A Division of Aggarwal Educare
Aggarwal CLASSES ™

JEE (MAIN)

TEST PAPER

SUBJECT : PHYSICS, CHEMISTRY, MATHEMATICS | *TEST CODE : TEST PAPER-2*

ANSWER PAPER

TIME : 3 HRS | *MARKS : 300*

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark(-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or “un- darken” a darkened bubble.

Part A - PHYSICS

Q.1 Electric conduction in a semiconductor takes place due to

(a) Electrons only (b) Holes only

(c) Both electrons and holes (d) Neither electrons nor holes

Ans: (c)

Sol: In, semiconductors, the charge carries are electrons and holes, so conduction is due to both.

Q.2 Two bodies at different temperatures are mixed in a calorimeter. Which of the following quantities remain conserved ?

(a) Sum of the temperature of the two bodies

(b) Total heat of the two bodies

(c) Total internal energy of the two bodies

(d) Internal energy of each body

Ans: (c)

Sol: Due to temperature difference, heat trnasfer is taking place from one body to another and no external source is supplying energy to the bodies. Thus, total internal energy of the two bodies remains conserved although indivdually it changes.

Q.3 The dimensional formula for torque is [ML^2T^{-2}], same as that of work or energy, its proper SI unit is

(a) Must be joule only (b) Either N-m or joule

(c) N-m (d) None of the above

Ans: (c)

Sol: As, $\tau = r \times F = rF\sin\theta$

Where, r is angle between r and F.

$\Rightarrow$ Unit of τ = N – m

Q.4 An alternating current having peak value 14 A is used to heat a metal wire. To produce the same heating effect, a constant current *i* can be used where *i* is

(a) 14 A (b) about 20 A (c) 7 A (d) about 10 A

Ans: (d)

Sol: As, $I_{DC} = I_{rms} = \frac{I_{peak}}{\sqrt{2}}$ ($\because$ constant current = direct current, DC)

$= \frac{14}{\sqrt{2}} = 9.9$ A

Q.5 The statement " current is defined as rate of flow of electrons through any cross-section" is

(a)Always true (b) Always false

(c) True in some cases (d) None of these

Ans: (c)

Sol: The given statement is true only for conductors where free electrons are the charge carriers. The exact definition for current is " The rate of flow of charge".

Q.6 Which of the following has more heat ?

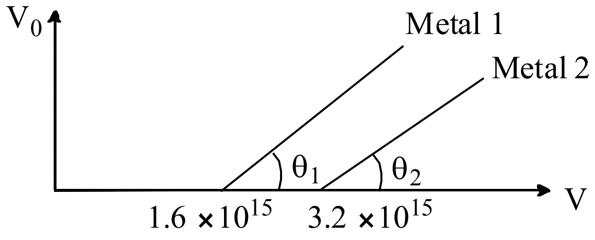
(a) Sun (b) A hot cup of tea

(c) A red hot iron (d) Question is irrevalant

Ans: (d)

Sol: Heat is defined as energy in transit due to temperature difference of two bodies. In all of the given cases, the objects have the energy but no heat.

Q.7 The graph between stopping potential versus frequency is given for two different metals. Then choose the most appropriate statement.



(a) $v_{01} = v_{02}$ **and** $\theta_1 = \theta_2$ **(b)** $v_{01} > v_{02}$ **and** $\theta_1 > \theta_2$

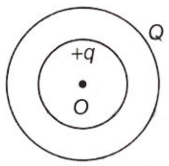
(c) $v_{01} < v_{02}$ **and** $\theta_1 < \theta_2$ **(d)** $v_{01} < v_{02}$ **and** $\theta_1 = \theta_2$

Ans: (d)

Sol: At threshold frequency, stopping potential $v_0 = 0$. More distance on X-axis will show more value of threshold frequency. From graph, it is clear that $v_{01} < v_{02}$ and $\tan\theta$ i.e slope of curve between v_0 versus v represents hv/e. From the equation, $v_0 = \frac{hv}{e} - \frac{\phi}{e}$ $\therefore$

$\tan\theta_1 = \tan\theta_2 \Rightarrow \theta_1 = \theta_2$

Q.8 A charged solid conductor having a cavity is a shown in figure. If a charg +q is placed asymmetrically within the cavity, then charge induced on outer surface of conductor would be



(a) -q **(b) +q** **(c) q - Q** **(d) Q - q**

Ans: (b)

Sol: Due to induction, -q charge induces on inner surface of cavity and from charge conservation for conductor, charge q + Q will appear on outer surface after redistribution of charge. Hence, due to induction +q charge will be induced on outer surface of conductor.

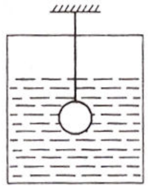
Q.9 A body of mass *m* is moving in a circle of radius *r* with a constant speed v. The force on the body is $\frac{mv^2}{r}$ **and is directed towards the centre. What is the work done by this force in moving the body over half the circumference of the circle ?**

(a) $\frac{mv^2}{r} \times \pi r$ **(b) Zero** **(c)** $\frac{mv^2}{r^2}$ **(d)** $\frac{\pi r^2}{mv^2}$

Ans: (b)

Sol: Work done by centripetal force is always zero.
[$\because$ force and displacements are perpendicular to each other].

Q.10 A solid sphere of steel has been dipped in a liquid. If the temperature is increased, then the force of buoyancy will



(a) Increase **(b) Decrease**
(c) May increase or Decrease **(d) Will remain constant**

Ans: (b)

Sol: Buoyant force $= F_B = V_0 \times \rho_0 \times g$

$$V_{st} = V_0\,(1+3\alpha_s\,\Delta T)$$

$$\rho = \frac{\rho_0}{(1+\gamma_l \Delta T)}$$

Where,

V_0, V_{st} = volume of steel sphere at temperature T and $T+\Delta T$, respectively

$\rho =$ density of liquid, at T and $T+\Delta T$,respectively

$\alpha_s =$ coefficient of linear exapnsion of steel

$\gamma_l =$ coefficient of cubical expansion of liquid

If temperature increases by ΔT then

$$F_B' = V_0\,\rho_0 g \times \left[\frac{1+3\alpha_s\,\Delta T}{1+\gamma_l \Delta T}\right]$$

Since $\gamma_l > 3\alpha_s$

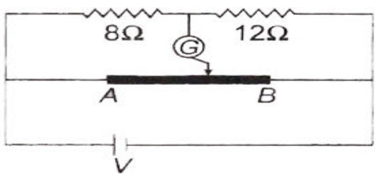
$\therefore$ $F_B' < F_B$

Q.11 The muscles of a normal eye are least strained when the eye is focused on an object
(a) Far away from the eye **(b) Very close to the eye**
(c) At about 25 cm from the eye **(d) At about 1 m from the eye.**

Ans: (a)

Sol: As we know that the shape (curvature) or the focal length of the eye lens can be modified by the ciliary muscles. For example, when the muscles are relaxed, the focal length is about 2.5 cm (diameter of eye) and this means that objects at infinity are in sharp focus on the retina.

Q.12 A potentiometer wire AB shown in figure is 40 cm long. Where should the free end of the galvanometer be connected on AB so that the galvanometer may show zero deflection ?



(a) 16 cm from B **(b) 20 cm from B** **(c) 16 cm from A** **(d) 20 cm from A**

Ans: (c)

Sol: If the upword part of the circuit form the balanced Wheastone bridge, only when no current will flow through the galvanometer. Let free end of galvanometer be connected to a point distant x from A, then

$$\frac{8}{12} = \frac{x}{(40-x)}$$

$\Rightarrow$ $80 - 2x = 3x$

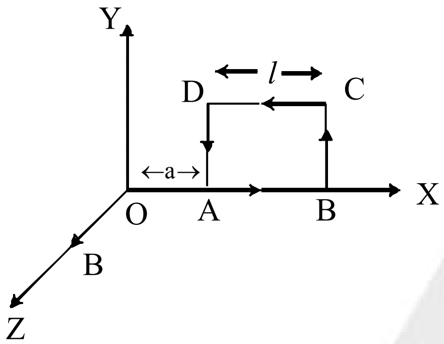
$\Rightarrow$ $x = 16$ cm

Q.13 The magnetic field existing in a region is given by $B = B0\,[1 + x/l]\,\hat{k}$, **A square loop of edge l and carriying a current, I is placed with its edges parallel to the x - y axis. Find the magnitude of the net magnetic force experienced by the loop.**

(a) $3lB_0I$ **(b) $2lB_0I$** **(c) lB_0I** **(d) None of the above**

Ans: (c)

Sol: The force experienced by the wire AB and CD are equal and opposite.



$$\text{Force on AD} = -B_0\left[1+\frac{a}{l}\right]/\,l\,\hat{i}$$

$$\text{Force on BC} = B_0\left[1+\frac{a+l}{l}\right]/\,l\,\hat{i}$$

$\therefore$ Net force $= IB_0l$

Q.14 **A body cool from 100°C to 90°C in 20 min, it will cool down from 110°C to 100°C in [assume same surroundings]**

(a) 20 min **(b) Less than 20 min**

(c) More than 20 min **(d) 30 min**

Ans: (b)

Sol: Hotter body emits radiation at a faster rate for the same temperature difference as compared to colder body [$\because$ Newton's law of cooling, $\frac{-d\theta}{dt} \propto T$]

Q.15 ***N* moles of an ideal diatomic gas are in a cylinder at temperature *T*. If we supply some heat to it, then *N*/3 moles of gas dissociates into atoms while temperature remains constant, Heat supplied to the gas is**

(a) *NRT*/6 **(b) 5 *NRT*/2** **(c) 5.6 *NRT*** **(d) 8 *NRT*/3**

Ans: (a)

Sol: According to first law of theromodynamics

Heat supplied = Change in internal energy

$= \Delta U = U_f - U_i$

$$U_f = \left(N - \frac{N}{3}\right) \times \frac{5RT}{2} + \frac{N}{3} \times 2 \times \frac{3RT}{2}$$

$$= \frac{5NRT}{3} + NRT = \frac{8}{3}NRT$$

$$U_i = 5 \times \frac{NRT}{2}$$

$$\therefore \quad \Delta Q = \frac{NRT}{6} \qquad [\because \Delta Q = \Delta V]$$

Q.16 A point source of light is taken away from the experimental set up of photoelectric effect, then which is the most appropriate statement ?
(a) Saturation photo current remains same, while stopping potential increases
(b) Saturation photo current and stopping potential both decreases
(c) Saturation photo current decreases while stopping potential remains same
(d) Saturation photo current decreases and stopping potential increases.

Ans: (c)

Sol: As the source of light is taken away, the intensity of light at the location of experiment setup decreases.
Photo current $\propto$ Intensity
So, photo current decreases.

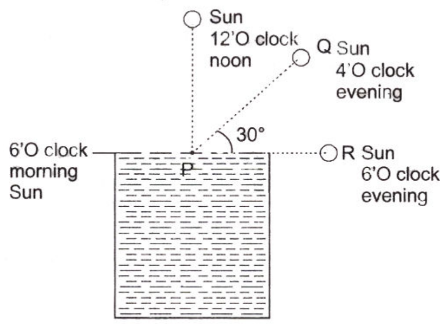
Now, $V_0 = \dfrac{hc}{\lambda_0 e} - \dfrac{\phi}{e}$ which is independent of intensity.

Q.17 A man in an empty swimming pool has a telescope focused at 4'O clock sun. When the swimming pool is filled with water, the man (now inside the water with his telescope undisturbed) observes the setting sun. Find the refrective index of water, if sun rises and sets at 6'O clock.

(a) $\frac{4}{3}$ **(b)** $\frac{2}{\sqrt{3}}$ **(c)** $\frac{8}{5}$ **(d)** $\frac{2}{5}$

Ans: (b)

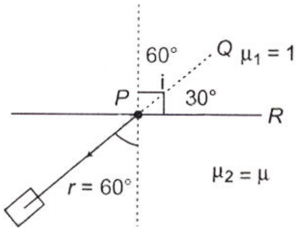
Sol: When no water is there in the swimming pool, the telescope is along the line PQ. When water has been filled into the pool, the person is able to see the setting sun i.e., refracted ray will be along QP while incident ray is along RP, as shown in figure, That is from the object (sun) to the telescope.



$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \qquad \text{[from Snell's law]}$$

Let μ be the refractive index of water, then by Snell's law

$$\left[\because\ 6h = 90^0 \Rightarrow 2h = \frac{90^0 \times 2}{6} = 30^0\right]$$



$$\frac{\sin 90^o}{\sin 60^o} = \frac{\mu}{1}$$

$$\Rightarrow \qquad \mu = \frac{1}{\sin 60^0} = \frac{2}{\sqrt{3}}$$

Q.18 A pistol fires a 3g bullet with a speed of 400 m/s. The pistol barrel is 13 cm long. How much energy is given to the bullet ? Also, calculate the average force acted on the bullet while it was moving down the barrel.

(a) 140 J, 1846 N **(b) 240 J, 184.6 N**

(c) 240 J, 1846 N **(d) 240 J, 1746 N**

Ans: (c)

Sol: The kinetic energy of the bullet on leaving the barrel is

$$K_f = \frac{1}{2}mv^2 = \frac{1}{2}(0.003)(400)^2 = 240\text{ J}$$

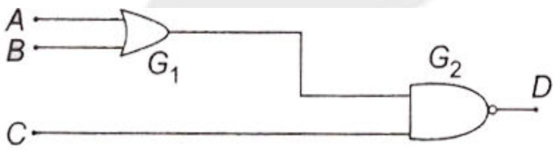
The work done on the bullet is equal to the change in its kinetic energy

$W = F \times X = K_f - K_i$ where F is the average force exerted on the bullet, Thus

$$F = \frac{K_f - K_i}{x} = \frac{240 - 0}{0.13} = 1846\text{ N}$$

Initial Kinetic energy is zero, since the bullet was at rest initially.

Q.19 For the given combination of gates, if the logic states of inputs A, B, C are as follows A = B = C = 0 and A = B = 1, C = 0, then the logic states of output D are



(a) 0, 0 **(b) 0, 1** **(c) 1, 0** **(d) 1, 1**

Ans: (d)

Sol: The output D for the given combination.

$$D = \overline{(A+B)\cdot C} = \overline{(A+B)} + \overline{C}$$

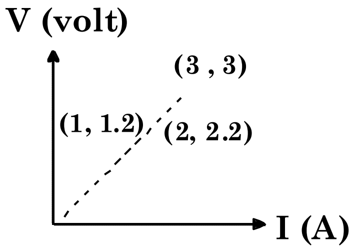
IF $A = B = C = 0$, then

$$D = \overline{(0+0)} + \overline{0} = \overline{0} + \overline{0} = 1 + 1 = 1$$

if $A = B = 1, C = 0$, then

$$D = \overline{1+1} + \overline{0} = \overline{1} + \overline{0} = 0 + 1 = 1$$

Q.20 In the measurement of resistance of a wire using Ohm's law, the plot between *V* and *I* is drawn as shown.



The resistance of the wire is

(a) 0.833Ω **(b) 0.9Ω**

(c) 1Ω **(d) None of these**

Ans: (c)

Sol: We, know that, V-I curve for a linear device is a straight line passing through origin. Due to some errors/ carelessness on the part of experimentar, all points may not come on the same line. In this situation, we draw the most appropriate curve.

From the diagram, $R = \frac{3}{3} = 1\Omega$.

Q.21 The equation of motion for a mass at the end of particular spring is $y = 0.30 \cos 0.50\, t$ metre. Find the displacement, velocity and acceleration of the mass at $t = 0$.

Sol: The speed

$$v = \frac{dy}{dt} = -0.15 \sin(0.50t)\, m/s$$

$$a = \frac{dv}{dt} = -0.075 \cos(0.50t)\, m/s^2$$

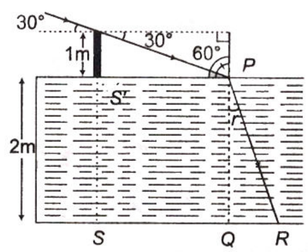
∴ $y = 0.30$ m, $v = 0$

and $a = -0.075\, m/s^2$.

Q.22 A pole standing in a pond stands 1m above the water surface, the pond is 2m deep. What is the length of the shadow thrown by the pole on the bottom of the pond, if the sun is 30^0 over the horizon ? [Refractive index for water is 4/3]

Sol: Shadow on the bottom of pond is

$$SR = SQ + QR = S'P + QR$$



From the figure, $S'P = \sqrt{3}\, m, \left(\frac{\sin 60^0}{\sin r}\right) = \frac{4}{3}$

$\Rightarrow \quad \sin r = \frac{3\sqrt{3}}{8} \qquad [\because i = 60^0]$

$QR = 2\tan r = 6\left(\sqrt{\frac{3}{37}}\right) \approx 1.71 \text{ m} \qquad$ [from ΔPQR]

$\therefore \quad SR = 3.44\,\text{m} \approx 3.5 \text{ m}$

Q.23 The electric field associated with a monochromatic beam becomes zero 2.4×10^{15} times per second. Find the maximum KE of the photoelectrons when this light falls on a metal surface whose work function is 2 eV.

Sol: The frequency of light wave would be same as that of electric field. The electric field become zero 2.4×10^{15} times per second, in one cycle it get zero twice, so frequency becomes

$\frac{2.4 \times 10^{15}}{2} = 1.2 \times 10^{15}$ Hz.

so, $\quad K_{max} = h\upsilon - \phi \qquad$ [Einstein's equation]

$= [4.96 - 2]\,\text{eV} = 2.96\,\text{eV}$

Q.24 The average kinetic energy of a gas molecule at 27°C is 6.21×10^{-21} J. Its average kinetic energy at 227°C will be

Sol: As, $\quad E \propto T$

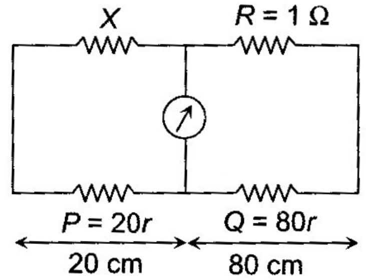
$\Rightarrow \quad \frac{E_1}{E_2} = \frac{T_1}{T_2}$

$\Rightarrow \quad \frac{6.21 \times 10^{-21}}{E_2} = \frac{(273+27)}{(273+227)} = \frac{300}{500}$

$\Rightarrow \quad E_2 = 10.35 \times 10^{-21}\,\text{J}$

Q.25 In meter bridge the balancing length from left and when standard resistance of 1 Ω is in right gap is found to be 20 cm. The value of unknown resistance is

Sol: The condition of Wheatstone bridge gives $\frac{X}{R} = \frac{20r}{80r}$, r resistance of wire per cm, X unknown resistance.



$\therefore X = \frac{20}{80} \times R = \frac{1}{4} \times 1 = 0.25\Omega$

Part - B - CHEMISTRY

Q.26 $\mathbf{RCONH_2 + 4\ NaOH + Br_2 \longrightarrow RNH_2 + 2NaBr + Na_2CO_3 + 2H_2O}$

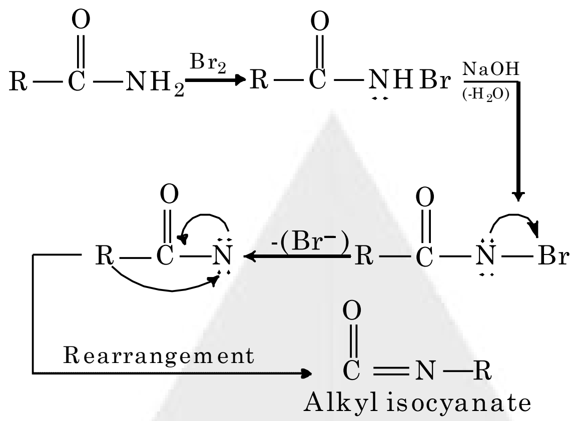
Reaction is said

(a) Hofmann-bromamide reaction **(b) Schmidt reaction**

(c) Curtius reaction **(d) Beckmann reaction**

Ans: (a)

Sol: Reaction is Hofmann-bromamide reaction. The mechanism is as



In all cases, alkyl isocyanates are converted into amines by reaction with alkali.

$$O{=}C{=}N{-}R \xrightarrow[H_2O]{OH^-} RNH_2 + CO_3^{2-}$$

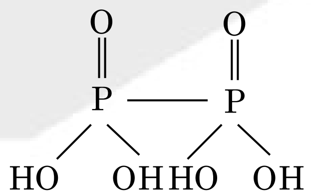
Q.27 **Which of the following does not contain P–O–P bond ?**

(a) Isohypophosphoric acid **(b) Diphosphorous acid**

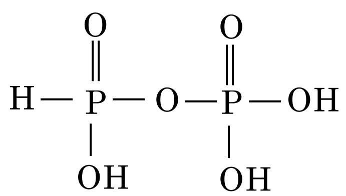
(c) Diphosphoric acid **(d) Hypophosphoric acid**

Ans: (d)

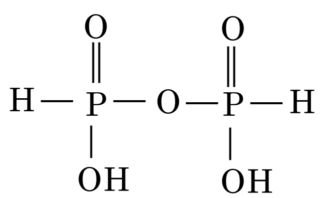
Sol: Hypophosphoric acid ($H_4P_2O_6$)



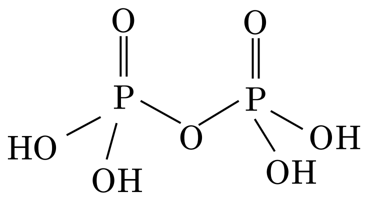
Isohyphosphoric acid ($H_4P_2O_6$)



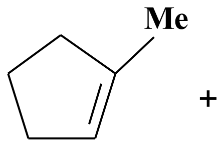
Diphosphorous acid (pyrophosphorous acid, $H_4P_2O_5$)



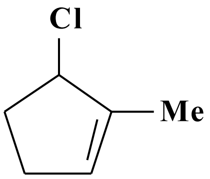
Diphosphoric acid (pyrophoshoric acid, $H_4P_2O_7$)

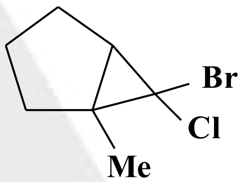


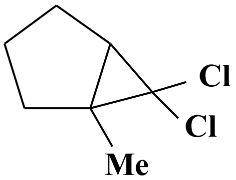
Hence, hypophosphoric acid contain $P-P$ bond instead of $P-O-P$ bond.

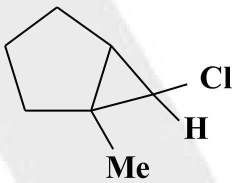
Q.28 

+ $CHClBr_2$ $\xrightarrow{Me_3COK}$ Product Predominant product is

(a) 

(b) 

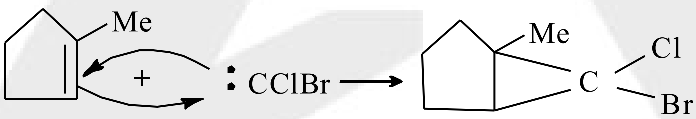
(c) 

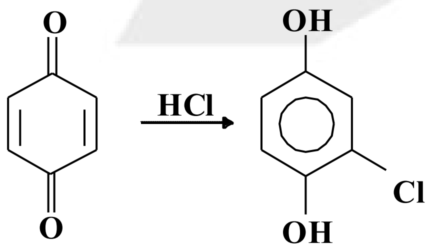
(d) 

Ans: (b)

Sol: $CHClBr_2 + Me_3COK \longrightarrow \overset{\ominus}{C}ClBr_2 \longrightarrow :CClBr$

Chlorobromocarbene (CClBr) undergoes addition.

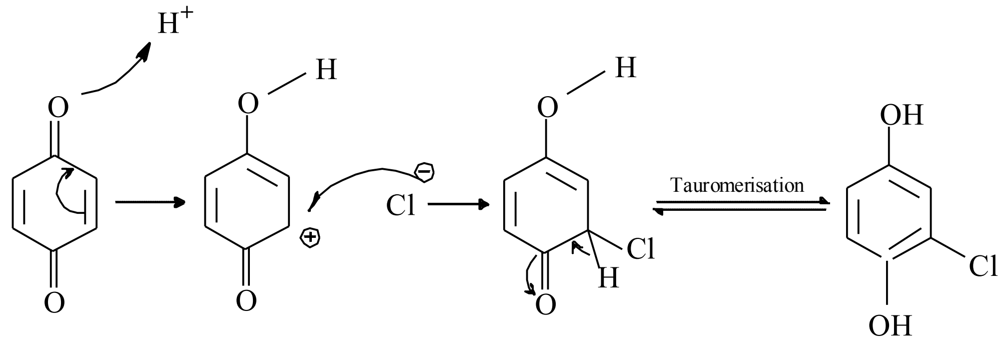


Q.29 

is an example of

(a) 1, 2 addition of HCl followed by tautomerism
(b) 1, 2 addition followed by reaction
(c) 1, 4 addition followed by tautomerism
(d) 1, 4 addition followed by oxidation

Ans: (c)
Sol:



The above reaction is an example of 1, 4 addition followed by tautomerism.

Q.30 At 20°C and 1.00 atm partial pressure of hydrogen, 18 mL of hydrogen, measured at STP, dissolves in 1L of water. If water at 20°C is exposed to a gaseous mixture having a total pressure of 1400 Torr (excluding the vapour pressure of water) and containing 68.5% H_2 by volume, find the volume of H_2, measured at STP, which will dissolve in 1L of water

(a) 18 mL **(b) 12 mL** **(c) 23 mL** **(d) 121 mL**

Ans: (c)

Sol: $p(H_2) = (1400 \text{ torr})\ (0.685)$

$= 959 \text{ torr} \equiv 959/760 \text{ atm} = 1.26 \text{ atm}$

According to Henry's law,
Amount of gas absorbed is directly proportional to pressure.

Hence, $\frac{V}{18 \text{ mL}} = \frac{1.26 \text{ atm}}{1 \text{ atm}}$

$V = 23 \text{mL}$

Q.31 Which of the following reaction takes place at the cathode in the electrolytic cell used for the cell extraction of aluminium from alumina ?

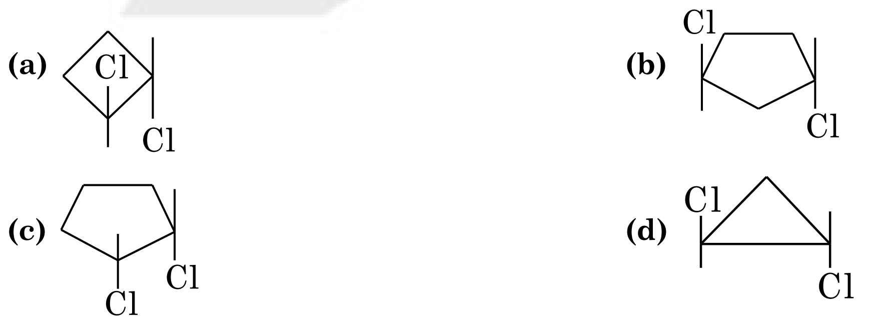
(a) $12F^- \rightarrow 12F + 12e^-$ **(b) $Al^{3+} + 3e^- \rightarrow Al$**

(c) $2C(s) + O_2 \rightarrow 2CO(g)$ **(d) $2Al_2O_3 + 12F \rightarrow 4Al^{3+} + 3O_2 + 12F^-$**

Ans: (b)

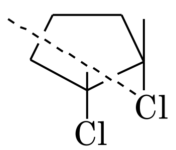
Sol: At cathode, reduction takes place i.e. $Al^{3+} + 3e^- \rightarrow Al$

Q.32 Which of the following molecule is optically inactive ?



Ans: (c)

Sol:



Plane of symmetry is present there, so optically inactive.

Q.33 When concentrated solution of KCl is shaken with wood charcoal, concentration decreases . This is

(a) Chemisortion **(b) Positive adosorption**
(c) Negative adsorption **(d) Occlusion**

Ans: (c)

Sol: Wood charoal adsorbs KCl solute, thus concentration of KCl decreases in solution and increases over adsorbent, wood charcoal contrary, if concentration increase in solution due to adsorbent is negative adsorption.

Q.34 The addition of sodium acetate in the solution of acetic acid

(a) Produces anhydride **(b) Increases acetate concetration**
(c) Increases undissociated acid (d) Increases K_a **(CH_3COOH)**

Ans: (c)

Sol: An acid buffer solution consists of a solution of a weak acid and its salt with strong base ($CH_3COOH + CH_3COONa$).

$$CH_3COOH \rightleftharpoons CH_3COO^- + H^+ \quad \text{(weak acid)}$$

$$CH_3COONa \rightleftharpoons CH_3COO^- + Na^+ \quad \text{(strong salt)}$$

Sodium acetate, being salt, ionises completely to form CH_3COO^- and Na^+ ions. On the other hand, acetic acid being a weak acid ionises very less. Moreover , its further ionisation suppressed by the acetate ions from sodium acetate (common ion effect).

Thus, addition of sodium acetate in the solution of acetic acid increases undissociated acid due to which pH of the solution remains unchanged.

Q.35 Mark out the most nucleophilic species at aliphatic trigonal carbon during substitution

(a) H_2O **(b) NH_3** **(c) H_2S** **(d) H_2Se**

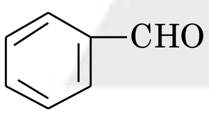
Ans: (b)

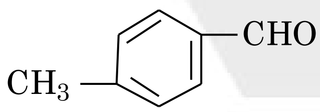
Sol: Aliphatic trigonal carbon is harder one, that's why harder base is better nucleophile.

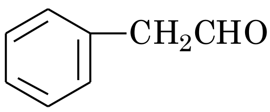
Q.36 Which of the following does not undergo benzoin condensation ?

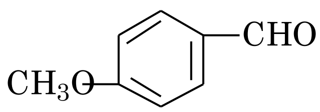
(a) Benzene carbaldehyde **(b) p-toluene carbaldehyde**
(c) Phenylethanal **(d) 4-methoxybenzaldehyde**

Ans: (c)

Sol: (a) 

(b) 

(c) 

(b) 

Compound (c) has α-H, i.e., it cannot undergo benzoin condensation.

Q.37 Which of the following combination of solute would result in the formation of a buffer solution ?

(a) HCl + NaCl
(b) $HCl + HC_2H_3O_2$
(c) $NaOH + HC_2H_3O_2$ (1 : 1 ratio) respectively
(d) NH_3 + HCl (2 : 1 ratio) respectively.

Ans: (d)

Sol: $H^+Cl^- + NH_3 \longrightarrow NH_4^+ + Cl^+$

NH_3 remains in excess.

Q.38 A compound X on heating gives a colourless gas. The residue is dissolved in water to obtain Y. Excess CO_2 is passed through aqueous solution of Y when Z is formed, Z on gentle heating gives back X. The compound X is

(a) $NaHCO_3$ **(b) Na_2CO_3**

(c) $Ca(HCO_3)_2$ **(d) $CaCO_3$**

Ans: (d)

Sol: $\underset{(X)}{CaCO_3} \xrightarrow{Heat} \underset{Residue}{CaO} + \underset{Colourless\ gas}{CO_2} \uparrow$

$$\underset{Residue}{CaO} + H_2O \longrightarrow \underset{(Y)}{Ca(OH)_2}$$

$$\underset{(y)}{Ca(OH)_2} + \underset{Excess}{2CO_2} \longrightarrow \underset{(Z)}{Ca(HCO_3)_2}$$

$$\underset{(Z)}{Ca(HCO_3)_2} \xrightarrow{Heat} \underset{(x)}{CaCO_3} + CO_2 + H_2O$$

Q.39 To observe the effect of concentration on the conductivity, electrolytes of different nature were taken in two vessels A and B. ' A' contains weak electrolyte NH_4OH and B contains strong electrolyte NaCl. In both container concentration of respective electrolyte was increased and conductivity observed.

(a) In A conductivity increase, In B conductivity decreases

(b) In A conductivity decreases, while in B conductivity increases

(c) In both A and B conductivity increases

(d) In both A and B conductivity decreases.

Ans: (d)

Sol: In container A (NH_4OH)

$$NH_4OH + H_2O \rightleftharpoons NH_4^+ + OH^-$$

Weak electrolyte NH_4OH usually ionizes very less. Ionization increase on dilution and increase in concentration adversely affects dilution. Thus low ionization and hence, less availability of ions and low condution

In container B (NaCl)

As concentration $\uparrow$ interionic attraction $\uparrow$

(ion pair attraction) $\uparrow$

Movement of ion shows down conductivity $\downarrow$

Q.40 The correct IUPAC name of complex Fe $(C_5H_5)_2$ is

(a) Cyclopentadienyl iron (II)

(b) Bis (cyclopentadienyl) iron (II)

(c) Dicyclopentadienyl ferrate (II)

(d) Ferrocene

Ans: (b)

Sol: Correct IUPAC name is bis (cyclopentadienyl) iron (II).

Q.41 An ore of tin containing $FeCrO_4$ is concentrated by

(a) Froth floatation process **(b) Magnetic separation method**

(c) Electrostatic method **(d) Gravity separation method**

Ans: (b)

Sol: $FeO \cdot Cr_2O_3$ is magnetic which is separated by magnetic separation method which are of non-magnetic tin.

Q.42 AgCl is colourless whereas AgI is yellow because of

(a) Ag^+ possess 18 electrons shell to screen the nuclear charge

(b) Ag^+ shows pseudo inert gas configuration

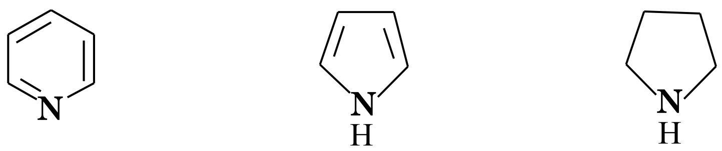
(c) distortion of I^- is more pronounced then Cl^- ion

(d) existance of d-d transition

Ans: (c)

Sol: Greater polarisation of electron cloud also cause colour.

Q.43 Which of the following is the correct order of basicity for these molecules ?



NH_3

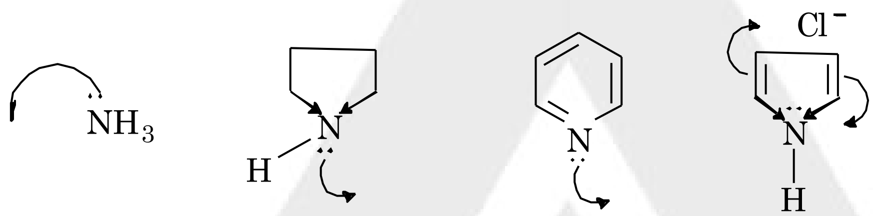
IV **I** **II** **III**

(a) IV > II > I > III **(b) III > IV > I > II**

(c) I > IV > III > II **(d) III > I > II > IV**

Ans: (b)

Sol:



Correct order III > IV > I > II.

Q.44 The salt of which one of the following weak acids will be the most hydrolysed ?

(a) HA : $K_a = 1 \times 10^{-8}$ **(b) HB : $K_a = 2 \times 10^{-6}$**

(c) HC : $K_a = 3 \times 10^{-8}$ **(d) HD : $K_a = 4 \times 10^{-10}$**

Ans: (d)

Sol: $K_h = \frac{K_w}{K_a}$, smaller the K_a greater the K_h or greater is hydrolysis

Q.45 When 2g of a gaseous substance A is introduced into an initially evacuated flask at 25^0C, the pressure was found to be 1.0 atm. 3 g of another gaseous substance B is added to it at the same temperature and pressure. The final pressure is found to be 1.5 atm. Assuming ideal gas behaviour, which of the following is the correct ratio of molecular weight of A and B ?

(a) 1 : 1 **(b) 1 : 2** **(c) 1 : 3** **(d) 1 : 4**

Ans: (c)

Sol: Suppose molecular weight of A and B are M_A and M_B.

Hence, number of moles of $A = \frac{2}{M_A}, B = \frac{3}{M_B}$

$P \propto n$ at constant temperature and pressure

pressure, $1 \propto 2/M_A$

On addition of 3g B, pressure increases by

$$(1.5-1) = 0.5\,\text{atm} = 0.5 \propto \frac{3}{M_B}$$

Hence, $$\frac{1}{0.5} = \frac{2}{M_A} \times \frac{M_B}{3}$$

$$\frac{2}{2}\frac{M_B}{M_A} = 2,\ \frac{M_B}{M_A} = \frac{3}{1}$$

Hence $$\frac{M_A}{M_B} = 1 : 3$$

Q.46 Molar conductance of a 1.5 M solution of an electrolyte is found to be 138.9 S cm². The specific conductance of this solution is

Sol: Molar conductance $= \dfrac{k \times 1000}{M}$

$$138.9 = \frac{k \times 1000}{1.5},\ k = 0.208\ \text{S cm}^{-1}$$

Q.47 In a certain polluted atmosphere containing O_3 at a steady state concentration of 2.0×10^{-8} mol / L, the hourly production of O_3 by all sources was estimated as 7.2×10^{-15} mol / L, If the only mechanism for the destruction of O_3 is the second order reaction $2O_3 \longrightarrow 3O_2$. What is the rate constant for the destruction reaction ?

Sol: At steady state, the rate of destruction of O_3 must be equal to rate of its generation.

7.2×10^{-15} L. mol^{-1}. h^{-1}.

From second order rate law, $-\Delta[O_3]/\Delta t = k[O_3]^2$

$k = (-\Delta[O_3]/\Delta t)/[O_3]^2 = 5 \times 10^{-3}\ \text{L mol}^{-1}\ \text{s}^{-1}$)

Q.48 $MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O\ \ E^0 = 1.51V$

$MnO_2 + 4H^+ + 2e^- \longrightarrow Mn^{2+} + 2H_2O\ \ E^0 = 1.23V$

$E^0_{MnO_4^-}\ |\ MnO_2$ **is**

Sol: $MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O\ \ E^0 = 1.51V$

$$\Delta G_1^0 = -5(1.51)\,F = -7.55\,F$$

$MnO_2 + 4H^+ + 2e^- \rightarrow Mn^{2+} + 2H_2O\ \ E^0 = 1.23V$

$$\Delta G_2^0 = -2(1.23)\,F = -2.46\,V$$

On substracting

$$MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$$

$$\Delta G_3^0 = -5.09\ F$$

$$E^0_{MnO_4^-|MnO_2} = \Delta G_3^0 / -n\ F$$

$= -5.09\ F/-3F = 1.70\,V$

Q.49 A litre of CO_2 gas at 15^0C and 1.00 atm dissolves in 1.00 L of water at the same temperature when the pressure of CO_2 is 1.00 atm. Compute the molar concentration of CO_2 in a solution over which the partial pressure of CO_2 is 150 Torr at this temperature

Sol: $n = \frac{pV}{RT} = \frac{(1.00\text{ atm})(1.00\text{ L})}{(0.0821\text{ L atm / mol K})(288\text{ K})}$

$= 0.0423$ mol

The concentration at 1.00 atm partial pressure is 0.0423 M At 150/760 atm partial pressure, the concentration is

$$0.0423 \times \frac{150}{760} = 8.35 \times 10^{-3}\text{ M}$$

$= 8.35$ mM

Q.50 The vapour pressure of pure liquid solvent A is 0.80 atm. When a non- volatile substance B is added to the solvent, it's vapour pressure drops to 0.60 atm. What is the mole-fraction of component B in the solution ?

Sol: $p = X_A P^0 \Rightarrow X_A = \frac{P}{P^0} = \frac{0.60\,\text{atm}}{0.80\text{ atm}} = 0.75$

Mole fraction of component B

$$X_B = 1 - 0.75 = 0.25$$

Part - C - MATHEMATICS

Q.51 The locus of the point z satisfying Re $\left(\frac{1}{z}\right)$ = k, Where k is a non-zero real number, is

(a) A straight line **(b) A circle**
(c) An ellipse **(d) A hyperbola**

Ans: (b)

Sol: Let $z = x + iy$ then

$$\frac{1}{z} = \frac{1}{x+iy} = \frac{x-iy}{(x+iy)(x-iy)} = \frac{x-iy}{x^2+y^2} = \frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}$$

$\therefore \quad \text{Re}\left(\frac{1}{z}\right) = \frac{x}{x^2+y^2}$

But $\quad \text{Re}\left(\frac{1}{z}\right) = k$

$\therefore \quad \frac{x}{x^2+y^2} = k$

$\Rightarrow \quad x^2 + y^2 - \frac{1}{k}x = 0$

Which is an equation of a circle. Hence, the required locus is a circle.

Q.52 The distance between the origin and the tangent to the curve $y = e^{2x} + x^2$ drawn at the point $x = 0$, is

(a) $\frac{1}{\sqrt{5}}$ **(b)** $\frac{2}{\sqrt{5}}$ **(c)** $-\frac{1}{\sqrt{5}}$ **(d)** $\frac{2}{\sqrt{3}}$

Ans: (a)

Sol: The equation of given curve is

$$y = e^{2x} + x^2 \qquad ....(i)$$

At $x = 0$, $y = e^0 + 0 = 1$

On differentiating Eq. (i) w.r.t.x we get

$$\frac{dy}{dx} = 2e^{2x} + 2x$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(0,1)} = 2e^0 + 0 = 2$$

The equation of the tangent at point $(0, 1)$ is $y - 1 = 2(x - 0) \Rightarrow 2x - y + 1 = 0$

$\therefore$ Required distance = Length of perpendicular from point $(0, 0)$ to $2x - y + 1 = 0$

$$= \frac{|0 - 0 + 1|}{\sqrt{4+1}} = \frac{1}{\sqrt{5}}$$

Q.53 If $\frac{3\pi}{2} \le x \le \frac{5\pi}{2}$, then $\sin^{-1}(\sin x)$ is equal to

(a) x **(b)** $x - 2\pi$ **(c)** $2\pi - x$ **(d)** $-x$

Ans: (b)

Sol: Given, $\frac{3\pi}{2} \le x \le \frac{5\pi}{2}$

$\therefore \quad \sin^{-1}(\sin x) = x - 2\pi$

Q.54 $\int \cos^3 x \, e^{\log(\sin x)} \, dx$ is equal to

(a) $-\frac{\sin^4 x}{4} + C$ **(b)** $-\frac{\cos^4 x}{4} + C$ **(c)** $\frac{e^{\sin x}}{4} + C$ **(d)** None of these

Ans: (b)

Sol: Let $I = \int \cos^3 x \, e^{\log \sin x} \, dx = \int \cos^3 x \sin x \, dx$

Put $\cos x = t \Rightarrow -\sin x \, dx = dt$

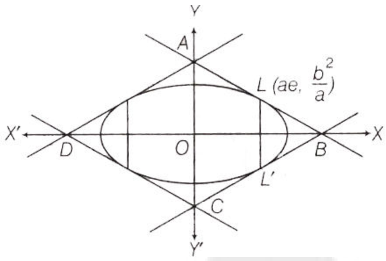
$\therefore \quad I = -\int t^3 \, dt = -\frac{t^4}{4} + C = -\frac{\cos^4 x}{4} + C$

Q.55 The area of the quadrilateral formed by the tangents at the end points of latusrectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is

(a) $\frac{27}{4}$ sq units (b) 9 sq units (c) $\frac{27}{2}$ sq units (d) 27 sq units

Ans: (d)

Sol: Given equation of ellipse is



So, total area is four times the area of the right angled triangle formed by the tangent and axis in the 1st quadrant.

Equation of tangent at $\left(2, \frac{5}{3}\right)$ is

$$\frac{2}{9}x+\frac{5}{3}\cdot\frac{y}{5}=1 \Rightarrow \frac{x}{9/2}+\frac{y}{3}=1$$

$\therefore$ Area of quadrilateral ABCD = 4 (Area of ΔAOB)

$$= 4\times\frac{1}{2}\times\frac{9}{2}\times 3 = 27 \text{ sq units}$$

Q.56 It is known that $\sum_{r=1}^{\infty}\frac{1}{(2r-1)^2}=\frac{\pi^2}{8}$, then $\sum_{r=1}^{\infty}\frac{1}{r^2}$

(a) $\frac{\pi^2}{24}$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{6}$ (d) None of these

Ans: (c)

Sol: $\because$ $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+......\infty=\frac{\pi^2}{8}$

Let $x=\frac{1}{1^2}+\frac{1}{2^2}+......\infty=\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+......\infty\right)+\left(\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}+......\infty\right)$

$$=\frac{\pi^2}{8}+\frac{1}{4}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+......\infty\right)$$

$$x=\frac{\pi^2}{8}+\frac{1}{4}x \Rightarrow \frac{3x}{4}=\frac{\pi^2}{8}$$

$\Rightarrow$ $x=\frac{\pi^2}{6}$

Q.57 If The sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ $(a, b, c \neq 0)$ is equal to sum of square of their reciprocals, then $\frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in

(a) AP **(b) GP** **(c) AGP** **(d) None of these**

Ans: (a)

Sol: Let α, β be the roots of the equation $ax^2 + bx + c = 0$, then

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

According to given condition,

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$$

$$\Rightarrow \quad \alpha + \beta = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\Rightarrow \quad -\frac{b}{a} = \frac{b^2 - 2ac}{c^2}$$

$$\Rightarrow \quad \frac{2a}{c} = \frac{b^2}{c^2} + \frac{b}{a} = \frac{ab^2 + bc^2}{ac^2}$$

$$\Rightarrow \quad 2a^2c = ab^2 + bc^2$$

$$\Rightarrow \quad \frac{2a}{b} = \frac{b}{c} + \frac{c}{a}$$

$$\Rightarrow \quad \frac{c}{a}, \frac{a}{b}, \frac{b}{c} \text{ are in AP.}$$

Q.58 The shortest distance between the lines $\mathbf{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\mathbf{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$ is

(a) $\frac{6}{5}$ **(b) $\frac{1}{\sqrt{5}}$** **(c) $\frac{6}{\sqrt{5}}$** **(d) None of these**

Ans: (c)

Sol: Here, $a_1 = 4\hat{i} - \hat{j}$,

$a_2 = \hat{i} - \hat{j} + 2\hat{k}$ $\quad b_1 = \hat{i} + 2\hat{j} - 3\hat{k}$

and $b_2 = 2\hat{i} + 4\hat{j} - 5\hat{k}$

Now, $a_2 - a_1 = -3\hat{i} + 0\hat{j} + 2\hat{k}$

and $b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j} + 0\hat{k}$

Again, now $(a_2 - a_1)\cdot(b_1 \times b_2)$

$$= (-3\hat{i} - 0\hat{j} + 2\hat{k})\cdot(2\hat{i} - \hat{j} + 0\hat{k}) = -6$$

and $|b_1 \times b_2| = \sqrt{4+1+0} = \sqrt{5}$

$\therefore$ Shortest distance,

$$d = \left|\frac{(a_2 - a_1)\cdot(b_1 \times b_2)}{|b_1 \times b_2|}\right| = \left|\frac{-6}{\sqrt{5}}\right| = \frac{6}{\sqrt{5}}$$

Q.59 The number of tangents to the curve $x^{3/2} + y^{3/2} = a^{3/2}$, where the tangents are equally inclined to the axes, is

(a) 2 **(b) 1** **(c) 0** **(d) 4**

Ans: (b)

Sol: The equation of given curve is

$$x^{3/2} + y^{3/2} = a^{3/2}$$

On differentiating w.r.t.x. we get

$$\frac{3}{2}x^{1/2} + \frac{3}{2}y^{1/2}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{x^{1/2}}{y^{1/2}}$$

Let (α, β) be the point of contact on the curve.

Since, $$\left(\frac{dy}{dx}\right)_{(\alpha,\beta)} = 1$$

$$\Rightarrow \quad \alpha^{1/2} + \beta^{1/2} = 0 \quad .....(i)$$

and $$\alpha^{3/2} + \beta^{3/2} = a^{3/2} \quad .....(ii)$$

On solving Eqs (i) and (ii) we do not get any value of α, β.

Now $$\left(\frac{dy}{dx}\right)_{(\alpha,\beta)} = -1$$

$$\Rightarrow \quad \alpha^{1/2} = \beta^{1/2} \quad ....(iii)$$

On solving Eqs. (ii) and (iii), we get

$$\alpha = \beta = \frac{a}{2^{2/3}}$$

$\therefore$ There is only one point.

Q.60 Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball of the total, 64 played both basketball and hockey, 80 played cricket and basketball and 40 played cricket and hockey, 24 played all the three games. The number of boys who did not play any game is

(a) 216 **(b) 240** **(c) 128** **(d) 160**

Ans: (d)

Sol: We have

$n(C) = 224$, $n(H) = 240$, $n(B) = 336$.

$n(H \cap B) = 64$, $n(B \cap C) = 80$

$n(H \cap C) = 40$, $n(C \cap H \cap B) = 24$

$\therefore$ $n(C^c \cap H^c \cap B^c) = n(C \cup H \cup B)^c$

$= n(U) - n(C \cup H \cup B)$

$= 800 - [n(C) + n(H) + n(B) - n(H \cap C) - n(H \cap B) - n(C \cap B) + n(C \cap H \cap B)]$

$= 800 - (224 + 240 + 336 - 40 - 64 - 80 + 24)$

$= 800 - 640 = 160$

Q.61 The area of the region bounded by the curves $y = 2^x$, $y = 2x - x^2$ and $x = 0$ is

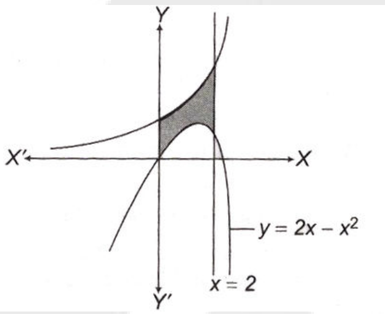
(a) $\left(\frac{3}{\log 2} - \frac{4}{3}\right)$ **sq unit** **(b)** $\left(\frac{3}{\log 2} + \frac{4}{3}\right)$ **sq unit**

(c) $\left(\frac{1}{\log 2} - \frac{4}{3}\right)$ **sq unit** **(d) None of these**

Ans: (a)

Sol: Given equation of curves are

$$y = 2^x,\ (x-1)^2 = -(y-1) \text{ and } x = 2$$



$\therefore$ Required area

$$= \int_0^2 [2^x - (2x - x^2)]\, dx = \int_0^2 (2^x - 2x + x^2)\, dx$$

$$= \left[\frac{2^x}{\log 2} - x^2 + \frac{x^3}{3}\right]_0^2 = \frac{4}{\log 2} - 4 + \frac{8}{3} - \frac{1}{\log 2}$$

$$= \left(\frac{3}{\log 2} - \frac{4}{3}\right) \text{ sq units.}$$

Q.62 The value of $\int_0^{1/2} \sin^{-1}\left(\frac{1}{x}\right) dx$ is

(a) $\pi/2$ **(b)** $\pi/4$ **(c)** $-\pi/2$ **(d) None of these**

Ans: (d)

Sol: $\int_0^{1/2} \sin^{-1}\left(\frac{1}{x}\right) dx$ Here, we see that for $0 < x < \frac{1}{x}$, then $2 < \frac{1}{2} < \infty$ i.e. domain of $\sin^{-1} x$ is $(2, \infty)$ which is not possible

Hence, we cannot determine it.

Q.63 **Given that the equation $z^2+(p+iq)z+r+is=0$, where p,q,r,s are real has non-zero root, then.**

(a) $pqr=r^2+p^2s$ **(b)** $prs=q^2+r^2p$

(c) $qrs=p^2+s^2q$ **(d)** $pqs=s^2+q^2r$

Ans: (d)

Sol: Given that, $z^2+(p+iq)z+r+is=0$ ...(i)

Let $z=\alpha$ be a root of Eq (i) then

$$\alpha^2+(p+iq)\alpha+r+is=0$$

$$\Rightarrow \quad \alpha^2+p\alpha+r+i(q\alpha+s)=0$$

On equating real and imaginary parts, we get

$$\alpha^2+p\alpha+r=0 \quad ...(ii)$$

and $$q\alpha+s=0\Rightarrow\alpha=-\frac{s}{q} \quad ...(iii)$$

On putting the value of α in Eq. (ii) we get

$$\left(\frac{-s}{q}\right)^2+p\left(\frac{-s}{q}\right)+r=0$$

$$\Rightarrow \quad pqs=s^2+q^2r$$

Q.64 **If a,b,c are the sides of a triangle, then $\frac{1}{b+c},\frac{1}{c+a},\frac{1}{a+b}$ are also the sides of the triangle, is**

(a) Always true **(b) Sometimes true**

(c) Cannot be discussed **(d) Never true**

Ans: (a)

Sol: Assume that $a\ge b\ge c$ we must have $b+c>a$. Alos, note that $b+c\le c.+a\le a+b$

$$\Rightarrow \quad \frac{1}{b+c}\ge\frac{1}{c+a}\ge\frac{1}{a+b}$$

To show that $\frac{1}{b+c},\frac{1}{c+a},\frac{1}{a+b}$ are sides of a triangle, it is sufficient to show that

$$\frac{1}{c+a}+\frac{1}{a+b}>\frac{1}{b+c} \quad ....(i)$$

As $a\ge b\ge c$ we get $2a\ge a+b$ and $2a\ge a+c$

$$\Rightarrow \quad \frac{1}{2a}\le\frac{1}{a+b},\frac{1}{2a}\le\frac{1}{a+c}$$

$$\Rightarrow \quad \frac{1}{a+b}+\frac{1}{a+c}\ge\frac{1}{2a}+\frac{1}{2a}=\frac{1}{a}>\frac{1}{b+c} \quad \text{[from Eq (i) } a<b+c\text{]}$$

$\therefore$ It represents a triangle.

Q.65 The solution of the differential equation $xdy - ydx = \sqrt{x^2 + y^2}\,dx$ is

(a) $x + \sqrt{x^2 + y^2} = Cx^2$

(b) $y - \sqrt{x^2 + y^2} = Cx$

(c) $x - \sqrt{x^2 + y^2} = Cx$

(d) $y + \sqrt{x^2 + y^2} = Cx^2$

Ans: (d)

Sol: Given that $xdy - ydx = \sqrt{x^2 + y^2}\,dx$

$\therefore \quad xdy = (\sqrt{x^2 + y^2} + y)dx$

$\Rightarrow \quad \frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} + y}{x}$

Now, put $y = \text{v}x \Rightarrow \frac{dy}{dx} = \text{v} + x\frac{dv}{dx}$

$\therefore \quad \text{v} + x\frac{dv}{dx} = \frac{\sqrt{x^2 + \text{v}^2x^2} + \text{v}x}{x}$

$\Rightarrow \quad x\frac{dv}{dx} = \sqrt{1 + \text{v}^2}$

$\Rightarrow \quad \frac{dv}{\sqrt{1 + \text{v}^2}} = \frac{dx}{x}$

$\Rightarrow \quad \log\left|(\text{v} + \sqrt{(1 + \text{v}^2})\right| = \log|x| + \log C$

$\Rightarrow \quad \text{y} + \sqrt{x^2 + y^2} = Cx^2$

Q.66 $\lim_{x \to 1} \frac{\sqrt{1 - \cos 2(x-1)}}{x - 1}$ **is equal to**

(a) exists and it equal $\sqrt{2}$

(b) exists and it equals $-\sqrt{2}$

(c) does not exist becuase $x - 1 \to 0$

(d) does not exist because left hand limt is not equal to right hand limit.

Ans: (d)

Sol: LHL $= \lim_{x \to 1^-} \frac{\sqrt{1 - \cos 2(x-1)}}{x - 1}$

$= \lim_{x \to 1^-} \frac{\sqrt{2\sin^2(x-1)}}{x - 1}$

$= \sqrt{2}\lim_{x \to 1^-} \frac{|\sin(x-1)|}{x - 1}$

Put $x = 1 - h, h > 0$ For $x \to 1^-, h \to 0$

$= \sqrt{2}\lim_{h \to 0} \frac{|\sin(-h)|}{-h}$

$$= \sqrt{2} \lim_{h \to 0} \frac{\sin h}{-h} = -\sqrt{2}$$

Again, RHL $= \lim_{x \to 1^+} \frac{\sqrt{1 - \cos 2(x-1)}}{x-1}$

$$= \lim_{x \to 1^+} \sqrt{2} \frac{|\sin(x-1)|}{x-1}$$

Put $x = 1 + h, h > 0$

For $x \to 1^+, h \to 0$

$$= \lim_{h \to 0} \sqrt{2} \frac{|\sin h|}{h}$$

$$= \lim_{h \to 0} \sqrt{2} \frac{\sin h}{h} = \sqrt{2}$$

LHL $\neq$ RHL

Therefore , $\lim_{x \to 1} f(x)$ does not exist.

Q.67 If $a > 2b > 0$, then positive value of m for which $y = mx - b\sqrt{1+m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x-a)^2 + y^2 = b^2$, is

(a) $\frac{2b}{\sqrt{a^2 - 4b^2}}$ **(b)** $\frac{\sqrt{a^2 - 4b^2}}{2b}$

(c) $\frac{2b}{a - 2b}$ **(d)** $\frac{b}{a - 2b}$

Ans: (a)

Sol: Given, $y = mx - b\sqrt{1+m^2}$ touches both the circles, so distance from centre = Radius of both the circles

$\therefore \quad \frac{|-b\sqrt{1+m^2}|}{\sqrt{m^2+1}} = b$ ....(i)

and $\frac{|ma - 0 - b\sqrt{1+m^2}|}{\sqrt{m^2+1}} = b$ ....(ii)

$\Rightarrow \quad |ma - b\sqrt{1+m^2}| = |-b\sqrt{1+m^2}|$ [from Eq (i) and (ii)]

$\Rightarrow \quad m^2a^2 - 2abm\sqrt{1+m^2} + b^2(1+m^2) = b^2(1+m^2)$

$\Rightarrow \quad ma - 2b\sqrt{1+m^2} = 0$

$\Rightarrow \quad m^2a^2 = 4b^2(1+m^2)$

$\Rightarrow \quad m = \frac{2b}{\sqrt{a^2 - 4b^2}}$

Q.68 If $a_r > 0$, r∈N and $a_1, a_2 ......a_{2n}$ are in AP, then

$$\frac{a_1 + a_{2n}}{\sqrt{a_1} + \sqrt{a_2}} + \frac{a_2 + a_{2n-1}}{\sqrt{a_2} + \sqrt{a_3}} + \frac{a_3 + a_{2n-2}}{\sqrt{a_3} + \sqrt{a_4}} + ... + \frac{a_n + a_{n+1}}{\sqrt{a_n} + \sqrt{a_{n+1}}}$$ **is equal to**

(a) $n - 1$

(b) $\frac{n(a_1 + a_{2n})}{\sqrt{a_1} + \sqrt{a_{n+1}}}$

(c) $\frac{n - 1}{\sqrt{a_1} + \sqrt{a_{n+1}}}$

(d) None of these

Ans: (b)

Sol: Let $a_1 + a_{2n} = a_2 + a_{2n-1} + .. = a_n + a_{n+1} = \text{k}$

$$\therefore \quad \frac{a_1 + a_{2n}}{\sqrt{a_1} + \sqrt{a_2}} + \frac{a_2 + a_{2n-1}}{\sqrt{a_2} + \sqrt{a_3}} + ... + \frac{a_n + a_{n+1}}{\sqrt{a_n} + \sqrt{a_{n+1}}}$$

$$= \text{k} \left\{ \frac{\sqrt{a_1} - \sqrt{a_2}}{a_1 - a_2} + \frac{\sqrt{a_2} - \sqrt{a_3}}{a_2 - a_3} + ... + \frac{\sqrt{a_n} - \sqrt{a_{n+1}}}{a_2 - a_{n+1}} \right\}$$

$$= -\frac{\text{k}}{\text{d}} \{\sqrt{a_1} - \sqrt{a_2} + \sqrt{a_2} - \sqrt{a_3} + ... + \sqrt{a_n} - \sqrt{a_{n+1}}\}$$

Where, d is common difference.

$$= -\frac{\text{k}}{\text{d}} \{\sqrt{a_1} - \sqrt{a_{n+1}}\}$$

$$= \frac{\text{k}}{\text{d}} \{\sqrt{a_{n+1}} - \sqrt{a_1}\}$$

$$= (a_1 + a_{2n}) \cdot \frac{-\text{nd}}{-\text{d}(\sqrt{a_1} + \sqrt{a_{n+1}})} = \frac{\text{n}(a_1 + a_{2n})}{\sqrt{a_1} + \sqrt{a_{n+1}}}$$

Q.69 A flag is standing vertically on a tower of height b on a point at a distance a from the foot of the tower, the flat and the tower subtend equal angles, The height of the flag is

(a) $b \cdot \frac{a^2 + b^2}{a^2 - b^2}$

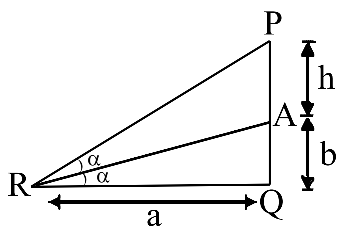
(b) $a \cdot \frac{a^2 - b^2}{a^2 + b^2}$

(c) $b \cdot \frac{a^2 - b^2}{a^2 + b^2}$

(d) $a \cdot \frac{a^2 + b^2}{a^2 - b^2}$

Ans: (a)

Sol: Let the height of the flag be h.



In ΔARQ $\tan\alpha = \frac{b}{a}$ ...(i)

and in ΔPRQ, $\tan 2\alpha = \frac{h+b}{a}$ ...(ii)

$$\Rightarrow \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{h+b}{a}$$

$$\Rightarrow \frac{2\times\frac{b}{a}}{1-\frac{b^2}{a^2}} = \frac{h+b}{a}$$ [from Eq (i)]

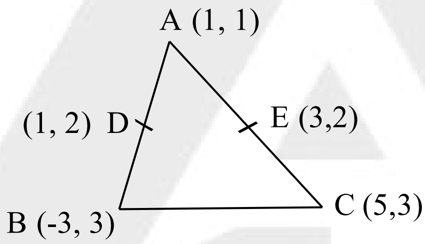
$$\Rightarrow \frac{2ab}{a^2-b^2} = \frac{h+b}{a} \Rightarrow h = \frac{b(a^2+b^2)}{a^2-b^2}$$

Q.70 If a vertex of a triangle is (1 ,1) and the mid-points of two sides through this vertex are (-1, 2) and (3, 2), then the centroid of the triangle is

(a) $\left(\frac{1}{3}, \frac{7}{3}\right)$ **(b)** $\left(1, \frac{7}{3}\right)$ **(c)** $\left(-\frac{1}{3}, \frac{7}{3}\right)$ **(d)** $\left(-1, \frac{7}{3}\right)$

Ans: (b)

Sol: Let D and E be the mid-point of AB and AC. So the coordinate of B and C are (-3, 3) and (5, 3) respectively.



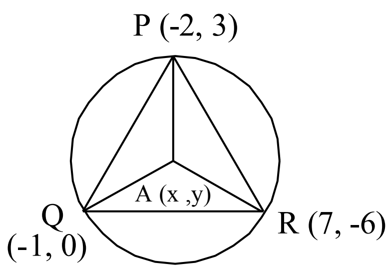
Centroid of triangle

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$= \left(\frac{1-3+5}{3}, \frac{1+3+3}{3}\right) = \left(1. \frac{7}{3}\right)$$

Q.71 The circumcentre of the triangle whose vertices are $(-2, -3), (-1, 0), (7, -6)$, is

Sol: Let the vertices of triangle be $P \equiv (-2, -3)$, $Q \equiv (-1, 0)$ and $R \equiv (7, -6)$. Let $A(x, y)$ be the circumcentre of ΔPQR.



$\therefore \quad AP^2 = AQ^2$

$\Rightarrow \quad (x+2)^2 + (y+3)^2 = (x+1) + y^2$

$\Rightarrow \quad 4x + 6y + 13 = 2x + 1$

$\Rightarrow \quad 2x + 6y + 12 = 0$

$\Rightarrow \quad x + 3y = -6$ ....(i)

Similarly $AP^2 = AR^2$

$\Rightarrow \quad (x+2)^2 + (y+3)^2 = (x-7)^2 + (y+6)^2$

$\Rightarrow \quad 4x + 6y + 13 = -14x + 12y + 85$

$\Rightarrow \quad 18x - 6y = 72$

$\Rightarrow \quad 3x - y = 12$ ....(ii)

From Eqs (i) and (ii) we get

$(x, y) \equiv (3, -3)$

Hence, circumcentre is $(3, -3)$

Q.72 If $\cos\frac{\pi}{7}, \cos\frac{3\pi}{7}, \cos\frac{5\pi}{7}$ are the roots of the equation $8x^3 - 4x^2 - 4x + 1 = 0.$ then find the value of $\sec\frac{\pi}{7} + \sec\frac{3\pi}{7} + \sec\frac{5\pi}{7}$.

Sol: Given, $\cos\frac{\pi}{7}, \cos\frac{3\pi}{7}, \cos\frac{5\pi}{7}$, are the roots of the equation

$8x^3 - 4x^2 - 4x + 1 = 0.$ ....(i)

Replacing x by $\frac{1}{x}$ in Eq (i), we get

$x^3 - 4x^2 - 4x + 8 = 0$ ...(ii)

Since, $\sec\frac{\pi}{7}, \sec\frac{3\pi}{7}, \sec\frac{5\pi}{7}$, are the roots of Eq (ii).

$\therefore \quad \sec\frac{\pi}{7} + \sec\frac{3\pi}{7} + \sec\frac{5\pi}{7} = 4$

Q.73 If $P = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ -2 & 0 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} -4 & -5 & -6 \\ 0 & 0 & 1 \end{bmatrix}$, then P_{22} is equal to

Sol: Given, $P = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ -2 & 0 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} -4 & -5 & -6 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}\begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 0 & 0 & -4 \end{bmatrix}$$

$$P_{22} = [2 \;\; 3 \;\; 4]\begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix} = 10 + 30 = 40$$

So, option (a) is corect.

Q.74 The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently, is

Sol: Given word is BANANA.
Here, presence of alphabet A = 3 times and N = 2 times.
Required number of arrangements

$$= \frac{6!}{2!3!} - \frac{5!}{3!} = 60 - 20 = 40$$

Q.75 If 25% of the items are less than 20 and 25% are more than 40, the quartile deviation is

Sol: We have, Q_1=20 and Q_3=40

$$\therefore \quad Q.D. = \frac{Q_3 - Q_1}{2} = \frac{40-20}{2} = \frac{20}{2} = 10$$

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