A Division of Aggarwal Educare
Aggarwal CLASSES ™

JEE (MAIN)

TEST PAPER

SUBJECT : PHYSICS,CHEMISTRY, MATHEMATICS ***TEST CODE : TEST*** *PAPER-4*

ANSWER PAPER

TIME : 3 HRS ***MARKS : 300***

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark(-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or “un- darken” a darkened bubble.

Part A - PHYSICS

Q.1 **From the equation $\tan\theta = rg/v^2$, one can obtain the angle of banking θ for a cyclist taking a curve (the symbol have their usual meanings). Then say, it is**
(a) Both dimensionally and numerically correct
(b) Neither numerically nor dimensionally correct
(c) Dimensionally correct only
(d) Numerically correct only

Ans: (c)

Sol: Given equation is dimensionally correct because both sides are dimensionless, but numberically wrong because the correct equation is $\tan\theta = v^2/rg$

Q.2 **A stone dropped from a building of height h and it reaches after t second on earth. From the same building if two stones are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 seconds, respectively, then**

(a) $t = t_1 - t_2$ **(b)** $t = \frac{t_1 + t_2}{2}$ **(c)** $t = \sqrt{t_1 t_2}$ **(d)** $t = t_1^2 t_2^2$

Ans: (c)

Sol: For first case of dropping $h = \frac{1}{2}gt^2$.

For second case of downward throwing,

$$h = -ut_1 + \frac{1}{2}gt_1^2 = \frac{1}{2}gt^2$$

$$\Rightarrow \quad -ut_1 = \frac{1}{2}g(t^2 - t_1^2)$$

For third case of upward throwing,

$$h = ut_2 + \frac{1}{2}gt_2^2 = \frac{1}{2}gt^2$$

$$\Rightarrow \quad ut_2 = \frac{1}{2}g(t^2 - t_2^2)$$

On solving these two equation : $-\frac{t_1}{t_2} = \frac{t^2 - t_1^2}{t^2 - t_2^2} \Rightarrow t = \sqrt{t_1 t_2}$

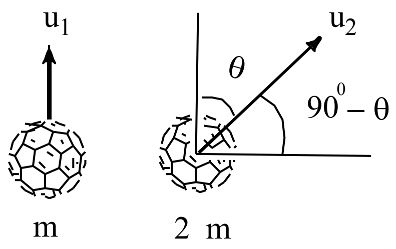
Q.3 **Pankaj and Sudhir are playing with two different balls of masses m and 2m, respectively. If Pankaj throws his ball vertically up and Sudhir at an angle θ, both of them stay in our view for the same period. The height attained by the two balls are in the ratio**
(a) 2 : 1 **(b) 1 : 1** **(c) 1 : $\cos\theta$** **(d) 1 : $\sec\theta$**

Ans: (b)

Sol: Time of flight for the ball thrown by Pankaj $T_1 = \frac{2u_1}{g}$

Time of flight for the ball thrown by Sudhir,

$$T_2 = \frac{2u_2 \sin(90^0 - \theta)}{g} = \frac{2u_2\cos\theta}{g}$$



According to the equation, $T_1 = T_2$

$$\Rightarrow \quad \frac{2u_1}{g} = \frac{2u_2\cos\theta}{g}$$

$$\Rightarrow \quad u_1 = u_2 \cos\theta$$

Height of the ball thrown by Pankaj, $H_1 = \frac{u_1^2}{2g}$

Height of the ball thrown by Sudhir,

$$H_2 = \frac{u_2^2 \sin^2(90^0 - \theta)}{2g} = \frac{u_2^2 \cos^2\theta}{2g}$$

$$\therefore \quad \frac{H_1}{H_2} = \frac{u_1^2/2g}{u_2^2\cos^2\theta/2g} = 1 \qquad [\text{as } u_1 = u_2\cos\theta]$$

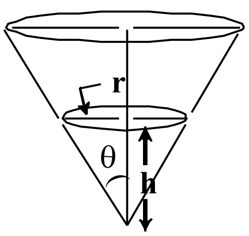
Q.4 A rocket has a mass of 100 kg. 90% of this is fuel. It ejects fuel vapors at the rate of 1 kg/s with a velocity of 500 m/s relative to the rocket. It is supposed that the rocket is outside the gravitational field. The initial upthrust on the rocket when it just starts moving upwards is

(a) Zero **(b) 500 N** **(c) 1000 N** **(d) 2000 N**

Ans: (b)

Sol: Upthrust force, $F = u\left(\frac{dm}{dt}\right) = 500 \times 1 = 500 \text{ N}$

Q.5 A particle describe a horizontal circle at the mouth of funnel type vessel as shown in fig . The surface of the funnel is frctionless . The velocity v of the particle in terms of r and θ will be

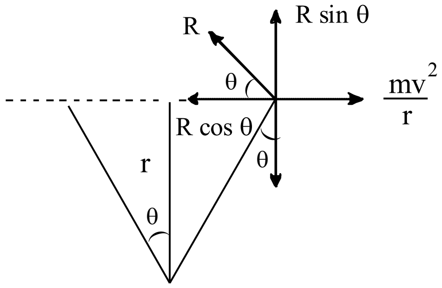


(a) $v = \sqrt{rg/\tan\theta}$ **(b)** $v = \sqrt{rg\tan\theta}$

(c) $v = \sqrt{rg\cot\theta}$ **(d)** $v = \sqrt{rg/\cot\theta}$

Ans: (c)

Sol: For uniform circular motion of a particle,



$$\frac{mv^2}{r} = R\cos\theta \qquad ...(i)$$

and $\quad mg = R \sin\theta \qquad ...(ii)$

Dividing (i) by (ii),

$$\frac{v^2}{rg} = \cot\theta \Rightarrow v = \sqrt{rg\cot\theta}$$

Q.6 **The ratio of masses of two balls is 2 : 1 and before collision the ratio of their velocities is 1 : 2 in mutually opposite direction. After collision each ball moves in an opposite direction to its initial direction. If e = (5/6), the ratio of speed of each ball before and after collision would be**

(a) (5/6) times

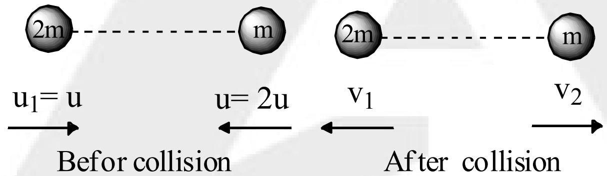
(b) Equal

(c) Not related

(d) Double for the first ball and half for the second ball

Ans: (a)

Sol: Let masses of the two ball are 2 m and m and their speeds are u and 2u respecitvely.



By conservation of momentum

$$m_1\overrightarrow{u_1} + m_1\overrightarrow{u_2} = m_1\overrightarrow{v_1} + m_1\overrightarrow{v_2}$$

$\Rightarrow \qquad 2mu - 2mu = mv_2 - 2mv_1 \Rightarrow v_2 = 2v_1$

Coefficient of restitution

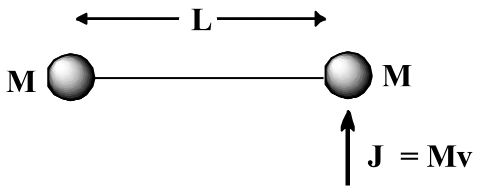
$$= \frac{-(\overrightarrow{v_2} - \overrightarrow{v_1})}{(\overrightarrow{u_2} - \overrightarrow{u_1})} = -\frac{(2v_1 + v_1)}{(-2u - u)} = \frac{-3v_1}{-3u} = \frac{v_1}{u} = \frac{5}{6} \quad [\text{as } e = 5/6 \text{ given}]$$

$\Rightarrow \qquad \frac{v_1}{u} = \frac{5}{6} =$ ratio of the speed of first ball before and after collision.

Similarly, we can calculate the ratio of second ball before and after collision,

$$\frac{v_2}{u_2} = \frac{2v_1}{2u} = \frac{v_1}{u} = \frac{5}{6}$$

Q.7 **Consider a body, shown in fig, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse J = Mv is imparted to the body at one of its ends, what would be its angular velocity ?**



(a) v/L **(b)** $2v/\text{L}$ **(c)** $v/3\text{L}$ **(d)** $v/4\text{L}$

Ans: (a)

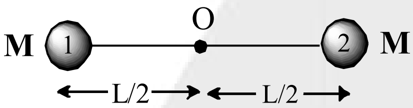
Sol: Initial angular momentum of the system about point O = Linear momentum ×

Perpendicular distance of linear momentum from the axis of rotation = $\text{M}v\left(\frac{\text{L}}{2}\right)$ ..(i)

Final angular momentum of the system about point O

$$= \text{I}_1\omega + \text{I}_2\omega = (\text{I}_1 + \text{I}_2)\omega = \left[\text{M}\left(\frac{\text{L}}{2}\right)^2 + \text{M}\left(\frac{\text{L}}{2}\right)^2\right]\omega \quad ..(iii)$$

Applying the law of conservation of angular momentum



$$\Rightarrow \quad \text{M}v\left(\frac{\text{L}}{2}\right) = 2\text{M}\left(\frac{\text{L}}{2}\right)^2\omega$$

$$\Rightarrow \quad \omega = \frac{v}{\text{L}}$$

Q.8 The length of an elastic string is a meters when the longitudinal tension is 4 N and b meters when the logitudinal tension is 5 N. The length of the string in meters when the longitudinal tension is 9 N is

(a) $a-b$ **(b)** $5b-4a$ **(c)** $2b-\frac{1}{4}a$ **(d)** $4a-3b$

Ans: (b)

Sol: Let the original length of elestic string is L and its force constant is k.
When longitudinal tension 4 N is applied on it,

$$\text{L} + \frac{4}{\text{k}} = a \quad ....(i)$$

and when longitudinal tenesion 5 N is applied on it,

$$\text{L} + \frac{5}{\text{k}} = b \quad ...(ii)$$

By solving (i) and (ii) we get $\text{k} = \frac{1}{b-a}$ and $\text{L} = 5a - 4b$.

Now when longitudinal tension 9 N is applied on elastic, string, then its length

$$= \text{L} + \frac{9}{\text{k}} = 5a - 4b + 9(b-a) = 5b - 4a$$

Q.9 Two capillary tubes of same diameter are kept vertically one each in two liquids whose relative densities are 0.8 and 0.6 and surface tensions are 60 and 50 dyne/cm. respectively, Ratio of heights of liquids in the two tubes h_1/h_2 is

(a) $\mathbf{\frac{10}{9}}$ **(b)** $\mathbf{\frac{3}{10}}$ **(c)** $\mathbf{\frac{10}{3}}$ **(d)** $\mathbf{\frac{9}{10}}$

Ans: (d)

Sol: $h = \frac{2T\cos\theta}{rdg}$

[If diameter of capillaries are same and taking value of θ same for both liquids.]

$\therefore \quad \frac{h_1}{h_2} = \left(\frac{T_1}{T_2}\right)\left(\frac{d_2}{d_1}\right)$

$= \left(\frac{60}{50}\right)\times\left(\frac{0.6}{0.8}\right) = \left(\frac{36}{40}\right) = \frac{9}{10}$

Q.10 Two samples A and B of a gas initially at the same pressure and temperature are compressed from volume V to V/2 (A isothermally and B adiabatically). The final pressure of A is
(a) Greater than the final pressure of B.
(b) Equal to the final pressure of B.
(c) Less than the final pressure of B.
(d) Twice the final pressure of B.

Ans: (c)

Sol: For isothermal process $P_1V = P'_2\frac{V}{2}$

$\Rightarrow \quad P'_2 = 2P_1 \quad$ ....(i)

For adiabatic process, $P_1V^{\gamma} = P_2\left(\frac{V}{2}\right)^{\gamma}$

$\Rightarrow \quad P_2 = 2^{\gamma}P_1$

Since $\gamma > 1$. Therefore, $P_2 > P'_2$

Q.11 The energy of all molecules of a monoatomic gas having a volume V and pressure P is 3PV/2. The total translational kinetic energy of all molecules of a diatomic gas as the same volume and pressure is

(a) $\mathbf{\frac{1}{2}PV}$ **(b)** $\mathbf{\frac{3}{2}PV}$ **(c)** $\mathbf{\frac{5}{2}PV}$ **(d)** $\mathbf{3PV}$

Ans: (b)

Sol: Energy of 1 mol of gas $= \frac{f}{2}RT = \frac{f}{2}PV$, where f = degree of freedom.

Monoatomic and diatomic gases both possess equal degree of freedom for translational motion and that is equal to 3, i.e $f = 3$.

$\therefore \quad E = \frac{3}{2}PV$

Although total energy will be different,

For monoatomic gas $E_{total} = \frac{3}{2}PV \quad$ [as $f = 3$]

For diatomic gas, $E_{total} = \frac{5}{2}PV$ [as $f = 5$]

Q.12 A man is watching two trains, one leaving and the other coming in with equal speed of 4 m/s. If they sound their whistles, each of frequency 240 Hz, the number of beats heard by the man (velocity of sound in air = 320 m/s) will be equal to

(a) 6 **(b) 3** **(c) 0** **(d) 12**

Ans: (a)

Sol: Apparent frequency due to train which is coming in, $n_1 = \frac{v}{v - v_s} \cdot n$

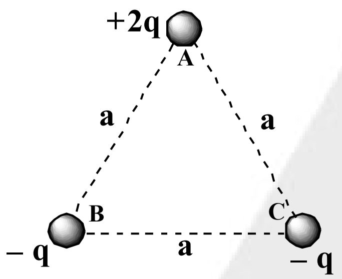
Apparent frequency due to train which is leaving, $n_2 = \frac{v}{v + v_s} \cdot n$

So number of beats $n_1 - n_2$

$$= \left(\frac{1}{316} - \frac{1}{324}\right) 320 \times 240$$

$\Rightarrow$ $n_1 - n_2 = 6$

Q.13 Three charges of (+2q), (-q) and (-q) are placed at the corners A, B and C of an equilateral triangle of side a as shown in fig, Then the dipole moment of this combination is

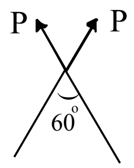


(a) q*a* **(b) Zero** **(c) $qa\sqrt{3}$** **(d) $\frac{2}{\sqrt{3}}qa$**

Ans: (c)

Sol: The charge +2q can be broken in +q, +q. Now as shown in fig, we have two equal dipoles inclined at an angle of 60^0. Therefore, resultant dipole moment will be

$$P_{net} = \sqrt{p^2 + p^2 + 2\,pp\cos 60} = \sqrt{3}p = \sqrt{3}qa$$



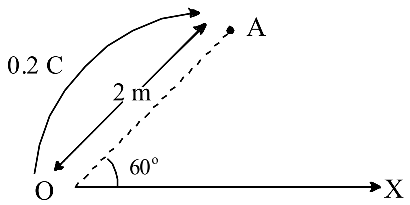
Q.14 There is an electric field E in x-direction. If the work done in moving a charge 0.2 C through a distance of 2 m along a line making an angle 60^0 with the x-axis is 4 J, what is the value of E ?

(a) 4 N/C **(b) 8 N/C** **(c) $\sqrt{3}$ N / C** **(d) 20 N/C**

Ans: (d)

Sol: By using $W = q \times \Delta V$ and $\Delta V = E\,\Delta r \cos\theta$

So, $W = qE\,\Delta r \cos\theta$



$$W = 4j = 0.2 \times E \times 2 \times \cos 60$$

$$\Rightarrow \quad E = 20\ N/C$$

Q.15 Condenser A has a capacity of 15μF when it is filled with a medium of dielectric constant 15. Another condenser B has a capacity 1μF with air between the plates. Both are charged separately by a battery of 100 V. After charging, both are connected in parallel without the battery and the dielectric material being removed. The common potential now is

(a) 400 V **(b) 800 V** **(c) 1200 V** **(d) 1600 V**

Ans: (b)

Sol: Charge on capacitor A is given by

$$Q_1 = 15 \times 10^{-6} \times 100 = 15 \times 10^{-4} C$$

Charge on capacitor B is given by

$$Q_2 = 1 \times 10^{-6} \times 100 = 10^{-4} C$$

Capacity of capcitor A after removing dielectric

$$= \frac{15 \times 10^{-6}}{15} = 1\mu F$$

Now when both capacitors are connected in parallel their equivalent capacitance will be

$$C_{eq} = 1 + 1 = 2\mu F$$

So, common potential

$$= \frac{(15 \times 10^{-4}) + (1 \times 10^{-4})}{2 \times 10^{-6}} = 800\,V$$

Q.16 At a specific instant, emission of radioactive compound is deflected in magnetic field. The compound can emit

(a) Electron **(b) Protons** **(c) He^{2+}** **(d) Neutrons**

Ans: (a)

Sol: Charged particles can be deflected by magnetic field.

Q.17 The rest mass of the deuteron is equivalent to an energy of 1876 MeV; the rest mass of proton is equivalent to 939 MeV and that of a neutron to 40 MeV.
A deuteron may disintegrate to a proton and a neutron if it.

(a) emits a X-ray photon of energy 2 Me V.
(b) captures a X-rays photon of energy 2 MeV.
(c) emits a X-ray photon of energy 3 Me V.
(d) capture a X-ray photon of energy 3 Me V.

Ans: (d)

Sol: Disintegration of deuteron to a proton and a neutron can be represented by

$$^2_1H + Q \rightarrow {}^1_1H + {}^1_0n$$

The energy captured is the γ - *ray* photon E_γ is given by

$$E_\gamma + 1876 = 939 + 940$$

$$\Rightarrow \quad E_\gamma = (990 + 940) - 1876 = 3\,\text{MeV}$$

Q.18 In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the Vernier scale. If the smallest division of the main scale is half -a-degree (= 0.5^0), then the least count of the instrument is

(a) One minute (b) Half minute (c) One degree (d) Half degree

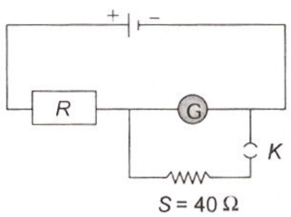
Ans: (a)

Sol: Least count $= \dfrac{\text{Value of main scale division}}{\text{Number of division on vernier scale}}$

$$= \frac{1}{30}\text{MSD}$$

$$= \frac{1}{30} \times \frac{1^0}{30} = \frac{1^0}{60} = 1\,\text{min}$$

Q.19 In the ciruit shown in fig, if key K is pressed, then the galvanometer reading becomes half. The resistance of galvanometer is



(a) 20Ω (b) 30Ω (c) 40Ω (d) 50Ω

Ans: (c)

Sol: Galvanometer reading becoming half means current distributes equally between galvanometer and resistance of 40Ω. Hence galvanometer resistance must be 40Ω.

Q.20 A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is 4 ms^{-1}. The kinetic energy of the other mass is

(a) 192 J (b) 96 J (c) 144 J (d) 288 J

Ans: (d)

Sol: Velocity of 4-kg mass : $v_2 = \dfrac{m_1 u_1}{m_2} = \dfrac{12 \times 4}{4} = 12\ \text{m/s}$

$$\text{KE of 4-kg mass} = \frac{1}{2} \times 4 \times (12)^2 = 288\ \text{J}$$

Q.21 A room is maintained at 20^{0}C by a heater of resistance 20Ω connected to 200 V mains. The temperature is uniform through out the room and heat is transmitted through a glass window of area 1 m^2 and thickness 0.2 cm. What will be the temperature outside ? Given that thermal conductivity K for glass is 0.2 cal/m^{0}Cs and J = 4.2 J/cal.

Sol: As the temperature of room remains constant, therefore the rate of heat generation from the heater should be equal to the rate of flow of heat through a glass window

$$\frac{1}{J}\left(\frac{V^2}{R}t\right) = KA\frac{\Delta\theta}{l}\cdot t$$

$$\Rightarrow \quad \frac{1}{4.2}\times\frac{(200)^2}{20} = \frac{0.2\times1\times(20-\theta)}{0.2\times10^{-2}}$$

$$\Rightarrow \quad \theta = 15.24\,^0C$$

[where $\theta =$ temperature of outside]

Q.22 A short bar magnet of magnetic moment 255 J/T is placed with its axis perpendicular to Earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45⁰ with Earth's field, H = 0.4×10^{-4} T.

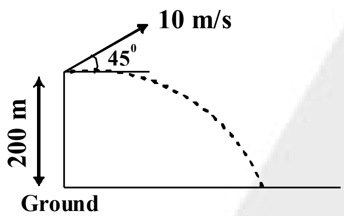
Sol: Since B and H are $\perp^r$ to each other and resultant field is inclined at an angle 45^0 with so,

$$B = H$$

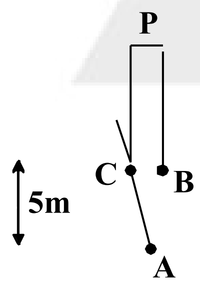
$$\frac{\mu_0}{4\pi}\frac{2M}{r^3} = H$$

$$\therefore \quad r^3 = \frac{\mu_0}{4\pi}\frac{2M}{H} = 0.5 \text{ m}$$

Q.23 A particle has been projected from the top of tower as shown in figure. Find the time taken by the particle to reach the ground. (Take g = 10 m/s²)



Sol: Consider point of projection as origin and horizontal direction as positive x-axis and verticalupward direction as positive y-axis.



For motion along y-axis, from

$$y = u_y t + \frac{a_y t^2}{2}$$

For particle to reach ground, $y = -200\,\text{m}$

$$\therefore \quad -200 = (10\sin 45^0)t - 200 = (10\sin 45^o)t - \frac{gt^2}{2}$$

The two roots of this quadratic equation are 7.07 s and -5.66s

Q.24 A uniform rectangular marble slab is 3.4 m long and 2.0 m wide. It has a mass of 180 kg. If it is originally lying on the flat ground, how much work is needed to stand it on an end ?

Sol: The work done by gravity is the work done, as if all the mass was concentrated at the centre of mass. The work necessary to lift the object can be thought of as the work done against gravity and is just, W = mgh, where h is the height throught which the centre of mass is raised.

$W = (180\text{ kg})\,(9.8\text{ m/s}^2)\,(1.7\text{ m}) = 3.0\text{ KJ}$

Q.25 A hemispherical bowl just floats without sinking in a liquid of density 1.2×10^3 kg/m³. If outer diameter and the density of the bowl are 1 m and 2×10^4 kg/m³ respectively, then the inner diameter of the bowl will be?

Sol: Weight of the bowl $= mg = V\rho g = \frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]\rho g$

where D is the outer diameter, d is the inner diameter, and ρ is the density of bowl. Weight of the liquid displaced by the bowl $= V\sigma g = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3\sigma g$

where σ is the density of the liquid.

For the flotation,

$$\frac{4}{3}\pi\left(\frac{D}{2}\right)^3\sigma g = \frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]\rho g$$

$$\Rightarrow \left(\frac{1}{2}\right)^3 \times 1.2\times10^3 = \left[\left(\frac{1}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]2\times10^4$$

Part - B - CHEMISTRY

Q.26 Nodal planes of $\pi-$bond (s) in $CH_2 = C = C = CH_2$ are located in :

(a) All are in molecular plane

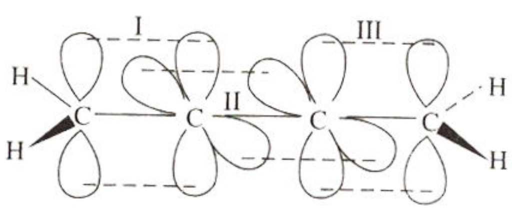
(b) Two in molecular plane and one in a plane perpendicular to the molecular plane which contains $C-C\ \sigma-$bond.

(c) One in molecular plane and two in plane perpendicular to molecular plane which contains $C-C\ \sigma-$bonds

(d) Two in molecular plane and one in a plane perpendicular to molecular plane which bisects $C-C\ \sigma-$bonds at right angle

Ans: (b)

Sol:



Q.27 For an f - orbital, the values of m are :

(a) $-2, -1, 0, +1, +2$ **(b) $-3, -2, -1, 0, +1, +2, +3$**

(c) $-1, 0, +1$ **(d) $0, +1, +2, +3$**

Ans: (b)

Sol: Value of $m = -l,$ including zero.

Q.28 At what temperature in the Celsius scale V (volume) of a certain mass of gas at 27°C will be doubled keeping the pressure constant ?

(a) 54°C **(b) 327°C** **(c) 427°C** **(d) 527°C**

Ans: (b)

Sol: $\frac{V_1}{V_2} = \frac{T_1}{T_2}$

$$\therefore \quad T_2 = \frac{T_1 V_2}{V_1} = 300^\circ \text{ K}, \ \frac{2V}{V} = 600^\circ K$$

$$T_2 = 600^\circ\text{K} = (600 - 273)^\circ\text{C} = 327^\circ\text{C}$$

Q.29 One mole of an ideal monoatomic gas at temperature T and volume 1 L expands to 2 L against a constant external pressure of 1 atm under adiabatic conditions, then the final temperature of gas will be :

(a) $T + \frac{2}{3 \times 0.0821}$ **(b) $T - \frac{2}{3 \times 0.0821}$**

(c) $\frac{T}{2^{5/3-1}}$ **(d) $\frac{T}{2^{5/3+1}}$**

Ans: (b)

Sol: $\Delta U = \Delta w$

$$nC_v(T_2 - T) = -P \times (V_2 - V_1)$$

$$\frac{3}{2}R(T_2 - T) = -1$$

$$\therefore \quad T_2 = T - \frac{2}{3 \times 0.0821}$$

Q.30 Which may be added to 1 L of water to act as a buffer ?

(a) One mole of $HC_2H_3O_2$ and **0.5 mole of NaOH**

(b) One mole of NH_4Cl and 1 mole of HCl

(c) One mole of NH_4OH and 1 mole of NaOH

(d) One mole of $HC_2H_3O_2$ and 1 mole of HCl

Ans: (a)

Sol: One mole oxalic acid and 0.5 mole of NaOH will be added to make a buffer.

Q.31 Salts of A (atomic weight : 7), B (atomic weight : 27), and C (atomic weight: 48) were electrolyzed under identical condition using the same quantity of electricity. It was found that when 2.1 g of A was deposited, the weights of B and C deposited were 2.7 and 7.2 g, respectively. The valencies of A, B and C, respectively are :

(a) 3, 1, and 2 **(b) 1, 3, and 2** **(c) 3, 1, and 3** **(d) 2, 3, and 2**

Ans: (b)

Sol: Eq of A = Eq. of B = Eq. of C

$$\frac{2.1}{7/n_1} = \frac{2.7}{27/n_2} = \frac{7.2}{48/n_3}$$

$$0.3n_1 = 0.1n_2 = 0.15n_3$$

$\therefore$ $n_1 = \frac{n_2}{3} = \frac{n_3}{2} = (n_1, n_2, n_3)$ are integers.

Q.32 A radioactive substance (parent) decays to its daughter element. The age of radioactive substance (t) is related to the daughter (d) / parent (p) ratio by the equation :

(a) $t = \frac{1}{\lambda}\ln\left(1+\frac{p}{d}\right)$ **(b)** $t = \frac{1}{\lambda}\ln\left(1+\frac{d}{p}\right)$

(c) $t = \frac{1}{\lambda}\ln\left(\frac{d}{p}\right)$ **(d)** $t = \frac{1}{\lambda}\ln\left(\frac{p}{d}\right)$

Ans: (b)

Sol: $N = N_0\, e^{-\lambda t}$, Where N = Parent remaining (p) and N_0 = Initial parent = Parent remaining (p) + daughter formed (d)

$$p = (p+d)\cdot e^{-\lambda t} \text{ or } \ln\frac{(p+d)}{p} = \lambda \cdot t$$

$$t = \frac{1}{\lambda}\ln\left(1+\frac{d}{p}\right)$$

Q.33 What will be the effect of increase in temperature on physical adsorption ?

(a) It will decrease **(b) It will increasese**

(c) First increases then decreases **(d) None of these**

Ans: (a)

Sol: Since adsorption is an exothermic process (taking place with the evolution of heat) therefore in accordance with Lechatelier's principle, the magnitude of physical adsorption will decrease with increase in temperature. In case of chemisorptions, the adsorption first increase and then decrease with increase in temperature.

Q.34 The correct order of decreasing stability of the carbanions is

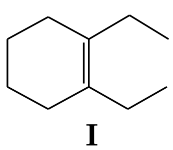
$\underset{(1)}{(CH_3)_3\overset{\ominus}{C}}$, $\underset{(2)}{(CH_3)_2\overset{\ominus}{C}H}$, $\underset{(3)}{CH_3\overset{\ominus}{C}H_2}$, $\underset{(4)}{C_6H_5\overset{\ominus}{C}H_2}$

(a) 1 > 2 > 3 > 4 **(b)** 4 > 3 > 2 > 1 **(c)** 4 < 1 > 2 > 3 **(d)** 1 > 2 > 4 > 3

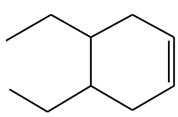
Ans: (b)

Sol: ------

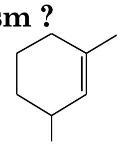
Q.35 Which of these cycloalkenes will exhibit geometrical isomerism ?



I



II



III

(a) I **(b) II** **(c) III** **(d) All of these**

Ans: (b)
Sol: -----

Q.36 $Me_2CH-\underset{\displaystyle OH}{\underset{|}{C}}H-Me \xrightarrow[350^\circ C]{Al_2O_3} (A) \xrightarrow[(ii)\,AgOH]{(i)\,HI} (B)$. **Product (B) of the given reaction is :**

(a) $Me_2C(OH)CH_2Me$

(b) $Me_2CH-\underset{\displaystyle OH}{\underset{|}{C}}H-Me$

(c) $Me-\underset{\displaystyle OH}{\underset{|}{C}}H-CMe_3$

(d) $HO-CH_2-(CH_2)Me$

Ans: (a)
Sol: ------

Q.37 If a mixture of two alkyl chlorides on treatment with sodium metal in ether solution gives isobutane as one of the product, then the reactants are :
(a) Methyl chloride and propyl chloride
(b) Methyl chloride and ethyl chloride
(c) Isopropyl chloride and ethyl chloride
(d) Isopropyl chloride and methyl chloride

Ans: (d)

Sol:

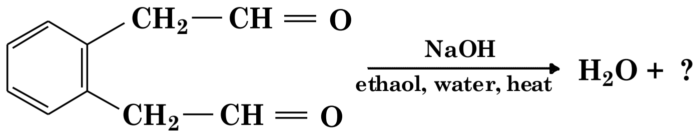
$$CH_3-\underset{\displaystyle CH_3}{\underset{|}{\overset{\displaystyle H}{\overset{|}{C}}}}-Cl + Na + Cl + CH_3 \longrightarrow CH_3-\underset{\displaystyle CH_3}{\underset{|}{\overset{\displaystyle H}{\overset{|}{C}}}}-CH_3$$

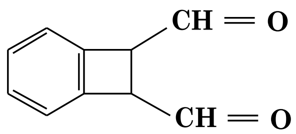
Isobutane

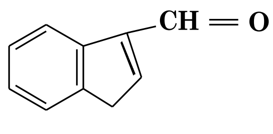
Q.38 Phenol $\xrightarrow{NaNO_2/H_2SO_4} B \xrightarrow{H_2O} C \xrightarrow{NaOH} D$ **Name of the above reaction is :**
(a) Liebermann's reaction **(b) Phthalein fusion test**
(c) Reimer - Tiemann reaction **(d) Schotten - Baumann reation**

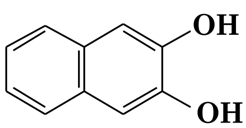
Ans: (a)
Sol: Liebermann's reaction

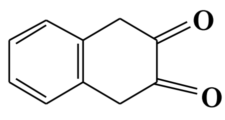
Q.39 What is the principal product of the following reaction ?



(a) 

(b) 

(c) 

(d) 

Ans: (b)
Sol: --------

Q.40 Alkyl isocyanides undergo addition reaction on heating with HgO or S. This is due to
(a) electron-deficient nature of carbon atom
(b) electron-rich nature of carbon atom
(c) carbon has strong affinity for O and sulphur
(d) multiple bonding between N and C atom.

Ans: (a)
Sol: -------

Q.41 In the estimation of sulphur, organic compound on treating with conc. HNO_3 is converted into :
(a) SO_2 (b) H_2S (c) H_2SO_4 (d) SO_3

Ans: (c)
Sol: In Carius method, sulphur of organic compound is converted into H_2SO_4

$$S + H_2O + 3O \xrightarrow[HNO_3]{\Delta} H_2SO_4$$

Q.42 If a native protein is subjected to physical or chemical treatment which may disrupt its higher structure without affecting primary structure, then this process is called :
(a) inversion of protein (b) denaturation of protein
(c) renaturation of protein (d) fermentation

Ans: (b)
Sol: -------

Q.43 PHBV is a biodegradable polymer of :
(a) 3-hydroxybutanoic acid and 2-hydroxypentanoic acid
(b) 3-hydroxybutanoic acid and 3-hydroxypentanoic acid
(c) 2-hydroxybutanoic acid and 2-hydroxypentanoic acid
(d) 2-hydroxybutanoic acid and 3-hydroxypentanoic acid

Ans: (b)
Sol: ------

Q.44 A red solid is insoluble in water. However, it becomes soluble if some KI is added to water. Heating the red solid in a test tube results in liberation of some violet colored fumes and droplets of a metal appear on the cooler part of the test tube. The red solid is :
(a) Pb_3O_4 (b) HgI_2 (c) HgO (d) $(NH_4)_2Cr_2O_7$

Ans: (b)
Sol: -------

Q.45 Which of the following ions forms most stable complex compound ?
(a) Cu^{2+} (b) Ni^{2+} (c) Fe^{2+} (d) Mn^{2+}

Ans: (a)
Sol: The magnitude of stability constant for some divalent metal ions of the first transition series with oxygen or nitrogen donar ligands increases in the order :

$$Mn^{2+} < Fe^{2+} < Co^{2+} < Ni^{2+} < Cu^{2+} < Zn^{2+}$$

Q.46 Two solution of a substance (nonelectrolyte) are mixed in the following manner ; 480 mL of 1.5 M first solution + 520 mL of 1.2 M second solution. What is the molarity of the final mixture ?

Sol: From the molarity equation,

$$M_1V_1 + M_2V_2 = MV$$

Let M be the molarity of final mixture,

$$M = \frac{M_1V_1 + M_2V_2}{V} \text{ (where } V = V_1 + V_2)$$

$$M = \frac{480 \times 1.5 + 520 \times 1.2}{480 + 520} = 1.344\ M$$

Q.47 Consider the reaction : $A_{(g)} + B_{(g)} \rightleftharpoons C_{(g)} + D_{(g)}$, which occurs in one step. The specific rate constant are 0.25 and 5000 for the forward and reverse reactions, repectively. The equilibrium constant is_____?

Sol: $K_f = 0.25, K_b = 5000$

$$\therefore \quad K_c = \frac{K_f}{K_p} = \frac{0.25}{5000} = 5.0 \times 10^{-5}$$

Q.48 An atomic solid crystallizes in a body centre cubic lattice and the inner surface of the atoms at the adjacent corner are separated by 60.3 pm. If the atomic weight of A is 48, then the density of the solid is nearly ?

Sol: Given $a - 2r = 60.3$ and for bcc, $4r = \sqrt{3}a$

$$\Rightarrow \quad a - \frac{\sqrt{3}}{2}a = 60.3 \Rightarrow a = 450 \text{ pm}$$

$$\text{Density } (\rho) = \frac{2 \times 48}{6.023 \times 10^{23} \times (4.5)^3 \times 10^{-24}} = 1.75 \text{ g / cc}$$

Q.49 K_f of 1,4-dioxane is 4.9 mol^{-1} for 1000 g. The depression in freezing point for a 0.001 m solution in dioxane is_____?

Sol: $\Delta T = K_f \times \text{Molality} = 4.9 \times 0.001 = 0.0049\,K$

Q.50 3.7 g of an oxide of a metal was heated with charcoal. The liberated CO_2 was absorbed in caustic soda solution and weighed 1.0 g. If the specific gravity of the metal is 0.095, the exact atomic weight of the metal is _____?

Sol: Weight of $CO_2 = 1g$ (as absorbed in KOH)

Weight of oxygen in oxide = Weight of oxygen in 1 g of $CO_2 = \frac{32}{44} = \frac{8}{11}g$

Weight of metal $= 3.7 - \frac{8}{11}$

Equivalent weight $= \frac{\text{Weight of metal}}{\text{Weight of oxygen}} \times 8 = 32.7$

According to Dulong Petits law,

Atomic weight (approx) $= \frac{6.4}{0.095} = 67.37$

Valency $= \frac{\text{Atomic weight}}{\text{Equivalent weight}} = 2$ (approx)

Exact atomic weight $= 32.7 \times 2 = 65.4$

Part - C - MATHEMATICS

Q.51 Let two number have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the equation

(a) $x^2+18x+16=0$ **(b)** $x^2-18x+16=0$

(c) $x^2+18x-16=0$ **(d)** $x^2-18x-16=0$

Ans: (b)

Sol: Let "a" and "b" be two numbers By the equation, $A = 9$ and $G = 4$

$\Rightarrow$ $\dfrac{a+b}{2}=9$ and $\sqrt{ab}=4$

$\Rightarrow$ $a+b=18$ and $ab=16$

$\therefore$ The required equation is $x^2-(a+b)x+ab=0$

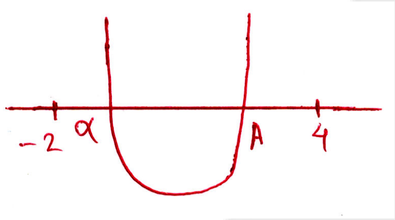
$\Rightarrow$ $x^2-18x+16=0$

Q.52 All the values of m for which both the roots of the equation $x^2-2mx+m^2-1=0$ are greater than -2 but less than 4 lie in the interval

(a) $-2 < m < 0$ **(b)** $1 < m < -3$ **(c)** $-1 < m < 3$ **(d)** $1 < m < 4$

Ans: (c)

Sol:



$f(x)=x^2-2mx+m^2-1=0$

i.e, $f(-2)>0$

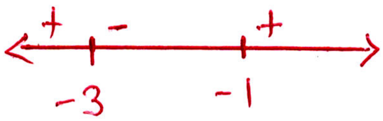
$4+4m+m^2-1>0$

$m^2+4m+3>0$

$m^2 3m+m+3>0$

$m(m+3)+1(m+3)>0$

$(m+3)(m+3)>0$



$m\in(-\infty,-3)\cup(-1,\infty)$

and $f(4)>0$

$16-8m+m^2-1>0$

$m^2-8m+15>0$

$m^2-5m-3m+15>0$

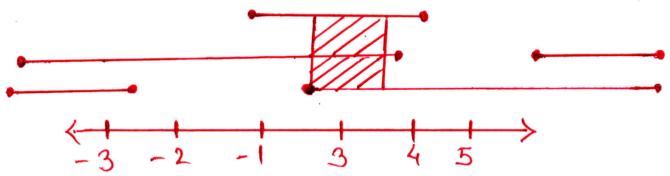
$(m-5)(m-3)>0$

$m \in (-\infty, 3) \cup (5, \infty)$

and $-2 < \dfrac{-b}{2a} < 4$

$-2 < \dfrac{2m}{2} < 4$

$m \in (-2, 4)$ Taking intersection



$m \in (-1, 3)$

$\boxed{-1 < m < 3}$

Q.53 The points representing the complex number z for which $|z+5|^2 - |z-5|^2 = 10$ lie on

(a) a straight line
(b) a circle
(c) a parabola
(d) the bisector of the line joining (5, 0) and (-5, 0)

Ans: (a)

Sol: $(z+5)(\bar{z}+5) - (z-5)(\bar{z}-5) = 10$

or $5(z+\bar{z}) + 25 + 5(z+\bar{z}) - 25 = 10$

$$2 \cdot 2x = 2 \Rightarrow x = \frac{1}{2} \Rightarrow \text{(a)}$$

Q.54 If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $A^2 + 2A$ equals

(a) A **(b) 2 A** **(c) 3 A** **(d) 4 A**

Ans: (c)

Sol: Here $A = I \Rightarrow A^2 + 2A = I^2 + 2I = I + 2I = 3I$

Q.55 In the which expansion of $\left(x^2 + 1 + \dfrac{1}{x^2}\right)^n$, $n \in N$, which is not true

(a) number of terms is $2n+1$

(b) coefficient of constant term is 2^{n-1}

(c) coefficient of x^{2n-2} is n

(d) none of these

Ans: (b)

Sol: $\left(x^2+1+\frac{1}{x^2}\right)^n = {}^nC_0 + {}^nC_1\left(x^2+\frac{1}{x^2}\right) + {}^nC_2\left(x^2+\frac{1}{x^2}\right)^2 + \ldots + {}^nC_n\left(x^2+\frac{1}{x^2}\right)^n$

This contains each of the term $x^0, x^2, x^4 \ldots.. x^{2n}$, x^{-2}, $x^{-4}, \ldots.. x^{-2n}$

Coefficient of constant term

$$= {}^nC_0 + ({}^nC_2)(2) + ({}^nC_4)({}^4C_2) + ({}^nC_6)({}^6C_3) + \ldots \neq 2^{n-1}$$

Coefficient of x^{2n-2} in ${}^nC_{n-1} = n$

Q.56 For $x > 0$, sum of the series $\frac{x-1}{x+1} + \frac{1}{2}\frac{x^2-1}{(x+1)^2} + \frac{1}{3}\frac{x^3-1}{(x+1)^3} + \frac{1}{4}\frac{x^4-1}{(x+1)^4} + \ldots$ is

(a) $\log_e(x-1)$ **(b) $\log_e x$** **(c) $\log_e(x+1)$** **(d) none of these**

Ans: (b)

Sol: We first split the given series into two series. We have

$$\frac{x-1}{x+1} + \frac{1}{2}\frac{x^2-1}{(x+1)^2} + \frac{1}{3}\frac{x^3-1}{(x+1)^3} + \ldots = \frac{x}{x+1} + \frac{1}{2}\left(\frac{x}{x+1}\right)^2 + \frac{1}{3}\left(\frac{x}{x+1}\right)^3 + \ldots$$

$$-\left[\frac{1}{x+1} + \frac{1}{2}\frac{1}{(x+1)^2} + \frac{1}{3}\left(\frac{1}{x+1}\right)^3 + \ldots..\right]$$

$$= -\log_e\left(1-\frac{x}{x+1}\right) - \left\{-\log_e\left(1-\frac{1}{x+1}\right)\right\}$$

$$= -\log_e\left(\frac{x+1-x}{x+1}\right) + \log_e\left(\frac{x+1-1}{x+1}\right)$$

$$= -\log_e\left(\frac{1}{x+1}\right) + \log_e\left(\frac{x}{x+1}\right)$$

$$= \log_e\left[\frac{x/(x+1)}{1/(x+1)}\right]$$

$$= \log_e x$$

Q.57 If for a variable line $\frac{x}{a}+\frac{y}{b}=1$, the condition $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$ (c is a constant) is satisfied, then locus of foot of perpendicular drawn from origin to the line is

(a) $x^2+y^2=c^2/2$ **(b) $x^2+y^2=2c^2$** **(c) $x^2+y^2=c^2$** **(d) $x^2-y^2=c^2$**

Ans: (c)

Sol: Equation of perpendicular drawn from the origin to the line $\frac{x}{a}+\frac{y}{b}=1$ is

$$y-0=\frac{a}{b}(x-0) \qquad \left[\begin{array}{l}\therefore\ m \text{ of given line } = \frac{-b}{a} \\ \therefore\ m \text{ of perpendicular } = \frac{a}{b}\end{array}\right]$$

$\Rightarrow \quad by-ax=0 \Rightarrow \frac{x}{b}-\frac{y}{a}=0$

Now, the locus of foot of perpendicular is the intersection point of line

$$\frac{x}{a}+\frac{y}{b}=1 \qquad \text{...(i)}$$

and $$\frac{x}{b}-\frac{y}{a}=0 \qquad \text{...(ii)}$$

To, find locus, squaring and adding (i) and (ii)

$$\left(\frac{x}{a}+\frac{y}{b}\right)^2+\left(\frac{x}{b}-\frac{y}{\text{a}}\right)^2=1$$

$$\Rightarrow \quad x^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)+y^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)=1$$

$$\Rightarrow \quad x^2\left(\frac{1}{c^2}\right)+y^2\left(\frac{1}{c^2}\right)=1 \quad \left[\therefore\ \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}\right]$$

$$\Rightarrow \quad x^2+y^2=c^2$$

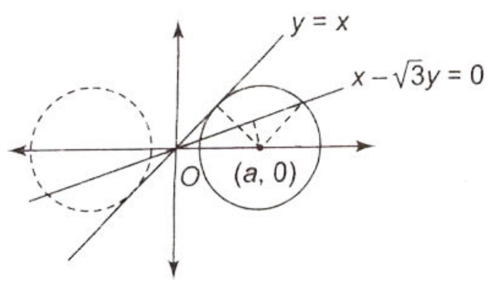
Q.58 The equation of the circle which is touched by $y=x$ has its centre on the positive direction of the x-axis and cuts off a chord of length 2 units along the line $\sqrt{3}\ y-x=0$ is

(a) $x^2+y^2-4x+2=0$ **(b)** $x^2+y^2-4x+1=0$

(c) $x^2+y^2-8x+8=0$ **(d)** $x^2+y^2-4y+2=0$

Ans: (a)

Sol: Since the required circle has its centre on the x-axis. So let the coordinates of the centre be $(a, 0)$. The circle touches $y=x$. Therefore, radius $=$length of the perpendicular from $(a, 0)$ on $x-y=0$ i $\frac{a}{\sqrt{2}}$. The circle cut off a chord of length 2 units along $x-\sqrt{3}\ y=0$.



$$\left(\frac{a}{\sqrt{2}}\right)^2 = 1^2 + \left(\frac{a - \sqrt{3} \times 0}{\sqrt{1^2 + (\sqrt{3})^2}}\right)^2$$

$\Rightarrow \quad \frac{a^2}{2} = 1 + \frac{a^2}{4} \Rightarrow \quad a = 2$

Thus, centre of the circle is at (2, 0) and radius

$= \frac{a}{\sqrt{2}} = \sqrt{2}$. So its equation is $x^2 + y^2 - 4x + 2 = 0$.

Q.59 A set of parallel chords of the parabola $y^2 = 4ax$ have their midpoint on
(a) any straight line through the vertex
(b) any straight line through the focus
(c) a straight line parallel to the axis
(d) another parabola

Ans: (c)

Sol: Let points $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ lie on the parabola $y^2 = 4ax$.
Here Point P and Q are variable but the slope of the chord is PQ.

$$m_{PQ} = \frac{2}{t_1 + t_2}$$

Now let midpoint PQ be $R(h, k)$

$$k = \frac{2at_1 + 2at_2}{2} \quad \text{or} \quad k = a(t_1 + t_2) = \frac{2}{m}$$

$\Rightarrow \quad y = \frac{2}{m}$, which is a line paralle to the axis of parabola.

Q.60 Let R be a relation on a set A such that $R = R^{-1}$, then R is
(a) reflexive **(b) symmetric** **(c) transitive** **(d) none of these**

Ans: (b)

Sol: Let $(a, b) \in R$

Then, $(a, b) \in R$

$\Rightarrow \quad (b, a) \in R^{-1}$ [By definition of R^{-1}]

$(b, a) \in R$ [$\because R = R^{-1}$]

So, R is symmetric.

Q.61 If $\sqrt{(1 - x^6)} + \sqrt{(1 - y^6)} = a(x^3 - y^3)$ and $\frac{dy}{dx} = f(x, y)\sqrt{\left(\frac{1 - y^6}{1 - x^6}\right)}$ then

(a) $f(x, y) = y / x$ **(b) $f(x, y) = y^2 / x^2$**

(c) $f(x, y) = 2y^2 / x^2$ **(d) $f(x, y) = x^2 / y^2$**

Ans: (d)

Sol: Let $x^3 = \cos p \quad y^3 = \cos q$

Given $\sqrt{(1 - x^6)} + \sqrt{(1 - y^6)} = a(x^3 - y^3)$

$\Rightarrow \quad \sqrt{(1-\cos^2 p)} + \sqrt{(1-\cos^2 q)} = a(\cos p - \cos q)$

$\Rightarrow \quad \sin p + \sin q = a(\cos p - \cos q)$

$\Rightarrow \quad 2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right) \Rightarrow \; = -2a\sin\left(\frac{p-q}{2}\right)\sin\left(\frac{p+q}{2}\right)$

$\Rightarrow \quad \tan\left(\frac{p-q}{2}\right) = -\frac{1}{a}$

$\Rightarrow \quad p - q = \tan^{-1}\left(-\frac{1}{a}\right)$

$\Rightarrow \quad \cos^{-1} x^3 - \cos^{-1} y^3 = \tan^{-1}\left(-\frac{1}{a}\right)$

Differntiating w.r.t.x we have

$$-\frac{3x^2}{\sqrt{1-x^6}} + \frac{3y^2}{\sqrt{1-y^6}}\frac{dy}{dx} = 0$$

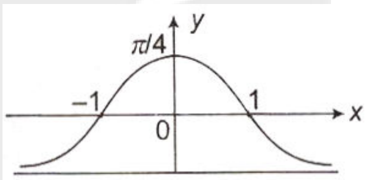
$$\Rightarrow \quad \frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$$

Q.62 The function $f(x) = \tan^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

(a) increasing in its domain
(b) decreasing in its domain
(c) decreasing in $(-\infty, 0)$ and increasing in $(0, \infty)$
(d) Increasing in $(-\infty, 0)$ and decreasing in $(0, \infty)$

Ans: (d)

Sol:



Putting $x^2 = \tan\theta$ to get

$$f(x) = \frac{\pi}{4} - \tan^{-1}(x^2)$$

$$\Rightarrow \quad f'(x) = -\frac{2x}{1+x^4}$$

Which is greater than zero for $x < 0$ and less than zero for $x > 0$.

Q.63 $\int \tan^{-1}\sqrt{x}\, dx$ equals

(a) $x \tan^{-1} x - \frac{1}{2}\log(1+x^2) + c$

(b) $x \tan^{-1}\sqrt{x} - \frac{1}{2}\log(1+x^2) + c$

(c) $x \tan^{-1}\sqrt{x} - \sqrt{x} + \log(1+x) + c$

(d) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + c$

Ans: (d)

Sol: $\int \tan^{-1}\sqrt{x}.1dx \; = \int 1-\tan^{-1}\sqrt{x}\; dx$

By using Integration by parts

$$I = \tan^{-1}\sqrt{x}.\int 1dx - \int \frac{d}{dx}(\tan{-}1\sqrt{x}).\int 1\; dx$$

$$= x\tan^{-1}\sqrt{x} - \int \frac{1}{1+x}\times\frac{1}{2\sqrt{x}}\times x$$

$$= x\tan^{-1}\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}}{1+x}dx$$

$$= x\tan^{-1}\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}}{1+(\sqrt{x})^2}\; dx$$

$let\ \sqrt{x} = t\ ,\ \ \frac{1}{2\sqrt{x}}\ dx = dt$

$$= x\tan^{-1}\sqrt{x} - \frac{2}{2}\int \frac{t^2}{1+t^2}\; dt$$

$$= x\tan^{-1}\sqrt{x} - \frac{2}{2}\int \frac{t^2+1-1}{t^2+1}\; dt$$

$$= x\ \tan^{-1}\sqrt{x} - \frac{2}{2}\int -1\ \int\frac{1}{t^2+1}\; dt$$

$$= x\ \tan^{-1}\sqrt{x} - t + \tan^{-1}t + c$$

$$= x\ \tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + c$$

$$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + c$$

Q.64 The equation of a curve passing through (2, 7/2) and having gradient $1-\frac{1}{x^2}$ at (*x, y*) is

(a) $y = x^2 + x + 1$ (b) $xy = x^2 + x + 1$ (c) $xy = x + 1$ (d) none of these

Ans: (b)

Sol: We have $\frac{dy}{dx} = 1 - \frac{1}{x^2} \Rightarrow y = x + \frac{1}{x} + C$

This passes through (2, 7/2) therefore

$$\frac{7}{2} = 2 + \frac{1}{2} + C \Rightarrow C = 1$$

Thus, the equation of the curve is

$$y = x + \frac{1}{x} + 1 \Rightarrow xy = x^2 + x + 1$$

Q.65 If $\frac{1+\sin 2x}{1-\sin 2x} = \cot^2(a+x)\,\forall\, x \in \mathbf{R} \sim \left(n\pi + \frac{\pi}{4}\right)$, n$\in$N. Then "*a*" is equal to

(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{2}$ **(c)** $\frac{3\pi}{4}$ **(d) none of these**

Ans: (c)

Sol: $\frac{1+\sin 2x}{1-\sin 2x} = \frac{(\sin x+\cos x)^2}{(\sin x-\cos x)^2}$

$$= \left(\frac{1+\tan x}{1-\tan x}\right)^2 = \left(\tan\left(\frac{\pi}{4}+x\right)\right)^2 = \tan^2\left(\frac{\pi}{4}+x\right)$$

$$= \cot^2\left(\frac{\pi}{2}+\frac{\pi}{4}+x\right) = \cot^2\left(\frac{3\pi}{4}+x\right)$$

Q.66 Let $\vec{a} = 2\hat{i} + \hat{j}+\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$, and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$ then $\vec{c}$ =

(a) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ **(b)** $\frac{1}{\sqrt{3}}(-\hat{i}-\hat{j}-\hat{k},)$ **(c)** $\frac{1}{\sqrt{5}}(\hat{i}-2\hat{j})$ **(d)** $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k},)$

Ans: (a)

Sol: As $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, we take

$\vec{c} = \alpha\,\vec{a} + \beta\vec{b}$ ...(i)

where α,β are scalars

As c is perpendicular to $\vec{a}$,

$\therefore$ From (i) we get $0 = \alpha\vec{a}\cdot\vec{a} + \beta\vec{b}\cdot\vec{a}$

$\Rightarrow \quad 0 = \alpha(6) + \beta\,(2+2-1) = 3\,(2\alpha+\beta)$

$\Rightarrow \quad \beta = -2\alpha$

Thus, $\vec{c} = \alpha(\vec{a}-2\vec{b}) = \alpha\,(-3\hat{j}+3\hat{k}) = 3\alpha\,(-\hat{j}+\hat{k})$

$\Rightarrow \quad |c|^2 = 9\alpha^2(1+1) = 18\alpha^2$

$\Rightarrow \quad 1 = 18\alpha^2$

$\Rightarrow \quad \alpha = \pm\frac{1}{3\sqrt{2}}$

$\therefore \quad \vec{c} = \pm\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$

Thus, we may take $c = \frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$

Q.67 Which of the following is logically equivalent to $\sim(\sim p \Rightarrow q)$?

(a) $p \wedge q$ **(b)** $p \wedge \sim q$ **(c)** $\sim p \wedge q$ **(d)** $\sim p \wedge \sim q$

Ans: (d)

Sol: It is clear from the table that $(\sim p \Rightarrow q)$ is equivalent to $\sim p \wedge \sim q$.

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$\sim p \wedge q$	$\sim p \wedge \sim q$	$\sim p \Rightarrow q$	$\sim(\sim p \Rightarrow q)$
T	T	F	F	T	F	F	F	T	F
T	F	F	T	F	T	F	F	T	F
F	T	T	F	F	F	T	F	T	F
F	F	T	T	F	F	F	T	F	T

Q.68 A and B are two events such that $P(A) > 0, P(B) \neq 1$, then, $P(\overline{A}/\overline{B})$ is equal to

(a) $1 - P(A/B)$ (b) $1 - P(\overline{A}/\overline{B})$ (c) $\frac{1 - P(A \cup B)}{P(\overline{B})}$ (d) $\frac{P(\overline{A})}{P(\overline{B})}$

Ans: (c)

Sol: $P(\overline{A}/\overline{B}) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{P(\overline{A \cup B})}{P(\overline{B})}$

$= \frac{1 - P(A \cup B)}{P(\overline{B})}$

Q.69 In the three dimensional xyz space the equation $x^2 - 5x + 6 = 0$ represents

(a) points (b) planes
(c) curves (d) pair of straight lines

Ans: (b)

Sol: $x^2 - 5x + 6 = 0$

$\Rightarrow \quad x - 2 = 0, x - 3 = 0$

Which represents a plane.

Q.70 The mean income of a group of workers is $\overline{X}$ and that of another group is $\overline{Y}$. If the number of worker in the second group is 10 times the number of workers in the first group, then the mean income of the combined group is

(a) $(\overline{X} + 10\overline{Y})/3$ (b) $(\overline{X} + 10\overline{Y})/11$ (c) $(10\overline{X} + \overline{Y})$ (d) $(\overline{X} + 10\overline{Y})/9$

Ans: (b)

Sol: Let the no of workers in group I be "2" i.e , x_1 and x_2

$\overline{X} = \frac{x_1 + x_2}{2}, x_1 + x_2 = 2\overline{X}$

Now, according to question, No of workers becomes 10 times.
i.e. 20 workers in Group-2

$\overline{X} = \frac{y_1 + y_2 + ... + y_{20}}{20}, (y_1 + y_2 + ... + y_{20}) = 20\overline{Y}$

for combined group,

$\text{Mean} = \frac{x_1 + x_2 + y_1 + y_3 + ... + y_{20}}{22}$

$\text{Mean} = \frac{2\overline{X} + 20\overline{Y}}{22}$

$\left[\text{Mean} = \dfrac{\overline{X}+10\overline{Y}}{11}\right]$

Q.71 A function $y = f(x)$ has a second -order derivative $f''(x) = 6(x-1)$. If its graph passes through the point (2, 1) and at that point tangent to the graph is $y = 3x-5$, then $f(2)$ is _____?

Sol: We have $f''(x) = 6(x-1)$ integrating

$$f'(x)=3(x-1)^2+c \qquad \text{...(i)}$$

At $(2, 1)$, $y = 3x-5$ is tangent to $y=f(x)$

$\therefore \quad f'(2) = 3$

From (i), $3= 3(2-1)^2+c \Rightarrow 3 = 3+c \Rightarrow c = 0$

$\therefore \quad f'(x) = 3(x-1)^2$

Integrating, $f(x) = (x-1)^3 + c'$

Since the curve passes through (2, 1)

$\therefore \quad 1 = (2-1)^3 + c' \Rightarrow c' = 0$

$\therefore \quad f(x) = (x-1)^3$

Q.72 Let $f : R \to R$ be a differentiable function having $f(2) = 6$, $f'(2) = \dfrac{1}{48}\cdot$ Then

$\lim\limits_{x\to 2}\displaystyle\int_6^{f(x)} \frac{4t^3}{x-2}\,dt$ equals ?

Sol: $\lim\limits_{x\to 2}\displaystyle\int_6^{f(x)} \frac{4t^3}{x-2}\,dt = \lim\limits_{x\to 2}\frac{\int_6^{f(x)} 4t^3\,dt}{x-2}$ $\qquad \left[\text{form } \dfrac{0}{0}\right]$

$$= \lim_{x\to 2}\frac{4(f(x))^3\ f'(x)}{1}$$

$$= 4(f(2))^3\ f'(2)$$

$$= 4(6)^3 \times \frac{1}{48} = 18$$

Q.73 Consider the five points comprising the vertices of a square and the intersection point of its diagonals. How many triangles can be formed using these points ?

Sol: To form a triangle, three points out of five can be chosen in ${}^5C_3 = 10$ ways. But of these, the three points lying on the two diagonals will be collinear. So $10-2=8$ triangle can be formed].

Q.74 The remainder when 1! + 2! + 3! + ...+ n! is divided by 5 is n ≥ 4 ?

Sol: In 1! + 2! + 3! + 4! + 5! ...+n! values 5!, 6!,...n!
are divisible by 5. Hence we have to find the remainder when
1! + 2! + 3! + 4! = 33 is divided by 5 which is 3.

Q.75 **Let $\vec{u} = \hat{i} + \hat{j}, \vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n} = 0$ and $\vec{v}.\hat{n} = 0$. $|\vec{w}\hat{n}|$ is equal to ?**

Sol: $\hat{n}$ is perpendicular to $\vec{u}$ and $\vec{v}$

$$[\because \vec{u}.\hat{n} = 0 \text{ and } \vec{v}.\hat{n} = 0]$$

$\therefore \hat{n} = \vec{u} \times \vec{v}$

$$\therefore \hat{n} = \frac{1}{\sqrt{2} \times \sqrt{2}} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = \frac{-2\hat{k}}{2} = -\hat{k}$$

$$\therefore |\vec{w}.\hat{n}| = (\hat{i} + 2\hat{j} + 3\hat{k}).(-\hat{k})| = |-3| = 3$$

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