\right)$$

Q.31 Assertion: All Arrehenius acids are also Bronsted acids but all Arrehenius bases are not Bronsted bases.

Reason: An Arrehenius base is a substance which gives OH^- ion in the solution. A Bronsted base is a substance which accepts a proton.
(a) Assertion and reason are true. Reason is the correct explanation of Assertion.
(b) Assertion and Reason are true. Reason is not the correct explanation of Assertion.
(c) Assertion is true. Reason is false.

(d) Assertion is false. Reason is true.

Ans: (a)

Q.32 Which of the following statement is CORRECT?

(a) Reduction is a substance which increases the oxidation number of other substance.
(b) Oxidising agent undergoes a decrease in its own oxidation number.
(c) Reduction means loss of electrons.
(d) Reducing agent is a substance that accepts electrons.

Ans: (b)

Q.33 Heavy water is ______________.

(a) known as hard water
(b) water of mineral springs
(c) water obtained by repeated distillation and condensation of odinary water
(d) odinary water containing dissolved salts and heavy metals.

Ans: (c)

Q.34 Choose the CORRECT order of metallic character from the options given below.

(a) $K<Ca>Ba$ **(b) $K>Ca>Ba$** **(c) $K<Ca<Ba$** **(d) $K>Ca<Ba$**

Ans: (d)

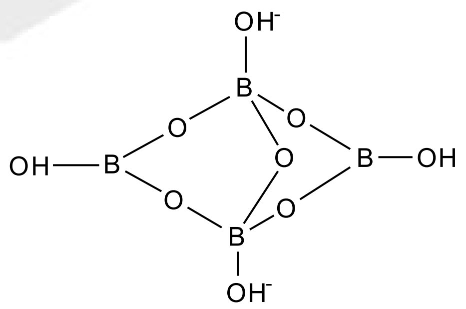
Sol: Alkali metals are more metallic than alkaline earth metals due to mobile ns^1 electrons. Among alkaline earth metals, the metallic character increases down the group with increasing size.

Q.35 In diborane, ______________.

(a) 2 bridged hydrogen atoms are present
(b) 3 bridged hydrogen atoms are present
(c) 4 bridged hydrogen atoms are present
(d) 5 bridged hydrogen atoms are present

Ans: (a)
Structure of diborane

Q.36 The following structural unit is present in ______________.

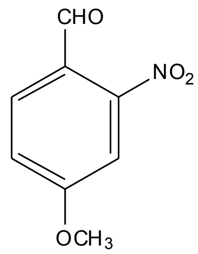


(a) borane **(b) borax** **(c) boric acid** **(d) silicates**

Ans: (b)

Sol: The structural unit is $[B_4O_5(OH)_4]^{2-}$ which is present in borax, $Na_2[B_4O_5(OH)_4].8H_2O$.

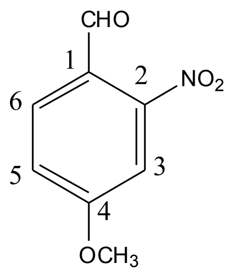
Q.37 IUPAC name of the compound is ______________.



(a) 2- Formyl-5-methoxynitrobenzene
(b) 4-Formyl-3-nitroanisole
(c) 4-Methoxy-2-nitrobenzaldehyde
(d) 4-Formyl-5-nitroanisole

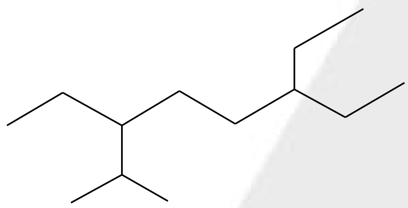
Ans: (c)

Sol:



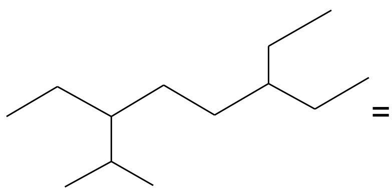
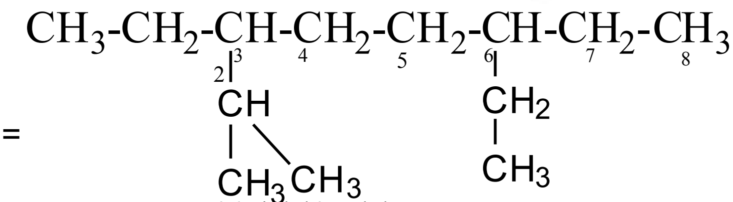
Principal functional group is -CHO
∴ IUPAC name is 4-Methoxy-2-nitrobenzaldehyde.

Q.38 The CORRECT IUPAC name of the following alkane is ______________.

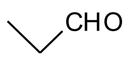
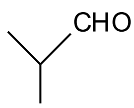


(a) 3,6- Diethyl-2-methyloctane
(b) 6-Isopropl-3-ethyloctane
(c) 2-Methyl-3-ethyloctane
(d)3-Isopropyl-6-ethyloctane

Ans: (d)

Sol:  = 

3,6-Diethyl-2-methyloctane

Q.39 Predict the IUPAC name of the alkene which would give  **and**  **on reductive ozonolysis?**
(a) 2-Methylhex-3-ene
(b) 4-Methylpent-2-ene
(c) 2-Methylpent-3-ene
(d) 5-Methylhex-3-ene

Ans: (a)

Sol: $CH_3\text{-}\underset{\substack{|\\CH_3}}{CH}\text{-}CH=CH\text{-}CH_2\text{-}CH_3 \xrightarrow[\text{(ii) } Zn/H_2O]{\text{(i)}O_3} CH_3\text{-}CH_2\text{-}CHO + CH_3\text{-}\underset{\substack{|\\CH_3}}{CH}\text{-}CHO$

2-Methylhex-3-ene → Propanol + 2-Methylpropanol

Q.40 The number of types of bonds between two carbon atoms in calcium carbide is __________.

(a) two sigma, one pi
(b) position isomer
(c) functional isomer
(d)optical isomer

Ans: (c)

Sol: Calcium carbide $[CaCl_2]$ is $Ca^{2+}[C \equiv C]^{2-}$

Q.41 Identify the INCORRECT match:

	Term		Unit
(A)	**Conductance**	→	Ω^{-1}
(B)	**Molar conductivity**	→	$\Omega^{-1}m^2mol^{-1}m$
(C)	**Resitivity**	→	Ωm^{-1}
(D)	**Conductivity**	→	$\Omega^{-1}m^{-1}$

Ans: (c)

Sol: The unit of resitivity is $\Omega^{-1}m^{-1}$ or Sm^{-1}

Q.42 When initial concentration of a reactant of a reactant is doubled in a reaction , its half-time period is not affected. The order of the reaction is __________.

(a) zero **(b) first**
(c) second **(d) more than zero but less than first**

Ans: (b)

Sol: The half-life of first order reaction is given by equation $t_{1/2} = \frac{0.693}{k}$

The equation implies that the half-life of a first order reaction is constant and is independent of the reactant concentration. Since, in given reaction, chnage in concentration does not affect its half-life; it must be a first order reaction.

Q.43 Which of the following adsorption willNOT show the below mentioned general characteristics?

(i) High enthalpy of adsorption.
(ii) High activation energy.
(iii)Formation of unilayer of adsorbate on adsorbent surface.
(a) Adsorption of acetic acid in solution by charcoal.
(b) Adsorption of oxygen on Pt.
(c) Adsorption of CO on W.
(d) Adsorption of H_2 on Ni.

Ans: (a)

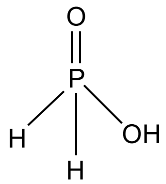
Sol: The given general characteristics are of chemisorption. Among the options, only option (A) is an example of physisorption. Hence, option (A) will not show the given characteristics.

Q.44 Strong reducing behaviour of H_3PO_2 is due to ____________.

(a) low/high oxidation state of phosphorous
(b) presence of two -OH groups and one P-H bond
(c) presence of one -OH group and two P-H bonds
(d) high electron gain enthalpy of phosphorous

Ans: (c)

Sol:



Hypophosphorous acid (H_3PO_2)

H_3PO_2 is a monobasic acid since it contains one P-OH bond. It shows strong reducing behaviour due to the presence of two P-H bonds.

Q.45 Identify the INCORRECT statement regarding 3d transition series.

(a)The strength of metallic bonds increases from Sc to Cr and then it decreases.
(b) The second ionization enthalpy of Cr is high because of stable d^5 configuration.
(c) The number of oxidation states increases from Sc to Mn and then decreases from Fe to Zn.
(d) The first ionization enthalpy decreases from Sc to Zn as the atomic number increases.

Ans: (d)

Sol: The first ionization enthalpy increases with some irregularities as the atomic number increases in the 3d transtition series. The irregularities are due to the screening effect of added (n-1) d-electrons.

Q.46 Praveen went to a hill station where he experienced some breathing problems due to low density of oxygen. If at NTP density of oxygen is 1.520 g L^{-1}, what is the difference in densities, if the temperature at the hill station is 3^0 C and pressure is 705 mm. The hill is at a height of 2173 m.

Sol: $d = \frac{PM}{RT}$ OR $\frac{d_1}{d_2} = \frac{P_1}{T_1} \times \frac{T_2}{P_2}$

$P_1 = 760\,\text{mm}, T_1 = 298\,\text{K}, P_2 = 705\ \text{mm}$

$T_2 = 276\,\text{K}$

$d_1 = 1.52\,\text{g}\,L^{-1}, d_2 = ?\, d_2 - d_1 = ?$

$$\frac{1.52}{d_2} = \frac{760}{298} \times \frac{276}{705}$$

$$d_2 = \frac{319336.8}{209760} = 1.522$$

$d_2 - d_1 = 1.522 - 1.52$

$d_2 - d_1 = 0.002$

Q.47 A metal crystallizes into two cubic phases, face centred cubic (FCC) and body centred cubic (BCC) whose unit cell lengths are 3.5 and 3.0 $\overset{0}{A}$, respectively. Calculate the ratio of the densities of FCC and BCC.

Sol: FCC has 4 atoms in a unit cell.
BCC has 2 atoms in a unit cell.

$$d = \frac{n \times M}{N_0 \times a^3}$$

$$\frac{d_{FCC}}{d_{BCC}} = \frac{4}{2}\frac{(3.0)^3}{(3.5)^3} = 1.26$$

Q.48 A mixture contains N_2O_4 and NO_2 in the ratio of 2 : 1 by volume. The vapour density of the mixture is _____?

Sol: Let the volume of mixture = 100 mL

Volume of $N_2O_4 = \frac{2}{3} \times 100 = \frac{200}{3}$

Volume of $NO_2 = \frac{100}{3}$

Vapour density of $N_2O_4 = \frac{\text{Mol.wt.}}{2} = \frac{92}{2} = 46$

Vapour density of $NO_2 = \frac{46}{2} = 23$

Vapour of density of mixture = d
hence,
mass of mixture = mass of NO_2 + Mass of N_2O_4

$$100 \times d = \frac{100}{3} \times 23 + \frac{200}{3} \times 46$$

Vapour density $(d) = \frac{115}{3} = 38.33$

Q.49 The pH 0.10 M NH_3 solution is [Given $K_b = 1.8 \times 10^{-5}$; log 1.35 = 0.13]

Sol: $H_2OH^+ \rightleftharpoons H^+ + OH^-$

$K_w = [H^+][OH^-]$

$10^{-14} = [H^+][OH^-]$

or $pH + pOH = 14$

Find the value of OH^- and calculate the values of pH and pOH.

$NH_3 + H_2O \rightleftharpoons \overset{+}{N}H_4 + OH^-$

$$K_b = \frac{[NH_4^+][OH^-]]}{[NH_3]} = \frac{x^2}{0.10}$$

$x^2 = 1.8 \times 10^{-6}$ $x = 1.35\times10^{-3} = [OH^-]$

$pOH = 2.87$ $pH = 11.13$

Q.50 In an aqueous solution $AgNO_3$ and $CuSO_4$ are connected in series. If Ag deposited at cathode is 1.08 g, Then Cu deposited is_____?

Sol: Eq. Of Ag = Eq. Of Cu

$$\frac{1.08}{108} = \frac{W_{Cu}}{63.5/2} \text{ or}$$

$$W_{Cu} = \frac{63.5 \times 1.08}{2 \times 108} = 0.3175 \text{ g}$$

Part - C - MATHEMATICS

Q.51 If P(x)=$ax^2 + bx + c$**, Q(x)=**$-ax^2 + dx + c$ **where** $ac \neq 0$**, then the equation P(x)Q(x)=0 has**
(a) all four real roots.
(b) atleast two real roots.
(c) exactly two real roots.
(d) all four non-real roots.

Ans: (b)

Sol: as, $ax^2 + bx + c = 0$

and $-ax^2 + dx + c = 0$

as, $ac + 0$

i.e

$ac > 0$ or $ac < 0$

If $ac < 0$, then,

b^2 $4ac$

b^2 is always + ve

$D > 0$ – so, two real roots and taking about,

for, $-ax^2 + dx + c = 0$

$D = b^2 - 4ac$

$= d^2 - 4(-a)c$

$= d^2 + 4ac$

If $ac < 0$,

$D < 0 \rightarrow$ may be

ac>0, for $ax^2 bx + c \rightarrow D < 0$

for $-ax^2 + dx + c \rightarrow D > 0$

So, at least two real roots.

Q.52 If $\tan^2\theta-(1+\sqrt{3})\tan\theta+\sqrt{3}=0$, then the general value of θ is

(a) $n\pi+\frac{\pi}{4}, n\pi+\frac{\pi}{3}$ **(b)** $n\pi-\frac{\pi}{4}, n\pi+\frac{\pi}{3}$ **(c)** $n\pi+\frac{\pi}{4}, n\pi-\frac{\pi}{3}$ **(d)** $n\pi-\frac{\pi}{4}, n\pi-\frac{\pi}{3}$

Ans: (a)

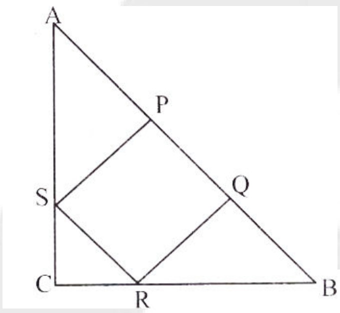
Sol: We have $\tan^2\theta-(1+\sqrt{3})\tan\theta+\sqrt{3}=0$

$\Leftrightarrow (\tan\theta-\sqrt{3})(\tan\theta-1)=0$

$\Rightarrow \tan\theta=\sqrt{3}$ or $\tan\theta=1$

$\Rightarrow \theta=n\pi+\frac{\pi}{3}, n\pi+\frac{\pi}{4}$

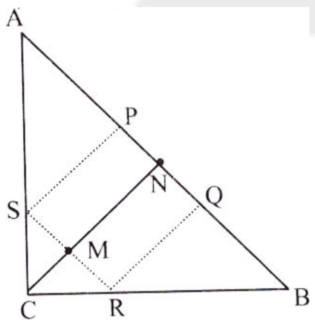
Q.53 PQRS is a square inscribed in a right triangle ACB. Let h be the distance of the vertex C from AB. If $\frac{1}{h}+\frac{1}{c}=\frac{2}{3}$, then the length of side of the square is when AB is equal to C



(a) $\frac{2}{3}$ **(b)** 3 **(c)** $\frac{3}{2}$ **(d)** 2

Ans: (c)

Sol:



Drop perpendicular from the vertex C to AB, that meets SR and PQ at M and N, respectively,

Let $|SR|$=x=side length of the square

$$\frac{|SR|}{|AB|}=\frac{|CM|}{|CN|}$$

$\Leftrightarrow \frac{x}{c} = \frac{h-x}{h} \Leftrightarrow 1 - \frac{x}{h} = \frac{x}{c}$

$\Leftrightarrow \frac{1}{h} + \frac{1}{c} = \frac{1}{x} \Rightarrow x = \frac{3}{2}$ $\quad ...\left[\frac{1}{h} + \frac{1}{c} = \frac{2}{3}(\text{given})\right]$

Q.54 If complex numbers z_1, z_2, and z_3 represent the vertices A, B and C respectively of an isosceles triangle ABC of which $\angle C$ is right angle, then correct statement is

(a) $z_1^2 + z_2^2 + z_3^3 = z_1 z_2 z_3$

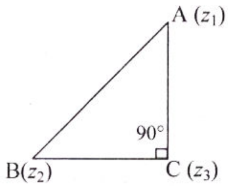
(b) $(z_3 - z_1)^2 = z_3 - z_2$

(c) $(z_1 - z_2)^2 = (z_1 - z_3)(z_3 - z_2)$

(d) $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$

Ans: (d)

Sol: **$|CB| = |CA|$ and $\angle C = \frac{\pi}{2}$**



$\Rightarrow (z_2 - z_3) = (z_1 - z_3)e^{\frac{i\pi}{2}}$

$= i(z_1 - z_3)$

$\Rightarrow (z_2 - z_3)^2 = -(z_1 - z_3)^2$

$\Rightarrow z_2^2 + z_3^2 - 2z_2z_3 = -z_1^2 - z_3^2 + 2z_1z_3$

$\Rightarrow z_1^2 + z_2^2 - 2z_1z_2 = 2z_1z_3 + 2z_2z_3 - 2z_3^2 - 2z_1z_2$

$\Rightarrow (z_1 - z_2)^2 = 2[(z_1z_3 - z_3^2) - (z_1z_2 - z_2z_3)]$

$\Rightarrow (z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$

Q.55 If m is A.M. of two distinct real number l and n(l, n> 1) and G_1, G_2 and G_3 are three geometric meated between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals

(a) $4l^2mn$ **(b) $4lm^2n$** **(c) $4lmn^2$** **(d) $4l^2m^2n^2$**

Ans: (b)

Sol: Since, G_1, G_2 and G_3 are three geometric means between l and n.

$\therefore \quad G_1 = l\left(\frac{n}{l}\right)^{\frac{1}{4}}, G_2 = l\left(\frac{n}{l}\right)^{\frac{2}{4}}, G_3 = l\left(\frac{n}{l}\right)^{\frac{3}{4}}$

$\therefore \quad G_1^4 + 2G_2^4 + G_3^4 = l^4\left(\frac{n}{l}\right) + 2l^4\left(\frac{n}{l}\right)^2 + l^4\left(\frac{n}{l}\right)^3$

$= nl^3 + 2n^2l^2 + n^3l$

$= 2n^2l^2 + nl(l^2 + n^2)$

$= nl(l + n)^2$

$= nl(2m)^2 \quad \ldots\left[\because m = \frac{l+n}{2} \Rightarrow 2m = l + n\right]$

$= 4lm^2n$

Q.56 If a>2b>0, then the positive value of m for which y=mx-b $\sqrt{1+m^2}$ is a common tangent to $x^2+y^2=b^2$ and $(x-a)^2+y^2=b^2$ is

(a) $\frac{2b}{\sqrt{a^2-4b^2}}$ (b) $\frac{\sqrt{a^2-4b^2}}{2b}$ (c) $\frac{2b}{a-2b}$ (d) $\frac{b}{a-2b}$

Ans: (a)

Sol: Any tangent to $x^2+y|=b^2$ is

$y = mx - b\sqrt{1+m^2}$

$(x-a)^2 + y^2 = b^2$

$\left|\frac{ma - b\sqrt{1+m^2}}{\sqrt{m^2+1}}\right| = b \Rightarrow ma = 2b\sqrt{1+m^2}$

$\Rightarrow m^2a^2 = 4b^2 + 4b^2m^2$

$\therefore m = \pm\frac{2b}{\sqrt{a^2-4b^2}}$

Q.57 Tangents are drawn through a point P to the ellipse $4x^2+5y^2=20$ having inclinations α and β such that $\tan(\alpha+\beta)$=k, the locus of P is

(a) $k(x^2-y^2-1) = 2xy$ (b) $k(x^2+y^2-1)=xy$ (c) $k(x^2+y^2)=2xy$ (d) none of these

Ans: (A)

Sol: We have $\frac{x^2}{5} + \frac{y^2}{4} = 1$

$\therefore$ The equation of a tangent in slope form is

$y = mx \pm \sqrt{5m^2+4}$

It passes through $P(x_1, y_1)$

$\Rightarrow y_1 = mx_1 \pm \sqrt{5m^2+4}$

Squaring both sides,

$y_1^2 - 2mx_1y_1 + m^2x_1^2 = 5m^2 + 4$

Now, $m_1 + m_2 = \frac{2x_1y_1}{x_1^2 - 5}$

and $m_1m_2 = \frac{y_1^2 - 4}{x_1^2 - 5}$

Now, $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=k$

$\Rightarrow \frac{m_1+m_2}{1-m_1m_2}=k$

$\Leftrightarrow m_1+m_2=k(1-m_1m_2)$

$\Leftrightarrow \frac{2x_1y_1}{x_1^2-5}=k\left(1-\frac{y_1^2-4}{x_1^2-5}\right)$ ...[From (i) and (ii)]

$\Leftrightarrow 2x_1y_1=k(x_1^2-y_1^2-1)$

$\Rightarrow$ the locus of $P(x^1,y^1)$ is $k(x^2-y^2-1)=2xy$

Q.58 Let S be the sum of the digits of the coefficient of x^6 in the expansion of $(1+2x-3x^2)^4$. Then which of the following statements is not correct?
(a)S is the square of an integer
(b)S is divisible by 3
(c)G.C.D of S and 6 is 6.
(d)S is divisible by 9

Ans: (c)

Sol: The general term of the expansion is

$\frac{4!}{p!q!r!}.1^p(2x)^q(-3x^2)^r$, where p+q+r=4

$=\frac{4!}{p!q!r!}(2)^q(-3)^r x^{q+2r}$

We require p, q, r such that
p+q+r=4 and q+2r=6
$\Rightarrow$ p=1, q=0 and r=3
or p=0, q=2 and r=2
$\Rightarrow$ The coefficient of x^6

$=\frac{4!}{1!0!3!}2^0(-3)^3+\frac{4!}{0!2!2!}(2)^2(-3)^2$

=-108+216=108

$\Rightarrow S=9$
S is perfect square of an integer (true)
S is divisible by 3 and 9 (true)
S and 6 have G.C.D.=3

Q.59 A purse contains two 50 paise coins, four 25 paise coins and six 10 paise coins, 5 coins are taken out from the purse (at random). The probability that the sum taken out is atleast Rs. 1.50 is

(a) $\frac{40}{^{12}C_5}$ **(b)** $\frac{42}{^{12}C_5}$ **(c)** $\frac{41}{^{12}C_5}$ **(d)42**

Ans: (b)

Sol:

$2-50\,P$
$4-25\,P$
$6-10\,P$

Total coins -12

Selection of 5 coin out og 12 coin is given by $12c_5$
Sum taken out a attent Rs150

$\text{Probability} = 2_{c2} \times 4_{c3} +$

$$\text{Probability} = \frac{2c_2 + 4c_3 + 2c_1 \times 4c_4 + 2c_2 \times 4c_2 \times 6c_3}{12c_5}$$

$$= \frac{92}{12c_5}$$

Q.60 For a moderately skewed distribution, quatile deviation and the standard deviation are related by

(a) S.D=$\frac{2}{3}$Q.D. **(b) S.D.=$\frac{3}{2}$Q.D.** **(c) S.D.=$\frac{3}{4}$Q.D.** **(d) S.D.=$\frac{4}{3}$Q.D.**

Ans: (b)

Q.61 One hundred identical coins, each with probability p of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is

(a) $\frac{1}{10}$ **(b) $\frac{49}{101}$** **(c) $\frac{50}{101}$** **(d) $\frac{51}{101}$**

Ans: (d)

Sol: We have ${}^{100}C_{50}p^{50}(1-p)^{50}$

$={}^{100}C_{51}p^{51}(1-p)^{49}$

$$\Rightarrow \frac{1-p}{p} = \frac{100!}{51!49!} \times \frac{50!.50!}{100!} = \frac{50}{51}$$

$$\Rightarrow 51 - 51p = 50p \Rightarrow p = \frac{51}{101}$$

Q.62 Let a covergent sequence $< b_n >$ of real numbers satisfy the recurrence relation:

$$b_{n+1} = \frac{1}{3}\left(2b_n + \frac{125}{b_n^2}\right), b_n \neq 0 \text{, then } \lim_{n\to\infty} b_n =$$

(a) is 0 **(b) does not exist**

(c) is 5 **(d) $\frac{2}{3}$**

Ans: (c)

Sol: Let $\lim_{n\to\infty} b_n = b.$

$$\Rightarrow \lim_{n\to\infty} b_{n+1} = \frac{1}{3}\left(2\lim_{n\to\infty} b_n + \frac{125}{\lim_{n\to\infty} b_n{}^2}\right)$$

$$\Rightarrow b = \frac{1}{3}\left(2b + \frac{125}{b^2}\right)$$

$$\Rightarrow b = \frac{125}{b^2} \Rightarrow b^3 = 125 \Rightarrow b = 5$$

Q.63 The derivative of $\tan^{-1}\dfrac{x}{\sqrt{a^2-x^2}}$ **is**

(a) $\dfrac{1}{a^2-x^2}$ **(b)** $\dfrac{a}{a^2-x^2}$ **(c)** $\dfrac{a}{\sqrt{a^2-x^2}}$ **(d)** $\dfrac{1}{\sqrt{a^2-x^2}}$

Ans: (d)

Sol: Let y= $\tan^{-1}\dfrac{x}{\sqrt{a^2-x^2}}$

Substituting $\theta = \sin^{-1}\left(\dfrac{x}{a}\right) \Rightarrow x = a\sin\theta$

$$y = \tan^{-1}\left[\frac{a\sin\theta}{\sqrt{a^2 - a^2\sin^2\theta}}\right] = \tan^{-1}\left[\frac{a\sin\theta}{a\cos\theta}\right]$$

$$\Rightarrow y = \tan^{-1}(\tan\theta) = \theta = \sin^{-1}\frac{x}{a}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin^{-1}\frac{x}{a}\right) = \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}.\frac{1}{a}$$

$$= \frac{a}{\sqrt{a^2-x^2}}.\frac{1}{2a} = \frac{1}{\sqrt{a^2-x^2}}$$

Q.64 The interval of values of a such that every tangent to the curve y=x³-ax²+x+1 has acute inclination, is

(a) $(-\sqrt{2}, \sqrt{2})$ **(b)** $\left(-\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right)$ **(c)** $\left(-\sqrt{3}, \sqrt{3}\right)$ **(d)** $\left(-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

Ans: (c)

Sol: $y=x^3-ax^2+x+1$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 2ax + 1$$

For acute inclination , $\frac{dy}{dx} > 0$

$\Leftrightarrow 3x^2 - 2ax + 1 > 0$ for all x

$\Rightarrow 4a^2 - 12 < 0 \Leftrightarrow a^2 < 3$

$\Leftrightarrow -\sqrt{3} < a < \sqrt{3}$

Q.65 $\int \frac{\cos^{-1} x}{x^3} dx$ **equals**

(a) $-\frac{1}{2x^2}\cos^{-1} x + \frac{1}{2}.\frac{\sqrt{1-x^2}}{x} + c$

(b) $-\frac{1}{2x^2}\cos^{-1} x + \frac{1}{2}.\frac{x^2}{\sqrt{1-x^2}} + c$

(c) $\frac{1}{2x^2}\cos^{-1} x - \frac{1}{2}.\frac{\sqrt{1-x^2}}{x} + c$

(d) $-\frac{1}{2x^2}\cos^{-1} x - \frac{1}{2}.\frac{x^2}{\sqrt{1-x^2}} + c$

Ans: (a)

Sol: Let $I=\int \frac{\cos^{-1} x}{x^3} dx$

Let $t=\cos^{-1} x \Rightarrow x=\cos t$

$\Rightarrow dx=-\sin t \, dt$

I= $-\int t(\tan t \sec^2 t)dt$

$= -\left[\frac{t \tan^2 t}{2} - \int 1.\frac{1}{2}\tan^2 t \, dt\right]$

$= -\frac{t}{2}\tan^2 t + \frac{1}{2}\int(\sec^2 t - 1)dt$

$= -\frac{t}{2}\tan^2 t + \frac{1}{2}(\tan t - t) + c$

$= -\frac{1}{2}(\cos^{-1} x)\left(\frac{1-x^2}{x^2}\right) + \frac{1}{2}.\frac{\sqrt{1-x^2}}{x} - \frac{1}{2}\cos^{-1} x + c$

$= -\frac{1}{2}(\cos^{-1} x)\left(1 + \frac{1-x^2}{x^2}\right) + \frac{1}{2}.\frac{\sqrt{1-x^2}}{x} + c$

$= -\frac{1}{2}(\cos^{-1} x)\left(\frac{1}{x^2}\right) + \frac{1}{2}.\frac{\sqrt{1-x^2}}{x} + c$

Q.66 The value of is $\int_{0}^{n\pi+v} |\sin x| dx$ is

(a) 2n+1+cosv **(b) 2n+1-cosv** **(c)2n+1** **(d) 2n+cosv**

Ans: (b)

Sol: $\int_{0}^{n\pi+v} |\sin x| dx = \int_{0}^{n\pi} |\sin x| dx + \int_{n\pi}^{n\pi+v} |\sin x| dx$

$= n\int_{0}^{\pi} |\sin x| dx + \int_{0}^{v} |\sin x| dx$ ...[$|\sin x|$ is periodic with period π]

$= n\int_{0}^{\pi} \sin x\, dx + \int_{0}^{v} \sin x\, dx$ ...[$0 < \sin x$ for $x \in (0, \pi)$]

$= n[-\cos x]_0^{\pi} + [-\cos x]_0^{v}$

$= 2n + 1 - \cos v$

Q.67 The area of the region

$\{(x, y) \in R : y \geq \sqrt{|x+3|}, 5y \leq x + 9 \leq 15\}$ is equal to

(a) $\frac{1}{6}$ **(b) $\frac{4}{3}$** **(c) $\frac{3}{2}$** **(d) $\frac{5}{3}$**

Ans: (c)

Sol: We draw the region

$y^2 \geq |x+3|$ and $5y \leq x + 9$

Solving $y^2 = -x - 3$ and $5y = x + 9$, we get

$y^2 = -(5y - 9) - 3 \Leftrightarrow y^2 + 5y - 6 = 0$

$\Leftrightarrow y = -6$ or 1

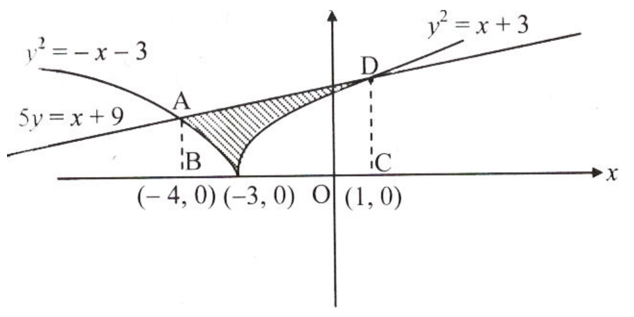
(y=-6 rejected as y is negative.)

y=1 $\Rightarrow$ x =-4

$\Rightarrow$ A=(-4,1)

Similarly, solving $y^2 = x + 3$ and 5y=x+9,

we get D=(1,2)



Required area

$=\text{area of } \square ABCD - \int_{-4}^{-3} \sqrt{-x-3}\,dx - \int_{-3}^{1} \sqrt{x+3}\,dx$

$= \frac{1}{2}(1+2)\times 5 - \left[\frac{(-x-3)^{\frac{3}{2}}}{-3/2}\right]_{-4}^{-3} - \left[\frac{(x+3)^{\frac{3}{2}}}{3/2}\right]_{-3}^{1}$

$= \frac{15}{2} - \frac{2}{3} - \frac{16}{3}$

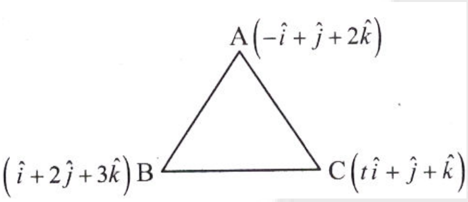
$= \frac{3}{2}$ sq. units

Q.68 The minimum are of the triangle whose vertices are $-\hat{i}+\hat{j}+2\hat{k}, \hat{i}+2\hat{j}+3\hat{k}$ and $t\hat{i}+\hat{j}+\hat{k}$, where t is a real number, is

(a) $\frac{1}{2}$ sq .unit **(b) $\frac{\sqrt{5}}{2}$ sq. unit** **(c)1 sq.unit** **(d) $\frac{\sqrt{3}}{2}$ sq. unit**

Ans: (d)

Sol: Let $-\hat{i}+\hat{j}+2\hat{k}, \hat{i}+2\hat{j}+3\hat{k}, t\hat{i}+\hat{j}+\hat{k}$ be the position vectors of A,B and C respectively.



$\overrightarrow{AB} = (\hat{i}+2\hat{j}+3\hat{k}) - (-\hat{i}+\hat{j}+2\hat{k})$

$\Rightarrow \overrightarrow{AB} = 2\hat{i}+\hat{j}+\hat{k}$

and $\overrightarrow{AC} = (t\hat{i}+\hat{j}+\hat{k}) - (-\hat{i}+\hat{j}+2\hat{k})$

$\Rightarrow \overrightarrow{AC} = (t+i)\,\hat{i} - \hat{k}$

Area of triangle ABC$= \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ (t+1) & 0 & -1 \end{vmatrix}$$

$\Rightarrow \text{Area} = \frac{1}{2}\left|[\hat{i}(-1-0)+\hat{j}(t+1+2)+\hat{k}(0-t-1)]\right|$

$= \frac{1}{2}\left|-\hat{i}+(t+3)\hat{j}-(t+1)\hat{k}\right|$

$= \frac{1}{2}\sqrt{1+(t+3)^2+(t+1)^2}$

$\Rightarrow$ Area of triangle ABC= $\frac{1}{2}\sqrt{2t^2+8t+11}$

Minimizing Area $\Leftrightarrow$ minimizing A,

where A= $2t^2+8t+11$

$\Rightarrow \frac{dA}{dt} = 4t+8 = 4(t+2)$

$\frac{dA}{dt} = 0$

$\Rightarrow 4t+8=0$

$\Rightarrow t=-2$ ...(a point of relative minima.)

(or We use second derivative test.

$\frac{d^2A}{dt^2} = 4 > 0$ for t = - 2)

$\Rightarrow$ Area is inimum at t = - 2.

$\Rightarrow$ Area of triangle ABC= $\frac{1}{2}\sqrt{2(-2)^2+8(-2)+11}$

$= \frac{\sqrt{3}}{2}$

Q.69 Consider the truth table of $p \oplus q$ given as

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Then which of the following has same truth value as $p \oplus q$?

(a) $(p \leftrightarrow q)$ **(b)** $\sim(p \leftrightarrow q)$ **(c)** $(p \rightarrow q)$ **(d)** $\sim(p \rightarrow q)$

Ans: (b)

Sol:

p	q	$p \oplus q$	$(p \leftrightarrow q)$	$\sim(p \leftrightarrow q)$
T	T	F	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	T	F

$p \oplus q \equiv \sim(p \leftrightarrow q)$

Q.70 The length of the perpendicular drawn from the point (3, -1, 11) to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is

(a) $\sqrt{33}$ **(b) $\sqrt{53}$** **(c) $\sqrt{66}$** **(d) $\sqrt{29}$**

Ans: (b)

Sol: Required distance

$$=\left\{(3-0)^2+(-1-2)^2+(11-3)^2-\left[\frac{2(3-0)+2(-1-2)+4(11-3)}{\sqrt{4+9+16}}\right]^2\right\}$$

$$=\sqrt{82-\left(\frac{29}{\sqrt{29}}\right)^2}=\sqrt{82-29}=\sqrt{53}$$

Aliter : Let $P=(3,-1,11)$

Let the point Q on the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$

be $(2\lambda, 3\lambda+2, 4\lambda+3)$ such that

$PQ \perp$ given line.

$\Rightarrow 2(2\lambda-3)+3(3\lambda+3)+4(4\lambda-8)=0$

$\Rightarrow \lambda=1 \Rightarrow Q=(2,5,7)$

$\Rightarrow |PQ|=\sqrt{1+36+16}=\sqrt{53}$

Q.71 P(x) is a polynomial with integer coeficients and has the lowest possible degree. If one of the zeroes of P(x) is $\sqrt[3]{11}+\sqrt[3]{121}$, then the product of all zeroes is_____?

Sol: Let a= $\sqrt[3]{11}+\sqrt[3]{121}$

$\Rightarrow a+\left(-\sqrt[3]{11}\right)+\left(\sqrt[3]{121}\right)=0$

$\Rightarrow a^3+(-\sqrt[3]{11})^3+\left(-\sqrt[3]{121}\right)^3=3a(-\sqrt[3]{11})(-\sqrt[3]{121})$

$\ldots[\because a+b+c=0 \Rightarrow a^3+b^3+c^3=3abc]$

$\Leftrightarrow a^3-11-121=3a(11)$

$\Leftrightarrow a^3-33a-132=0$

$\Rightarrow$ 'a' satisfies the cubic equation

$x^3-33x-132=0$

If the other two roots are b and c, then

$x^3-33x-132=(x-a)(x-b)(x-c)$

$\Rightarrow abc=132$

Q.72 In a town of 10,000 familie, it was found taht 40% family buy newspaper A, 20% buy newspaper Band 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, then number of families which buy A only is_____?

Sol: n(A)=40% of 10,000=4,000
n(B)=20% of 10,000=2,000
n(C)=10% of 10,000=1,000
$n(A\cap B)$ =5% of 10,000=500
$n(B\cap C)$ = 3% of 10,000=300
$n(C\cap A)$ =4% of 10,000=400
$n(A\cap B\cap C)$ =2% of 10,000=200
$\therefore$ n(A only)
$=n(A)-n(A\cap B)-n(A\cap C)+n(A\cap B\cap C)$
=4000-500-400+200
=3300

Q.73 If $^{28}C_{2r} : {}^{24}C_{2r-4} = 225:11$, then the value of r is _____?

Sol: $^{28}C_{2r} : {}^{24}C_{2r-4} = 225:11$

$$\Leftrightarrow \frac{28!}{(2r)!(28-2r)} : \frac{24!}{(2r-4)(24-2r+4)!} = \frac{225}{11}$$

$$\Leftrightarrow \frac{28!}{(2r)!} \times \frac{(2r-4)!}{24!} = \frac{225}{11}$$

$$\Leftrightarrow \frac{28\times 27\times 26\times 25}{2r(2r-1)(2r-2)(2r-3)} = \frac{225}{11}$$

$$\Leftrightarrow 2r(2r-1)(2r-2)(2r-3) = \frac{11}{225}\times 28\times 27\times 26\times 25$$

$$\Leftrightarrow 2r(2r-1)(2r-2)(2r-3)$$
$$=11\times 28\times 3\times 26$$
$$=11\times 14\times 2\times 3\times 13\times 2$$
$$=14\times 13\times 12\times 11$$

By comparing we get 2r=14

$\Leftrightarrow r = 7$

Q.74 The greatest value of $f(x) = (x+1)^{1/3} - (x-1)^{1/3}$ on [0, 1] is _____?

Sol: $f(x) = (x+1)^{\frac{1}{3}} - (x-1)^{\frac{1}{2}}$

$$f(x) = \frac{1}{3}(x+1)^{\frac{-2}{3}} - \frac{1}{3}(x-1)^{\frac{-2}{3}}$$

now, $f'(x) = 0$

$$\Rightarrow \frac{1}{3}(x+1)^{\frac{-2}{3}} - \frac{1}{3}(x-1)\frac{-2}{3} = 0$$

$$\Rightarrow (x+1)^{\frac{-2}{3}} = (x-1)^{\frac{-2}{3}}$$

$$(x+1)^{\frac{2}{3}} = (x-1)^{\frac{2}{3}}$$

Clearly, $f'(x) \pm 0$, for any other value of x. except. In interval i.e

$x \in [0,1]$

we have to check at

$x = 0$ and $x = 1$

$$f'(x) = \frac{1}{3}\left[\frac{1}{(x+1)^{2/3}} - \frac{1}{(x-1)^{2/3}}\right]$$

$$= \frac{1}{3}\left[\frac{(x-1)^{2/3} - (x+1)^{2/3}}{(x-1)^{2/3}}\right]$$

f '(x) doesnot exist x = 1, thus, only x = " 0 " is the value of which, we have to check

$$f(0) = (x+1)^{\frac{1}{3}} - (0-1)^{\frac{1}{3}}$$

$$= 1 + 1 = 2$$

$\left(f''(x)_{x=0} < 0\right)$ so, it is gretest.

Q.75 A GP consists of an even number of terms. If the sum of all the terms is 5 times the sum of the terms occupying odd places, then the common ratio will be equal to ?

Sol: Let the even number of terms in GP be 2 n, with first term a and common ratio r. Then, Sum of all terms = 5 (sum of odd terms)

$\Rightarrow a_1 + a_2 + \ldots.. + a_{2n} = 5(a_1 + a_3 + \ldots. + a_{2n-1})$

$\Rightarrow a_1 + ar + ar^2 \ldots + ar^{2n-1} = 5(a + ar + ar^2 + \ldots + ar^{2n-2})$

$\Rightarrow \dfrac{a(r^{2n} - 1)}{(r-1)} = \dfrac{5a(r^{2n} - 1)}{r^2 - 1} \Rightarrow r + 1 = 5 \Rightarrow r = 4.$

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