Aggarwal CLASSES
A Division of Aggarwal Educare

JEE (MAIN)

TEST PAPER

SUBJECT : PHYSICS,CHEMISTRY, MATHEMATICS ***TEST CODE : TEST PAPER-3***

ANSWER PAPER

TIME : 3 HRS ***MARKS : 300***

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark(-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

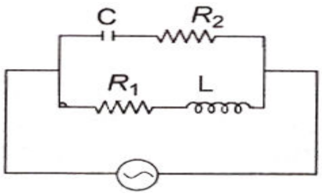
11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or "un- darken" a darkened bubble.

Part A - PHYSICS

Q.1 If a body is positively charged, then
(a) It may have more number of protons
(b) It may have only protons
(c) It may have only electrons
(d) Both (a) and (b)

Ans: (d)
Sol: Charge on a body is due to excess of deficiency of electron. If number of electrons is greater than number of protons, then it is negative charged. Again if number of electrons is less than number of protons then it is positive charged.
Example in support of option (a) is Li^+ and in support of option (b) is H^+.

Q.2 In the circuit shown in figure, $R_1 = 10\Omega$, $L = \frac{\sqrt{3}}{10}$ H, $R_2 = 20\Omega$ and $C = \frac{\sqrt{3}}{2}$ mF. The current in LR_1 circuit is I_1, in CR_2 circuit is I_2 and the main current is I.



$V_t = 200\sqrt{2}\sin(100t)$ volt The phase difference between I_1 and I_2 is
(a) 90^0 (b) 45^0 (c) 30^0 (d) 75^0

Ans: (a)
Sol: As inductive reactance,

$$X_L = \omega L = 100\left(\frac{\sqrt{3}}{10}\right) = 10\sqrt{3}\Omega$$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{X_L}{R_1}\right) = \tan^{-1}\left(\frac{10\sqrt{3}}{10}\right) = \tan^{-1}\sqrt{3} = 60^0$$

i.e. In LR1 circuit, current I_1 lags behind the voltage be 60^0.
Now capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times \frac{\sqrt{3}}{2} \times 10^{-3}} = \frac{20}{\sqrt{3}}\Omega$$

$$\Rightarrow \quad \phi_2 = \tan^{-1}\frac{X_C}{R_2} = \tan^{-1}\left(\frac{20/\sqrt{3}}{20}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^0$$

i.e. in CR_2 circuit, current I_2 leads the voltage by 30^0.
Hence, phase difference between I_1 and I_2 is 90^0.

Q.3 The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimation of kinetic energy obtained by measuring mass and speed ?
(a) 11 % (b) 8 % (c) 5 % (d) 1 %

Ans: (b)

Sol: As kinetic energy, $E = \frac{1}{2}mv^2$

$\therefore$ % error in KE

= % error in mass $+2\times$% error in velocity

= $2 + 2 \times 3 = 8\%$

Q.4 In determination of refractive index of glass slab using travelling microscope, first of all we took a reading when the microscope is focussed on a mark. This reading comes out to be s_1, then we place a glass slab on the surface covering the mark. Now, the microscope is readjusted to focus the mark through the slab and this time reading come out to be s_2. Then we place an opaque object on the glass slab and adjust the microscope to focus on opaque object. This time the reading of microscope is s_3. The refreactive index of the glass slab is

(a) $\frac{s_3 - s_1}{s_2 - s_1}$ **(b)** $\frac{s_3 - s_2}{s_2 - s_1}$ **(c)** $\frac{s_3 - s_1}{s_3 - s_2}$ **(d)** $\frac{s_3}{s_3 - s_2}$

Ans: (c)

Sol: The quantity $s_3 - s_1$ corresponds of thickness of slab or actual depth of the mark while $s_3 - s_2$ corresponds to the apparent depth of the object.

Now, $\mu = \frac{\text{Actual depth}}{\text{Apparent depth}}$

$= \frac{s_3 - s_1}{s_3 - s_2}$

Q.5 Why the capacitance of a capacitor is not affected by surrounding conducting bodies?

(a) From the formula $C = Q / V$, if Q changes, V changes and hence C remains same

(b) The plates of the capacitor are arrenged in such a way that the field created by charges accumulated on the plate is concentrated alomost completely inside the capacitor

(c) Capacitance changes due to induction phenomenon

(d) Both (a) and (b)

Ans: (b)

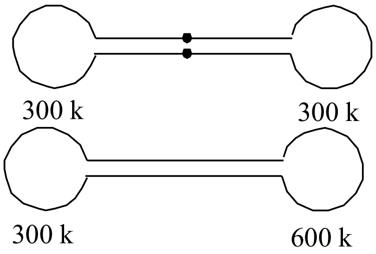
Sol: With respect to option (b) as field is not coming out, so they will not redistribute the charges of surrounding bodies and hence no change in potential of capacitor and hence C has not been affected by surrounding bodies.

Q.6 Two spherical vessels of equal volume are connected by a narrow tube. The apparatus contains an ideal gas at 1 atm and 300 K. Now if one vessel is immersed in a bath of constant temperature 600 K and other in a bath of constant temperature 300 K, then common pressure will be

(a) 1 atm (b) 4/5 atm (c) 4/3 atm (d) 3/2 atm

Ans: (c)

Sol: Initial pressure, $p_1 = p_2 = p_0 = \frac{nRT_0}{V_0}$ When $p_0 = 1\,\text{atm}$, $T_0 = 300\,\text{K}$. finally let common pressure be P.



So, $\frac{pV_0}{R \times 2\ T_0} = n_1$ $\quad$ [$\because$ pV = nRT]

$$\frac{pV_0}{RT_0} = n_2$$

and $\quad n_1 + n_2 = 2\ n$

$$\Rightarrow \quad \frac{pV_0}{2\ RT_0} + \frac{pV_0}{RT_0} = 2\left(\frac{p_0 V_0}{RT_0}\right)$$

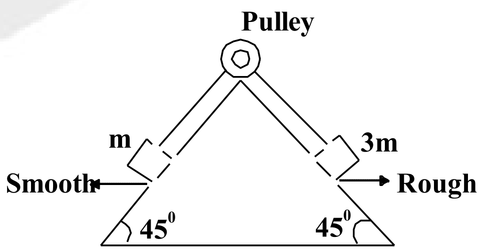
$$\Rightarrow \quad = \frac{p}{p_0} = \frac{4}{3}\ \text{atm}$$

Q.7 A block in moving with constant velocity on a horizontal surface, then which one is incorrect ?
(a) Some force must act on the body
(b) Some forces may act on the body
(c) Net force on the body must be zero
(d) the body is in equilibrium

Ans: (a)

Sol: This is the case of dynamic equilibrium where $F_{net} = 0$, but system is moving with constant speed. In this example equal and opposite forces may act on the body.

Q.8 Acceleration of each block is given as $g / 5\sqrt{2}$. Find the magnitude and direction of force exerted by string on pulley

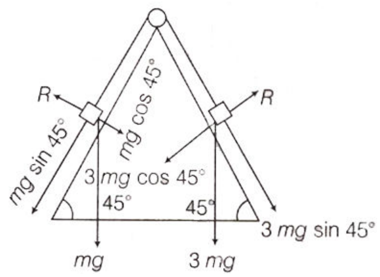


(a) 6 mg/5, downward direction **(b) 6 mg/5, upward direction**
(c) 6 mg/5, horizontal direction **(d) Not possible to have acceleration**

Ans: (a)

Sol: Let coefficient of friction be μ and 3m block be moving down the incline, then|

$$\text{Acceleration} = \frac{\text{Net pulling force}}{\text{Total mass}}$$



$$= \frac{3\,mg\sin 45^0 - 3\mu mg\cos 45^0 - mg\sin 45^0}{4m} \quad [\because R = \mu mg]$$

$$\Rightarrow \quad \frac{(2-3\mu)g}{4\sqrt{2}} = \frac{g}{5\sqrt{2}}$$

$$\Rightarrow \quad \mu = 0.4$$

$$\Rightarrow \quad T - mg\sin 45^0 = \frac{mg}{5\sqrt{2}}$$

$$\Rightarrow \quad T = 6mg/(5\sqrt{2})$$

Force exerted by string on pulley is $\sqrt{2}$ T as shown in figure.

$$F = \frac{6\ mg}{5}$$

Q.9 A particle is moving along *x*-axis, under the action of a variable force which is providing position varying acceleration described by the equation, $a = 3x - 4$ at $t = 0$, $x = \frac{4}{3}$m and $v = 0$. Find the velocity and position of particle at $t = 5$ s.

(a)0 m/s, 0 m **(b) 0 m/s, $-\frac{8}{3}$m** **(c) 0 m/s, 4/3 m** **(d) None of these**

Ans: (c)

Sol: In this particular question, acceleration at $t = 0$ is $3 \times \frac{4}{3} - 4 = 0$ and velocity is also zero at $t = 0$. So, particle will remain at rest.

Hence, $v = 0$ and $x = \frac{4}{3}m$ (initial position).

Q.10 A pulse or a wave train travels along a stretched string and reaches the fixed end of the string. It will be reflected back with

(a) The same phase as the incident pulse but with velocity reversed
(b) A phase change of 180^0 with no reversal of velocity
(c) The same phase as the incident pulse with no reversal of velocity
(d) A phase change of 180^0 with velocity reversed

Ans: (d)

Sol: On reflection from fixed end (denser medium), a phase difference of π is introduced.

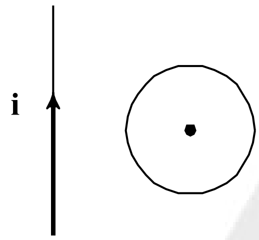
Q.11 If we consider the Rutherford model of atom, then from classical theory the electron will follow a spiral path due to continuously radiated energy and finally falls to nucleus, then the wavelength of continously emitted electromegnetic radiation

[Assume speed of electron to be constant as electron is going towards the centre]
(a) remains constant
(b) continuously decreasing as electron is coming closer to nucleus
(c) continuously increasing as electron is coming closer to nucelus
(d) randomly changes

Ans: (b)

Sol: As n decreases, frequency increases. So wavelength decreases. $[\because v \propto \frac{1}{n}]$

Q.12 If *i* is increasing continuously, then find the direction of induced current in the loop.



(a) Anti-clockwise **(b) Clockwise**
(c) In any direction **(d) No current will be induced**

Ans: (a)
Sol: At the location of loop, magnetic field is perpendicular to plane of paper and going into it, which is increasing with time as current is increasing. Therefore, emf will be induced in the loop in such a way so that induced current will produce magnetic field as such it opposes the original magnetic field in accordance with Lenz's law. Thus current is induced in the loop in anticlockwise direction.

Q.13 A body of mass m is placed on the earth's surface. It is taken from the earth's surface to a height $h = 3R$. The change in gravitational potential energy of the body is

(a) $\frac{2}{3}mgR$ **(b)** $\frac{3}{4}mgR$ **(c)** $\frac{mgR}{2}$ **(d)** $\frac{mgR}{4}$

Ans: (b)
Sol: Gravitational potential energy at surface of the earth

$$= mgh = mgR = -\frac{GM_eM}{R_e}$$

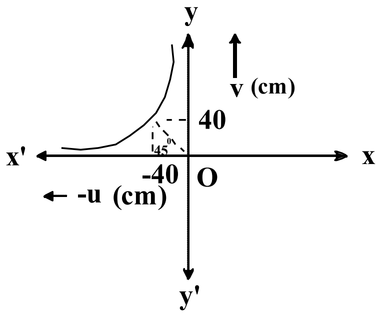
Gravitational potential energy at height

$$(h = 3R) = \frac{-GM_eM}{R_e + 3R_e} = \frac{-GM_eM}{4R_e}$$

$$\text{Change in PE} = +\frac{GM_eM}{R_e} - \frac{GM_eM}{4R_e} = \frac{3GM_eM}{4R_e}$$

$$= \frac{3}{4}\frac{gR_e^2m}{R_e} = \frac{3}{4}mgR_e \qquad [\because GM_e = gR_e^2]$$

Q.14 Consider the following u-v diagram regarding the experiment to determine the focal length of a convex lens.



At the point A, the values of u and v are equal. The focal length of the lens is
(a) 40 cm **(b) 20 cm** **(c) 10 cm** **(d) 15 cm**

Ans: (b)

Sol: Clearly, the coordinates of A are $(2f, 2f)$ [$\because$ at C only, the object and image sizes are equal]

$$f = \frac{40}{2} = 20 \text{ cm}$$

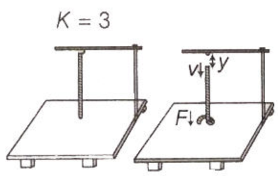
Q.15 A uniform rope, of mass m per unit length, hangs vertically from a support so that the lower end just touches the table top. If it is released, then at the time a length y of the rope has fallen, the force on the table is equivalent to the weight of the length Ky of the rope. Find the value of K.
(a) 1 **(b) 2** **(c)3** **(d)3.5**

Ans: (c)

Sol: The descending part of the rope is in free fall. it has speed, $v = \sqrt{2gy}$ at the instant all its points have descended by a distance y. The length of the rope which Lands on the table during an interval dt following this intant is *vdt*.
The increment of momentum inparted to the table by this length in coming to rest is m (*vdt*) v. Thus, the rate at which momentum is transferred to the table is

$$\frac{dp}{dt} = mv^2 = (2my)g$$

and this is the force arising from stopping the downward fall of the rope. Since a length of rope y, of the weight (my) g already lies on the tabletop, the total force on the tabletop is $(2\text{ my})g + (\text{my})g = (3\text{my})\text{ g}$ or the weight of a length 3y of rope.
So,



Q.16 A body when projected vertically up covers a total distance s, during its time of flight. If we neglect gravity then how much distance, the particle, will travel during the same time. Will it fall back ?
(a) s, Yes **(b) s, No** **(c) 2 s, Yes** **(d) 2 s, No**

Ans: (d)

Sol: Let particle is projected with speed u, so total time of flight

$$T = \left(\frac{2u}{g}\right) \text{ and } s = 2 \times \text{maximum height}$$

$$= 2 \times \frac{u^2}{2g} = \frac{u^2}{g}$$

If there is no gravity, then $s' = u \times T = \frac{2u^2}{g} = 2s$

If gravity is not there, then it will never fall back.

Q.17 Electric flux crossing the surface *S*, which encloses a charge *q*, will be maximum if surface *S* is
(a)Spherical (b) Cube (c)Cylindrical (d) Same in all three

Ans: (d)

Sol: Flux crossing the surface does not depend on shape of surface, it only depeneds on value of charge eneclosed, which in present question is same in all cases, so flux crossing is same in all cases.

Q.18 Regarding ideal gas, mark the incorrect option.
(a) Internal energy is comprising of PE and KE
(b) Monatomic gas has zero PE, because it is having single atom
(c) Vibration energy may have non-zero value for monoatomic gas
(d) At room temperature, the internal energy is comprising of translational and rotational KE only.

Ans: (c)

Sol: Since, ideal gas atoms are consider as of having negligible size. so the vibrational energy possessed by them will be zero.

Q.19 $dQ = nCdT$ represents the
(a) Change in amount of heat contained in a body, as a result of temperature change
(b) Amount of heat energy, which transits from one body to other due to temperature difference
(c) Both (a) and (b) are correct
(d) None of the above

Ans: (b)

Sol: Heat is defined as energy in transit, which takes place due to temperature difference for option (a), heat is not the energy contained in the body.

Q.20 If an isolated charge particle is moving in a magnetic field, then
(a) Its kinetic energy remains constant
(b) Work done by magnetic force is zero
(c) Velocity of particle must very with time
(d) All of the above

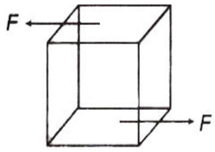
Ans: (d)

Sol: If some other force in addition to magnetic force is acting, then none of the options is correct. But, if only magnetic force is acting, then all the three options are right. But here charge is isolated means no other force is acting on it.

Q.21 Two parallel and opposite forces, each 4000 N, are applied tangentially to the upper and lower faces of a cubical metal block 25 cm on a side. Find the angle of shear and the displacements of the upper surface relative to the lower surface. The shear modulus for the metal is 80 Gpa?

Sol: We use the approximate formula $S = F/(A\phi)$, with $S = 8 \times 10^{10}\ N/m^2$,

and $A = (0.25\,m)^2 = 6.25 \times 10^{-2}\,m^2$



On solving for ϕ we get,

$$\phi = \frac{(4000\ \text{N})}{(6.25 \times 10^{-2}\ \text{m}^2)(8 \times 10^{10}\ \text{N}/\text{m}^2)}$$

$$= 8.0 \times 10^{-7}\ \text{rad}$$

The displacement of the upper surface is given by $d = L\phi$ where L is an edge of the cube.

$\therefore$ $d = (8.0 \times 10^{-7}\ \text{rad})\ (25\ \text{cm})$

$$= 2.0 \times 10^{-5}\ \text{cm}$$

Q.22 The magnetic field in a region between the poles of an electromagnet is uniform at any time, but it's magnitude is increasing at the rate of 0.02 T/s. A conducting loop is placed in this region, whose plane is perpendicular to direction of magnetic field. Calculate the emf induced and induced current in the loop. Take cross-section area of loop as 120 cm² and resistance of loop as 5Ω ?

Sol: As $e = -\frac{d\phi}{dt} = -A \times \frac{dB}{dt}$

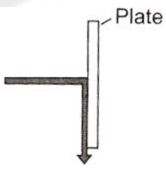
$$= -120 \times 10^{-4} \times 0.02 = -0.24\text{mV}$$

Negative sign tells about the direction (polarity).

Also, $i = \frac{e}{R} = \frac{0.24}{5} = 0.048\ \text{mA}$

Q.23 What force is exerted on a stationary flat plate held perpendicular to jet of water ? The horizonatal speed of the water is 80 cm/s and 30 cm³ of the water hits the plate each second. Assume that the water moves parallel to the plate after striking it. One cubic centimetre of water has a mass of one gram.

Sol: The plate exerts and impulse on the water and changes its horizontal momentum.



Impulse in x-direction = Change in X-direction momentum

Let us take t to be 1s, so that $m = 30\,\text{g}$ will be the mass that strikes in 1s.

Impulse $= p_f - p_i$

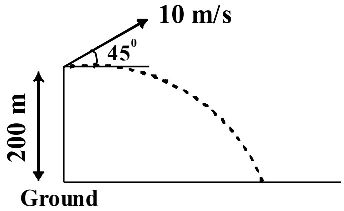
$$= (0.030\,\text{kg}) \times 0 - (0.030\,\text{kg})\ (0.8\ \text{m/s}) \quad [\because v = 0]$$

$$Ft = -0.024$$

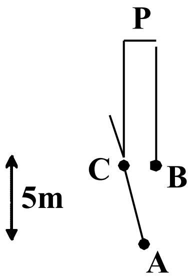
So, $F = -0.024\,\text{N}$

This is the force of the plate on the water. By Newton's third law, the force on the plate is $+0.024\,\text{N}$.

Q.24 A particle has been projected from the top of tower as shown in figure. Find the time taken by the particle to reach the ground. (Take g = 10 m/s²)



Sol: Consider point of projection as origin and horizontal direction as positive x-axis and verticalupward direction as positive y-axis.



For motion along y-axis, from

$$y = u_y t + \frac{a_y t^2}{2}$$

For particle to reach ground, $y = -200\,m$

$\therefore$ $$-200 = (10\sin 45^0)\,t - 200 = (10\sin 45^o)\,t - \frac{gt^2}{2}$$

The two roots of this quadratic equation are 7.07 s and -5.66s

Q.25 A uniform rectangular marble slab is 3.4 m long and 2.0 m wide. It has a mass of 180 kg. If it is originally lying on the hat ground, how much work is needed to stand it on one end ?

Sol: The work done by gravity is the work done, as if all the mass was concentrated at the centre of mass. The work necessary to lift the object can be thought of as the work done against gravity and is just, W = mgh, where h is the height throught which the centre of mass is raised.

$$W = (180\text{ kg})\,(9.8\text{ m/s}^2)\,(1.7\,m) = 3.0\,KJ$$

Part - B - CHEMISTRY

Q.26 The separation of lanthanides by ion exchange method is based on
(a) Size of the ions **(b) Oxidaion state of the ions**
(c) The solubility of their nitrates **(d) Basicity of hydroxides of lanthanides**

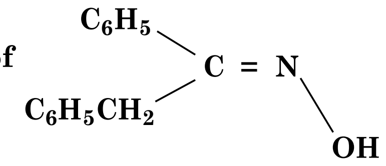
Ans: (a)
Sol: Separation of lanthanides by ion exchange method is based on size of the ions.

Q.27 Dimethyl glyoxime in a suitable slovent was refluxed for 10 min with pure pieces of Nickel sheet, it will result in
(a) Red precipitate **(b) Blue precipitate**
(c) Yellow precipitate **(d) No preicpitate**

Ans: (a)

Sol: DMG gives red ppt. with Ni^{2+} ions but not with Ni Metal.

Q.28 Beckman transformation of



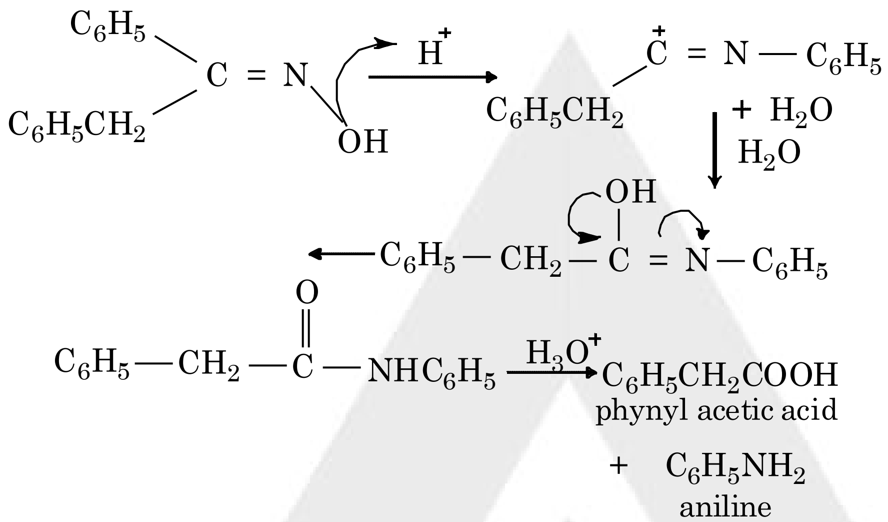
Followed by hydrolysis will yield

(a) Benzoic acid + benzylamine **(b) Phenylacetic acid + benzylamine**

(c) Aniline + phenylacetic acid **(d) Benzoic acid + aniline**

Ans: (c)

Sol:



Q.29 The wavelength of spectral line for an electronic transition is inversely related to

(a) The number of electrons undergoing transition

(b) The nuclear charge of the atom

(c) The difference in the energy levels involved in the transition

(d) The velocity of electron undergoing transition

Ans: (d)

Sol: $\Delta E = hv = h\frac{C}{\lambda}$ or $\lambda \infty \frac{1}{v}$

Q.30 A deliquescent white crystalline hydroxide X reacts with a nitrate Y to form another hydroxide which decomposes to give a insoluble brown layer of it's oxide. X is a powerful cautery and breaks down the proteins of skin flesh to a pasty mass. X and Y are.

(a) NaOH, $AgNO_3$ **(b) NaOH, $Zn(NO_3)_2$**

(c) NaOH, $Al(NO_3)_3$ **(d) $Ca(OH)_2$, $HgNO_3$**

Ans: (a)

Sol: $\underset{\substack{(X)\\ \text{Deliq}\\ \text{White crystal}}}{NaOH} + \underset{(Y)}{AgNO_3} \longrightarrow \underset{\text{Unstable}}{AgOH} + NaNO_3$

$$2AgOH \longrightarrow \underset{\text{brown}}{Ag_2OH} + H_2O$$

(X) is a powerful cautery and breaks down the proteins of skin flesh to a pasty mass i.e. X is caustic soda.

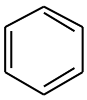
Q.31 Substances like sulphur and phospherus are fairly soluble in alcohol but less soluble in water. If their alcholic solution is poured in water, colloidal solutions of sulphur and phosphorus are obtained. This method can be called

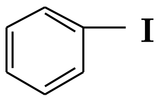
(a) Peptisation **(b) Disintegration**

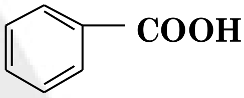
(c) Condensation **(d) Dispersion**

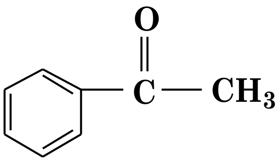
Ans: (c)

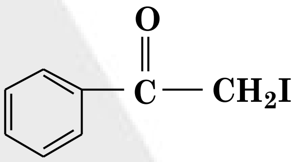
Sol: Fairly soluble solution means sulphur and phosphorous particules are much smaller but on pouring into heated solvent. These particles collapse together to form bigger aggregates, which fell into colloidal particles size range.

Q.32  **+ $(CH_3CO)_2O \xrightarrow{AlCl_3} \xrightarrow[H_3O^+]{I_2,\ OH^-}$ A A is predominantly**

(a) 

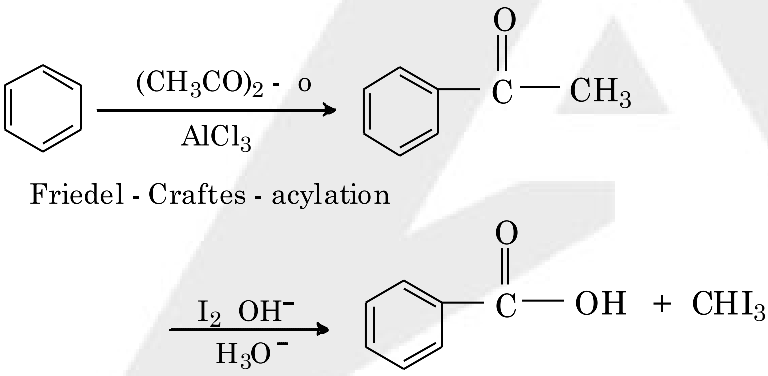
(b) 

(c) 

(d) 

Ans: (b)

Sol:



Haloform reaction

Q.33 Two hybrid orbitals have a bond angle of 120^0, The percentage of s-character in the hybrid orbital is nearly

(a) 25% **(b) 33%** **(c) 50%** **(d) 66%**

Ans: (b)

Sol: Since bond angle is 120^o orbital is sp^2 hybridised. In sp^2 hybridised orbital, % of s - character

$$= \frac{1}{1+2} \times 100 = 33\%.$$

Q.34 The only cations present in a slightly acidic solution are Fe^{3+}, Zn^{2+} and Cu^{2+}. The reagent that when added in excess to this solution would identify and separate Fe^{3+} in one step is

(a) 2M HCl **(b) 6 M NH_3**

(c) 6 M NaOH **(d) H_2S gas**

Ans: (b)

Sol: $Fe^{3+} + Zn^{2+} + Cu^{2+} \xrightarrow{6\text{ M }NH_3} \underset{\text{Brown ppt.}}{Fe(OH)_3} + \underset{\text{Soluble}}{[Zn(NH_3)_4]^{2+}} + \underset{\text{Soluble}}{[Cu(NH_3)_4]^{+}}$

Q.35 In analogy to $O_2^+[PtP_6]^-$ a compound $N_2^+[PtP_6]^-$ will not be formed because

(a) The ionisation enthalpy of N_2 gas is higer than that of O_2 gas
(b) The ionisation enthalpy of N_2 gas is lower than that O_2 gas
(c) The ionisation enthalphy of N_2 gas is higher than that of N atom
(d) None of the above

Ans: (a)

Sol: Ionisation enthalpy of N_2 gas (1503 kJ mol^{-1}) is higher than that O_2 gas (1175 kJ mol^{-1}) and it cannot lose its electron so easily as O_2 does in forming $O_2^+ + [PtF_6]^-$ compound.

Q.36 Gold (I) thiomaleate is used as medicine for the treatment of which one of the following ?

(a) Malaria **(b) Arthritis** **(c) Diabetes** **(d) Ulcer**

Ans: (b)

Sol: Gold (I) thiomaleate is used for the treatment of arthritis.

Q.37 Which of the following in not a functional isomer of C_3H_6O ?

(a) Prop -2-en-l-ol **(b) 1 , 2-epoxy propane**
(c) Propanol **(d) Prop - 2 - one**

Ans: (c)

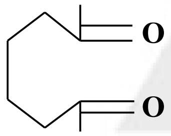
Sol: Propanol $CH_3CH_2CH_2OH$, C_3H_8O cannot be isomer of C_3H_6O.

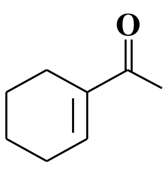
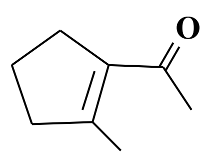
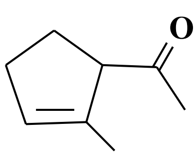
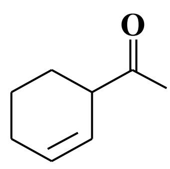
Q.38 An element (X) forms compounds of the formula XCl_3, X_2O_5, and Ca_3X_2, but does not form XCl_5. Which of the following is the element X ?

(a) B **(b) A** **(c) N** **(d) P**

Ans: (c)

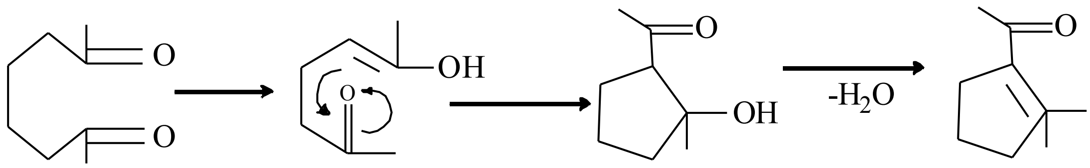
Sol: The element is N which forms NCl_3, N_2O_5, Ca_3N_2 but not NCl_5.

Q.39  $\xrightarrow[\Delta]{\overset{\ominus}{OH}}$ **Product, major predominant product of the reaction can be**

(a)  **(b)**  **(c)**  **(d)** 

Ans: (b)

Sol: Internal aldol condensation



Q.40 A hydrogen gas electrode has potential of -0.118 V when H_2 gas is bubbled at 298 K and 1 atm, in HCl solution. The pH of HCl solution is.

(a) 2 (b) 1 (c) 7 (d) 2.7

Ans: (a)

Sol: $pH = -\log[H+] = \log\frac{1}{[H^+]}$

$E_{cell} = E^o{}_{cell} - 0.0594pH$

So, calculate the pH by this formula

$H^+ + e^- \rightarrow \frac{1}{2}H_2$

$E_{cell} = E^o{}_{cell} - 0.0591\log\frac{1}{H^+}$

$-0.118\,V = 0 + 0.0591\ \log[H^+]$

$-0.118\,V = -0.0591(-\log[H^+])$

$-0.118\,C = -0.0591\,pH$

$pH = \frac{0.118}{0.0591} = 2$

Q.41 In the photo-electric effect, the number of photoelectrons emitted per unit time depends upon the

(a)Energy of incident radiations

(b)Frequency of incident radiations

(c)Intensity of incident radiations

(d)Both frequency and intensity of incident radiations

Ans: (c)

Sol: Number of photoelectrons emitted depends upon intensity of incident radiations.

Q.42 On the following reactions $C_2H_2 \longrightarrow C_2H_4 \xrightarrow{H_2} C_2H_6$ the state of hydridisation of carbon changes from

(a) $sp \rightarrow sp^2 \rightarrow sp^3$ (b) $sp^3 \rightarrow sp^2 \rightarrow sp$

(c) $sp^2 \rightarrow sp^2 \rightarrow sp$ (d) $sp^3 \rightarrow sp \rightarrow sp^2$

Ans: (d)

Sol: As CN^- is the strongest ligand, it will form the most stable complex.

Q.43 The correct order of dipole moment is

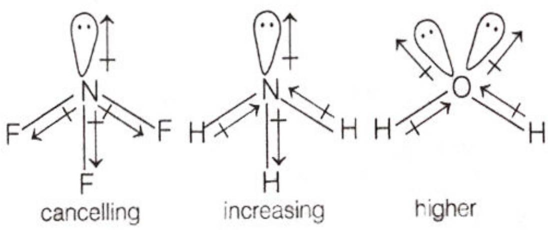
(a) $CH_4 < NF_3 < NH_3 < H_2O$ (b) $NF_3 < CH_4 < NH_3 < H_2O$

(c) $NH_3 < NF_3 < CH_4 < H_2O$ (d) $H_2O < NH_3 < NF_3 < CH_4$

Ans: (a)

Sol: When a molecule contains more than one polar bond, its dipole moment depends not only on the individual bond moment but also on the arrangement of the bonds in space. If a molecule has symmetrical structure then its dipole moment will be zero.

CH_4 is regular tetrahedral, hence net dipole moment $\mu_R = 0$.



Q.44 An aqueous solution of a substance gives a white precipitate on treatment with dil. HCl which dissolves on heating. When H_2S is passed through the hot acidic solution, a black precipitate is obtained. The substance is a

(a) Hg_2^{2+} salt (b) Cu^{2+} salt (c) Ag^{+} salt (d) Pb^{2+} salt

Ans: (d)

Sol: $Pb^{2+}(aq) + HCl\ (dill.) \rightarrow PbCl_2$ (White ppt.)

White precepitate of $PbCl_2$ is soluble only in hot water. Pb^{2+} ions give back precipitate of PbS with H_2S.

Q.45 Bicyclohexane was found to increase two parallel first order rearrangements . At 730 K, the first order rate constant for the formation of cyclohexane was measured as $1.26 \times 10^{-4}\ s^{-1}$, and for the formation of methylcyclopentane the rate constant was $3.8 \times 10^{-5}\ s^{-1}$. The percentage of the cyclohexane is _______?

(a) 20 % (b) 77 % (c) 23 % (d) 50%

Ans: (b)

Sol:
$$\frac{1.26 \times 10^{-4}\ s^{-1}}{(1.26 \times 10^{-4}\ s^{-1} + 3.8 \times 10^{-5}\ s^{-1})} = 77\%$$

Q.46 The pH 0.10 M NH_3 solution is [Given $K_b = 1.8 \times 10^{-5}$; log 1.35 = 0.13]

Sol: $H_2OH^+ \rightleftharpoons H^+ + OH^-$

$K_w = [H^+][OH^-]$

$10^{-14} = [H^+][OH^-]$

or $pH + pOH = 14$

Find the value of OH^- and calculate the values of pH and pOH.

$NH_3 + H_2O \rightleftharpoons \overset{+}{N}H_4 + OH^-$

$$K_b = \frac{[NH_4^+][OH^-]]}{[NH_3]} = \frac{x^2}{0.10}$$

$x^2 = 1.8 \times 10^{-6}$ $x = 1.35 \times 10^{-3} = [OH^-]$

pOH = 2.87 pH = 11.13

Q.47 3.7 g of an oxide of a metal was heated with charcoal. The liberated CO_2 was absorbed in caustic soda solution and weighed 1.0 g. If the specific gravity of the metal is 0.095, the exact atomic weight of the metal is _______?

Sol: Weight of $CO_2 = 1g$ (as absorbed in KOH)

Weight of oxygen in oxide = Weight of oxygen in 1 g of $CO_2 = \frac{32}{44} = \frac{8}{11}$ g

Weight of metal $= 3.7 - \frac{8}{11}$

Equivalent weight $= \frac{\text{Weight of metal}}{\text{Weight of oxygen}} \times 8 = 32.7$

According to Dulong Petits law,

Atomic weight (approx) $= \frac{6.4}{0.095} = 67.37$

Valency $= \frac{\text{Atomic weight}}{\text{Equivalent weight}} = 2$ (approx)

Exact atomic weight $= 32.7 \times 2 = 65.4$

Q.48 The standard heat of combustion of carbon(s), sulphur (s) and carbon disulphide (*l*) are - 393.3, - 293.72 and -1108.76 kJ/mol respectively. The standard heat of formation of carbon disulphide (*l*) is Given?

Sol: (i). $C(s) + O_2 \longrightarrow CO_2$; $\Delta H = -393.3$ kJ / mol

(ii) $S(s) + O_2 \longrightarrow SO_2$; $\Delta H = -293.72$ kJ / mol

(iii) $CS_2(I) + 3O_2(g) \longrightarrow CO_2(g) + 2SO_2(g)$; $\Delta H = -1108.76$ kJ / mol

On putting various enthalpy of formation in equation III

$\Delta H = \Delta H$ (prouducts) $- \Delta H$ (reactants)

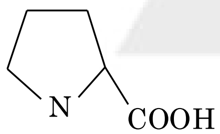
$-1108.76 = [-393.3 + 2(-293.72)] - [\Delta H_f(CS_2) + 3 \times 0]$

$-1108.76 = -393.3 - 2 \times 293.72 - \Delta H_f(CS_2)$

$\Delta H_f(CS_2) = 128.02$ KJ

Q.49 The reaction of primary aliphatic amines with nitrous acid gives a quantitative yield of nitrogen gas and is the basis of the van. Slyke determination of amino nitrogen. What volume of nitrogen gas at STP would be liberated from 0.001 mole of proline ?

Sol:



It consists of 2^0 amino group that's why diazotiation does not proceed resultantly no evolution of N_2. It is not proper for van slyke method

Q.50 In an aqueous solution $AgNO_3$ and $CuSO_4$ are connected in series. If Ag deposited at cathode is 1.08 g, Then Cu deposited is

Sol: Eq. Of Ag = Eq. Of Cu

$\frac{1.08}{108} = \frac{W_{Cu}}{63.5/2}$ or

$W_{Cu} = \frac{63.5 \times 1.08}{2 \times 108} = 0.3175$ g

Part - C - MATHEMATICS

Q.51 **If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then**

(a) $-4 < a < 2$ **(b) $2 \le a \le 3$** **(c) $3 < a \le 4$** **(d) $a > 4$**

Ans: (a)

Sol:

$f(x)x^2 - 2ax + a^2 + a - 3 = 0$

as roots ore real,

$D \ge 0$

$b^2 - 4ac \ge 0$

$4a^2 - 4(a^2 + a - 3) \ge 0$

$a^2 - a^2 - a + 3 \ge 0$

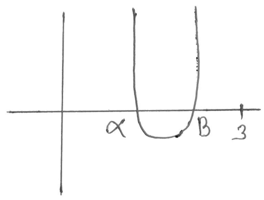
$-a + 3 \ge 0$

$a - 3 \le 0$

$a \le 3$

$a \in (-\infty, 3)$

As roots are less than 3

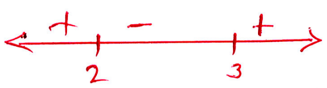


f(3)>3

$9 - 6a + a^2 a - 3 > 0$

$a^2 - 5a + 6 > 0$

$(a-3)(a-2) > 0$

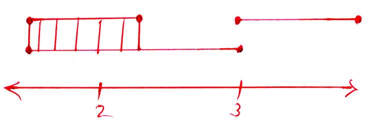


$a \in (-\infty, 2) \cup (3, \infty)$

$\frac{-b}{2a} < 3$

$\frac{2a}{2} < 3$

$a < 3$



so, $a \in (-\infty, 2)$

Q.52 If $\int \frac{2^x}{\sqrt{1-4^x}}dx = k\ \sin^{-1}(2^x) + C,$ then k is equal to ?

(a) log 2 **(b) $\frac{1}{2}$log 2** **(c) $\frac{1}{2}$** **(d) $\frac{1}{\log 2}$**

Ans: (d)

Sol: Let $I = \int \frac{2^x}{\sqrt{1-4^x}}dx$

Put $2^x = t$

$\Rightarrow \quad 2^x \log 2\, dx = dt$

$\therefore \quad I = \frac{1}{\log 2}\int \frac{1}{\sqrt{1-t^2}}dt$

$= \frac{1}{\log 2}\sin^{-1}(t) + C$

$= \frac{1}{\log 2}\sin^{-1}(2^x) + C$

But $\quad I = k\sin^{-1}(2^x) + C$

$\Rightarrow \quad k = \frac{1}{\log 2}$

Q.53 Let P be any point on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ then, the length of the segment of the tangent between the coordinates axes of length
(a) 3a **(b) 4a** **(c) 5a** **(d) a**

Ans: (d)

Sol: Let the coordinates of the point P be $(x_1\ y_1)$. This point lies on the curve

$x^{2/3} + y^{2/3} = a^{2/3}$ ....(i)

$\therefore \quad x_1^{2/3} + y_1^{2/3} = a^{2/3}$ ...(ii)

On differentiating Eq (i) w.r.t.x, we get

$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}}$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(x_1\ y_1)} = -\left(\frac{y_1}{x_1}\right)^{1/3}$

The equation of the tangent at $(x_1\ y_1)$ to the given curve is

$$y - y_1 = -\frac{y^{1/3}}{x^{1/3}}(x - x_1)$$

$$\Rightarrow \quad \frac{x}{x_1^{1/3}} + \frac{y}{y_1^{1/3}} = x_1^{2/3} + y_1^{2/3}$$

$$\Rightarrow \quad \frac{x}{x_1^{1/3}} + \frac{y}{y_1^{1/3}} = a^{2/3}$$ From Eq (ii)

This tangent meets the coordinate axes at

$A(a^{2/3}x_1^{1/3}, 0)$ and $B(0, a^{2/3}, y_1^{1/3})$.

$$\therefore \quad AB = \sqrt{(0 - a^{2/3}x_1^{1/3})^2 + (a^{2/3}y_1^{1/3} - 0)^2}$$

$$= \sqrt{a^{4/3}(x_1^{2/3} + y_1^{2/3})} = \sqrt{a^{4/3}.a^{2/3}}$$

$$= \sqrt{a^2} = a$$ [From eq.(ii)]

Q.54 If y = f(x) ia an even function such that $f'(0)$ exists, then $f'(0)$ equals

(a) 0 **(b) -1** **(c) 1** **(d) none of these**

Ans: (a)

Sol: Since $f(x)$ *is even* $\therefore f(-x) = f(x)$

$\therefore f'(-x)(-1) = f'(x)$

$\Rightarrow \quad f'(-x) = -f'(x)$

$\therefore \quad f'(-0) = -f'(0)$

$\Rightarrow \quad f'(0) = -f'(0) \Rightarrow 2f'(0) = 0 \Rightarrow f'(0) = 0$

Q.55 The set of points where $x^2|x|$ is thrice differentiable is

(a) R **(b) $R - \{0, \pm 1\}$** **(c) R - { 0 }** **(d) none of these**

Ans: (c)

Sol: Let f(x) = $x^2|x|$ which could be expressed as

$$f(x) = \begin{cases} -x^3, & x < 0 \\ 0, & x = 0 \\ x^3, & x > 0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} -3x^2, & x < 0 \\ 0, & x = 0 \\ 3x^2, & x > 0 \end{cases}$$

So $f'(x)$ exists for all real x.

$$f''(x) = \begin{cases} -6x, & x < 0 \\ 0, & x = 0 \\ 6x & x > 0 \end{cases}$$

So, $f''(x)$ exists for all real x.

$$f'''(x)\begin{cases}-6, & x<0\\ 0, & x=0\\ 6, & x>0\end{cases}$$

However , $f'''(0)$ does not exist since $f'''(0^-)=-6$ and $f'''(0^+)=6$ which are not equal.
Thus, the set of points where f(x) is thrice differentiable at R - { 0 }.

Q.56 There is a point P(a, a, a) on the line passing through the origin and equally inclined with axes. The equation of plane perpendicular to OP and passing through P cuts the intercepts on axes the sum of whose reciprocal is

(a) $\frac{3}{2a}$ **(b)** $\frac{3a}{2}$ **(c)** a **(d)** $\frac{1}{a}$

Ans: (d)

Sol: The DCS of a line which is equally Inclined to its coordinate axes are $\left(\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}}\pm\frac{1}{\sqrt{3}}\right)$

Now, $OP=\sqrt{a^2+a^2+a^2}=\sqrt{3a}$

$\therefore$ DC's of OP are $<\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}>$

Equation of plane is

$$x+y+z=3a$$

$$\Rightarrow \frac{x}{3a}+\frac{y}{3a}+\frac{z}{3a}=1$$

$\therefore$ Intercept on axes are $3a$, $3a$ and $3a$ respectively.

Sum of their reciprocals $\frac{1}{3a}+\frac{1}{3a}+\frac{1}{3a}=\frac{1}{a}$.

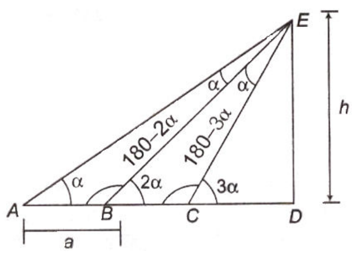
Q.57 The angle of elevation of the top of a TV tower from three points A, B and C in a straight line through the foot of the tower are α, 2α and 3α, respectively. If AB = a, then height of the tower is

(a) $a\tan\alpha$ **(b)** $a\sin\alpha$ **(c)** $a\sin 2\alpha$ **(d)** $a\sin 3\alpha$

Ans: (c)

Sol: In ΔABE ,

$$\angle BAE=\angle AEB \ [AB = BE]$$



In, ΔBCE Using sine rule

$$\frac{BE}{\sin(180^0 - 3\alpha)} = \frac{CE}{\sin 2\alpha}$$

$\Rightarrow$ $CE = \dfrac{a\sin 2\alpha}{\sin 3\alpha}$ ..(i) $\left[\begin{array}{l}\because \angle BAE = \angle BEA \\ \Rightarrow AB = BE = a\end{array}\right]$

Now, ΔDCE, $\sin 3\alpha = \dfrac{h}{CE}$

$$\sin 3\alpha = \frac{h}{a\sin 2\alpha / \sin 3\alpha}$$ [From eq. (i)]

$\Rightarrow$ $h = a\sin 2\alpha$

Q.58 Let R be set of real numbers. If **$f : R \to R$ is defined by $f(x) = e^x$, then f is**

(a) Surjective but not injective **(b) injective but not surjective**
(c) Bijective **(d) Neither surjective nor inejective**

Ans: (b)

Sol: Now, assume f is injective (i.e. one - one), since $x_1, x_2 \in R$ and $x_1 \neq x_2$

$\Rightarrow$ $e^{x_1} \neq e^{x_2}$

$\Rightarrow$ $f(x_1) \neq f(x_2)$

$\therefore$ It is true.

Again assume f is not surjective (onto), since $e^x > 0$ for all x and so negative real number cannot be the image of any real number.

For example, there is no real x such that $f(x) = -2$. Hence, f is injective but not surjective.

Q.59 A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance *a*, so that it slides a distance b down the wall making an angle β with the horizontal. Then, *a* is equal to

(a) $b \tan \frac{1}{2}(\alpha + \beta)$ **(b) $b \tan \frac{1}{2}(\alpha - \beta)$**

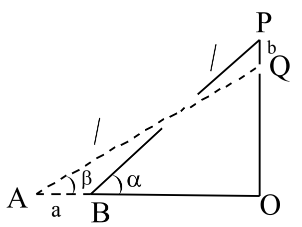
(c) $a \tan \frac{1}{2}(\alpha - \beta)$ **(d) None of these**

Ans: (a)

Sol: Let I be the length of the ladder.

Now, $a = OA - OB = (\cos\beta - \cos\alpha)\ell$

and $b = OP - OQ = (\sin\alpha -) - \sin\beta)\ell$



$$\therefore \quad \frac{a}{b} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$$

$$\Rightarrow \quad \frac{a}{b} = \frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}$$

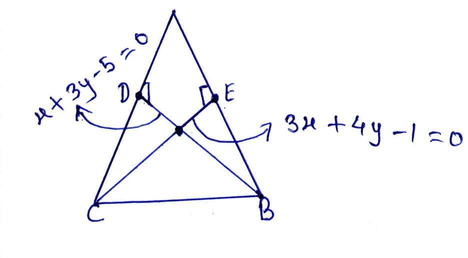
$$\Rightarrow \quad a = b\tan\left(\frac{\alpha+\beta}{2}\right)$$

Q.60 If the coordinates of the vertex A of a ΔABC are $(1, 2)$ and equation of the perpendicular bisectors of AB and AC are $3x+4y-1=0$ and $4x+3y-5=0$, then the area of ΔABC is

(a) 2 sq units (b) 3 sq units (c) 1 sq units (d) None of these

Ans: (d)

Sol:



As, eqn^n AB and CE are perpendicular

so, $m_1 m_2 = -1$

Slope line CE $= (m_1) = \frac{-3}{4}$

Slope AB(m_2)=

$m_1 m_2 = -1$

$m_2 = \frac{4}{3}$

Solving eqn^n of AB is,

$y + 2 = \frac{4}{3}(x-1)$

$3y + 6 = 4x - 4$

$4x - 3y = 10 \quad ....(1)$

AB and BD,

$4x - 3y = +10$

$4x + 3y = 5$

$8x = 15$

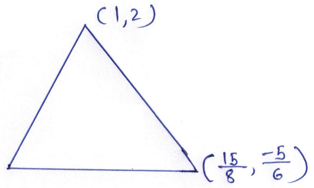
$x = \frac{15}{8}$,

$4\times\frac{15}{8}-3y=10$

$3y=\frac{15}{2}-10$

$y=\frac{-5}{6}$

$3y=\frac{-5}{2}$



eqn^n of BD,

$4x+3y=5$

$3y=-4x+5$

$y=\frac{-4}{3}x+\frac{5}{3}$

$m_1=\frac{-4}{3}.$

Slope of AC is,

$m_2=\frac{3}{4}$

eqn^n of AC is,

$y-2=\frac{3}{4}(x-1)$

$4y-8=3x-3$

$3x-4y=-5$ ____ (2)

$3x+4y=1$

$3x-4y=-5$

$6=-4$

$x=\frac{-2}{3}.$

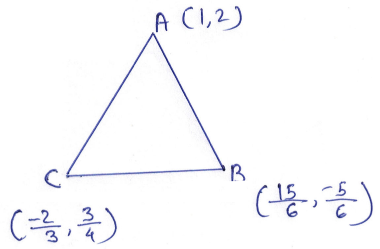
Putting in eqn^n (2)

$8\left(\frac{2}{5}\right)-4y=-5$

$-2-4y=-5$

$y=\frac{3}{4}$

$\left(\frac{-2}{3}, \frac{3}{4}\right)$



$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ \frac{15}{8} & \frac{-5}{6} & 2 \\ \frac{-2}{3} & \frac{3}{4} & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[\left(\frac{-5}{6} - 1\right) - 2\left(\frac{15}{8} + \frac{2}{3}\right) + \left(\frac{45}{32}\right) - \frac{10}{8}\right]$$

$$= \frac{1}{2}\left[\frac{-11}{6} - \frac{122}{24} + 0.85\right]$$

$$= \frac{1}{2}[-1.83 - 5.08 + 0.85]$$

$$= 3.03$$

Q.61 $\int \sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^3 x}}\, dx$ **is equal to**

(a) $\frac{2}{3}\sin^{-1}(\cos^{3/2} x) + C$ **(b)** $\frac{3}{2}\sin^{-1}(\cos^{3/2} x) + C$

(c) $\frac{2}{3}\cos^{-1}(\cos^{3/2} x) + C$ **(d) None of the above**

Ans: (c)

Sol: Let $I = \sqrt{\frac{\cos x(1 - \cos^2 x)}{1 - \cos^3 x}}\, dx$

$$= \int \frac{\sqrt{\cos x}\, \sin x}{1 - (\cos^{3/2} x)^2}\, dx$$

Put $\cos^{3/2} x = t$

$\Rightarrow \quad \frac{3}{2}\sqrt{\cos x}\,(-\sin x)dx = dt$

$\therefore \quad I = \frac{2}{3}\int \frac{-dt}{\sqrt{1-t^2}}$

$= \frac{2}{3}\cos^{-1}(t) + C$

$= \frac{2}{3}\cos^{-1}(\cos^{3/2} x) + C$

Q.62 If $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$ then sin x is equal to

(a) $\cot^2\left(\frac{\alpha}{2}\right)$ **(b) $\tan^2\left(\frac{\alpha}{2}\right)$** **(c) $\tan\alpha$** **(d) $\cot\left(\frac{\alpha}{2}\right)$**

Ans: (b)

Sol: Using formula

$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left[\frac{x-y}{1+xy}\right]$

Given that, $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

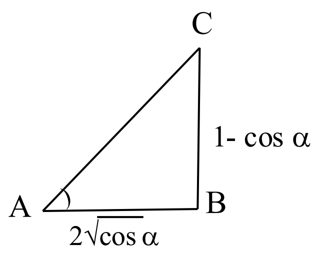
$\therefore \quad \tan^{-1}\left(\frac{1}{\sqrt{\cos\alpha}}\right) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$\Rightarrow \quad \tan^{-1}\left(\frac{\frac{1}{\sqrt{\cos\alpha}} - \sqrt{\cos\alpha}}{1 + \frac{1}{\sqrt{\cos\alpha}}\cdot\sqrt{\cos\alpha}}\right) = x$

$\Rightarrow \quad \tan^{-1}\left(\frac{1-\cos\alpha}{2\sqrt{\cos\alpha}}\right) = x$

$\Rightarrow \quad \tan x = \frac{1-\cos\alpha}{2\sqrt{\cos\alpha}}$

In ΔABC,



$AC^2 = AB^2 + CB^2$

$= (2\sqrt{\cos\alpha})^2 + (1-\cos\alpha)^2$

$= 4\cos\alpha + 1 + \cos^2\alpha - 2\cos\alpha$

$\Rightarrow \quad AC = 1 + \cos\alpha$

$\therefore \qquad \sin x = \dfrac{1-\cos\alpha}{1+\cos\alpha} = \dfrac{2\sin^2\dfrac{\alpha}{2}}{2\cos^2\dfrac{\alpha}{2}}$

$\therefore \qquad \sin x = \tan^2\dfrac{\alpha}{2}$

Q.63 If $x > 1$, $y > 1$, $z > 1$ are in GP, then $\dfrac{1}{1+\text{In}\,x}, \dfrac{1}{1+\text{In}\,y}, \dfrac{1}{1+\text{In}\,z}$ are in

(a) AP **(b) HP** **(c) GP** **(d) None of these**

Ans: (b)

Sol: Let the common ratio of the GP be r. then, $y = xr$ and $z = xr^2$

$\Rightarrow$ In y = In x + In r and In z = In x + 2 In r

Again, let A = 1 + In x, D = In r, then

$$\frac{1}{1+\text{In}\,x} = \frac{1}{A},$$

$$\frac{1}{1+\text{In}\,y} = \frac{1}{1+\text{In}\,x+\text{In r}} = \frac{1}{A+D} \text{ and } \frac{1}{1+\text{In}\,z} = \frac{1}{1+\text{In}\,x+2\text{In r}} = \frac{1}{A+2D}$$

Hence $\dfrac{1}{1+\text{In}\,x}, \dfrac{1}{1+\text{In}\,y}, \dfrac{1}{1+\text{In}\,z}$ are in HP.

Q.64 The value of the integral $\int_0^{\pi/2} \dfrac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}\,dx$ is

(a) $\dfrac{\pi}{4}$ **(b) $\dfrac{\pi}{2}$** **(c) π** **(d) None of these**

Ans: (a)

Sol: Let $\quad I = \int_0^{\pi/2} \dfrac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}\,dx \qquad$ ..... (i)

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\sqrt{\cot\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cot\left(\frac{\pi}{2}-x\right)}+\sqrt{\tan\left(\frac{\pi}{2}-x\right)}}\,dx$$

$\Rightarrow \quad I = \int_0^{\pi/2} \dfrac{\sqrt{\tan x}}{\sqrt{\cot x}+\sqrt{\tan x}}\,dx \qquad$ .... (ii)

On adding Eqs. (i) and (ii) we get

$$2I = \int_0^{\pi/2} \frac{\sqrt{\tan x}+\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}\,dx$$

$\Rightarrow \quad 2I = \int_0^{\pi/2} 1dx = [x]_0^{\pi/2}$

$\Rightarrow \quad 2I = \frac{\pi}{2} \quad \Rightarrow \quad I = \frac{\pi}{4}$

Q.65 The vector b which is collinear with the vector $a = 2\hat{i} + \hat{j} - \hat{k}$ and satisfies the condition $a \cdot b = 3$, is _______?

(a) $\frac{1}{2}a$ (b) a (c) 2a (d) None of these

Ans: (a)

Sol: Let,

$b = \lambda a = \lambda\ (2\hat{i} + \hat{j} - \hat{k})$

$\therefore \quad a \cdot b = 3 \Rightarrow a \cdot \lambda\ (a) = 3$

$\Rightarrow \quad \lambda\left(|a|^2\right) = 3$

$\Rightarrow \quad \lambda(2^2 + 1^2 + 1^2) = 3$

$\Rightarrow \quad \lambda = \frac{3}{6} = \frac{1}{2}$

Hence $b = \frac{1}{2}a$

Q.66 The general solution of the differential equation $\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$ is

(a) $\log\tan\left(\frac{y}{2}\right) = C - 2\sin x$ (b) $\log\tan\left(\frac{y}{4}\right) = C - 2\sin\left(\frac{x}{2}\right)$

(c) $\log\tan\left(\frac{y}{2} + \frac{\pi}{4}\right) = C - 2\sin x$ (d) $\log\tan\left(\frac{y}{4} + \frac{\pi}{4}\right) = C - 2\sin\left(\frac{x}{2}\right)$

Ans: (b)

Sol: Given differential equation is

$\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$

$\Rightarrow \quad \frac{dy}{dx} = \sin\left(\frac{x-y}{2}\right) - \sin\left(\frac{x+y}{2}\right)$

$\Rightarrow \quad \frac{dy}{dx} = -2\sin\left(\frac{y}{2}\right)\cos\left(\frac{x}{2}\right)$

$\Rightarrow \quad \operatorname{cosec}\left(\frac{y}{2}\right)dy = -2\cos\left(\frac{x}{2}\right)dx$

On integrating both sides, we get

$$\frac{\log\tan\left(\frac{y}{4}\right)}{\frac{1}{2}} - \frac{-2\sin\left(\frac{x}{2}\right)}{\frac{1}{2}} + C$$

$\Rightarrow \quad \log\left(\tan\frac{y}{4}\right) = C - 2\sin\left(\frac{x}{2}\right)$

Q.67 If p : it rains today, q : I go to school, r : I shall meet any friends and s : I shall go for a movie, then which of the following is the proportion, 'if it does not rain or if it I do not go to school, then I shall meet my friend and go for a movie' ?

(a) $(\sim p \wedge \sim q) \Rightarrow (r \wedge s)$ **(b) $\sim(p \wedge q) \Rightarrow (r \wedge s)$**

(c) $\sim(p \vee q) \Rightarrow (r \vee s)$ **(d) None of these**

Ans: (b)

Sol: $\therefore$ Correct result is $(\sim p \vee \sim p) \Rightarrow (r \wedge s)$ or $\sim(p \wedge q) \Rightarrow r \wedge s$.

Q.68 For $a \in [\pi, 2\pi]$ n $\in$ I, the critical points of

$f(x) = \frac{1}{3}\sin a \tan^3 x + (\sin a - 1)\tan x + \sqrt{\frac{a-2}{8-a}}$ are

(a) $x = nx$ **(b) $x = 2n\pi$** **(c) $x = (2n+1)\pi$** **(d) None of these**

Ans: (d)

Sol: Given that

$$f(x) = \frac{1}{3}\sin a \tan^3 x + (\sin a - 1)\tan x + \sqrt{\frac{a-2}{8-a}}$$

On differentiating w.r.t.x. we get

$$f'(x) = \sin a \tan^2 x \sec^2 x + (\sin a - 1)\sec^2 x$$

$$= (\sin a \tan^2 x + \sin a - 1)\sec^2 x$$

At critical point, we must have

$$f'(x) = 0$$

$\Rightarrow \quad \sin a \tan^2 x + \sin a - 1 = 0 \qquad [\because \sec^2 x \neq 0 \text{ for any } x \in R]$

$\Rightarrow \quad \tan^2 x = \frac{1-\sin a}{\sin a}$

Since, $a \in [\pi, 2\pi]$, therefore $\frac{1-\sin a}{\sin a} < 0$, So the eqution $\tan^2 x = \frac{1-\sin a}{\sin a}$ does not have solution in R. Hence, $f(x)$ has no ciritical points.

Q.69 If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3 then

(a) $a < 2$ **(b) $2 \le a \le 3$** **(c) $3 < a \le 4$** **(d) $a > 4$**

Ans: (a)

Sol: Let $f(x) = x^2 - 2ax + a^2 + a - 3 = 0$

Since $f(x)$ has real roots both less than 3.

Therefore,

$D > 0,\ f(3) > 0$ and $-\dfrac{B}{2A} < 3$

$\Rightarrow \quad 4a^2 - 4(a^2 + a - 3) > 0,\ a^2 - 5a + 6 > 0$

and $\quad \dfrac{2a}{2(1)} < 3$

$\Rightarrow \quad a < 3,\ (a-2)(a-3) > 0$

and $\quad a < 3$

$\Rightarrow \quad a < 3\ \&\ a < 2$ or $a > 3\ \&\ a < 3$

$\Rightarrow \quad a < 2$

Q.70 The equation of the common tangent touching the circle $(x-3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the X-axis is

(a) $\sqrt{3}\ y = 3x + 1$ **(b)** $\sqrt{3}\ y = -(x+3)$

(c) $\sqrt{3}\ y = x + 3$ **(d)** $\sqrt{3}\ y = -(3x+1)$

Ans: (c)

Sol: Any tangent to $y^2 = 4x$ is of the form $y = mx + \dfrac{1}{m}$, and his touches the circle

$$(x-3)^2 + y^2 = 9, \text{ if } \left|\frac{m(3) + \frac{1}{m} - 0}{\sqrt{m^2+1}}\right| = 3.$$

Since, centre of the circl is (3, 0) and radius is 3.

$\therefore \quad \dfrac{3m^2+1}{m} = \pm\, 3\sqrt{m^2+1}$

$\Rightarrow \quad 3m^2 + 1 = \pm\, 3m\sqrt{m^2+1}$

$\Rightarrow \quad (3m^2+1)^2 = (3m\sqrt{m^2+1})^2$

$\Rightarrow \quad 9m^4 + 1 + 6m^2 = 9m^2(m^2+1)$

$\Rightarrow \quad 9m^4 + 1 + 6m^2 = 9m^4 + 9m^2$

$\Rightarrow \quad 3m^2 = 1 \Rightarrow m = \pm\dfrac{1}{\sqrt{3}}$

If the tangent touches the parabola and circle above the X-axis, slope m should be positive

$\therefore \quad m = \dfrac{1}{\sqrt{3}}$ and the equation is $y = \dfrac{1}{\sqrt{3}}x + \sqrt{3}$ i.e. $\sqrt{3y} = x + 3$.

Q.71 If $3^{49}(x + iy) = \left(\dfrac{3}{2} + \dfrac{\sqrt{3}}{2}i\right)^{100}$ and $x = ky$, then k is _______?

We know that, $i = \sqrt{-1}$

Sol: Since, $3^{49}(x+iy) = \left(\frac{3}{2}+\frac{\sqrt{3}}{2}i\right)^{100} = \left[i\sqrt{3}\left(\frac{1-i\sqrt{3}}{2}\right)\right]^{100}$

$\Rightarrow \quad 3^{49}(x+iy) = i^{100}\cdot 3^{50}(-\omega)^{100}$

$\Rightarrow \quad 3^{49}(x+iy) = 3^{50}\cdot\omega = 3^{50}\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)$

$\Rightarrow \quad x+iy = -\frac{3}{2}+i\frac{3\sqrt{3}}{2}$

$\Rightarrow \quad x = -\frac{3}{2},\ y = \frac{3\sqrt{3}}{2}$

$\Rightarrow \quad x = -\frac{1}{\sqrt{3}}y,$

But $\quad x = ky$

$\Rightarrow \quad k = -\frac{1}{\sqrt{3}}$

Q.72 Let y be an implicit function of x defined by $x^{2x} - 2x^{x}\cot y - 1 = 0$ then y (1) equals?

Sol: $x^{2x} - 2x^{x}\cot y - 1 = 0$

Now $x = 1$

$1 - 2\cot y - 1 = 0 \Rightarrow \cot y = 0$

$\Rightarrow y = \frac{\pi}{2}$

Now differentiating (i) w.r.t. "x", we get

$2x^{2x}(1+\log x) - 2$

$\left(x^{x}(-\operatorname{cosec}^2 y)\frac{dy}{dx} + \cot y\, x^{x}(1+\log x)\right) = 0$

Now at $\left(1, \frac{\pi}{2}\right)$

$2(1+\log 1) - 2\left(1(-1)\left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)}^{+0}\right) = 0$

$\Rightarrow 2 + 2\left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)}^{=0}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)} = -1$

Q.73 A five digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways this can be done is _______?

Sol: Since, a five digit number is formed by using the digits, 0, 1, 2, 3, 4 and 5, divisible by 3 i.e. only possible when sum of digit is multiple of three which gives two cases.

Cases I using digit 0, 1, 2, 4, 5 the number of ways $= 4 \times 4 \times 3 \times 2 \times 1 = 96$.

Case II Using digit 1, 2, 3, 4, 5 the number of ways $= 5 \times 4 \times 3 \times 2 \times 1 = 120$.

$\therefore$ Total number formed $= 120 + 96 = 216$

Q.74 The value of $\lim_{x\to 0} \frac{\int_0^{x^2} \cos^2 t\,dt}{x \sin x}$ is _______?

Sol: $\lim_{x\to 0} \frac{\int_0^{x^2} \cos^2 t\,dt}{x \sin x}$

Using L' Hospital rule,

$$= \lim_{x\to 0} \frac{\cos^2(x^2)\cdot 2x - 0}{x\cos x + \sin x}$$

$$= \lim_{x\to 0} \frac{2\cos^2(x^2)}{\cos x + \frac{\sin x}{x}}$$

$$= \frac{\lim_{x\to 0} 2\cos^2(x^2)}{\lim_{x\to 0}\left[\cos x + \frac{\sin x}{x}\right]} = \frac{2}{2} = 1$$

Q.75 Let L={x : x $\in$ club cards in a pack of cards}
M={y : y appears on the face of a die}
The number of elements in the cartesian product of L and M is _______?

Sol: L={A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K}
M={1, 2, 3, 4, 5, 6}
$n(L \times M) = 13 \times 6 = 78$

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