A Division of Aggarwal Educare
Aggarwal CLASSES™

JEE (MAIN)

TEST PAPER

SUBJECT : PHYSICS,CHEMISTRY, MATHEMATICS ***TEST CODE : TEST PAPER-1***

ANSWER PAPER

TIME : 3 HRS ***MARKS : 300***

INSTRUCTIONS

GENERAL INSTRUCTIONS :

1. This test consists of 75 questions.
2. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 25 questions in each part.
3. 20 questions will be Multiple choice questions & 5 quetions will have answer to be filled as numerical value.
4. Marking scheme :

Type of Questions	Total Number of Questions	Correct Answer	Incorrect Answer	Unanswered
MCQ's	20	+4	Minus One Mark(-1)	No Mark (0)
Numerical Values	5	+4	No Mark (0)	No Mark (0)

5. There is only one correct responce for each question. Filling up more than one responce in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

OPTICAL MARK RECOGNITION (OMR) :

6. The OMR will be provided to the students.
7. Darken the appropriate bubbles on the OMR sheet by applying sufficient pressure.
8. The OMR sheet will be collected by the invigilator at the end of the examination.
9. Do not tamper with or mutilate the OMR. Do not use the OMR for rough work.
10. Write your name, Batch name, name of the center, Test Code, roll number and signature with pen in the space provided for this purpose on the OMR. Do not write any of these details anywhere else on the OMR.

DARKENING THE BUBBLES ON THE OMR :

11. Use a BLACK BALL POINT PEN to darken the bubbles on the OMR.
12. Darken the bubble COMPLETELY.
13. Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or "un- darken" a darkened bubble.

Part A - PHYSICS

Q.1. Electromagnetic waves are produced by
(a) a static charge **(b) a moving charge**
(c) an accelerating charge **(d) chargeless particle**

Ans: (c)

Sol: It is fact, that an accelerated charge produces electric and magnetic fields as well as emit electromagnetic radiation.

Q.2 During charging and discharging of a capacitor
(a) current flows in the circuit, which is constant during charging or discharging duration
(b) no current flows in the circuit
(c) current flows in the circuit and is varying with time
(d) during charging current is constant but while discharging current is variable

Ans: (c)

Sol: For charging, $i=\frac{E}{R}e^{-\frac{t}{RC}}$

For discharging, $i=\frac{Q}{RC}e^{-\frac{t}{RC}}$ Clearly, both are functions to time t.

Q.3 Internal energy of gas (ideal) depends only on
(a) pressure **(b) temperature** **(c) volume** **(d) temperature and pressure**

Ans: (b)

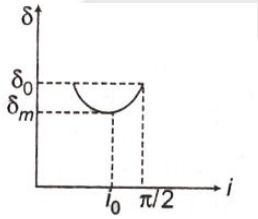
Sol: Internal energy of an ideal gas depends only on temperature.

Q.4 A stone has been thrown in vertical upward direction, from a balloon going up with an acceleration *a*. The acceleration of the stone ofter the throw is
(a) (g -a) upward **(b) (g + a) upward** **(c) g upward** **(d) g downward**

Ans: (d)

Sol: After throwing, the stone is under gravity, so acceleration of stone is g downward.

Q.5 In the diagram, a plot between δ (deviation) versus i (angle of incidence) for a triangular prism is given. From the observed plot, some conclusions can be withdrawn.



Mark out the correct conclusions.
(a) the range of deviation for which two angles of incidence are possible for same deviation is $\delta_0 - \delta_m$
(b) The curve is unsymmetrical about I_0
(c) For a given δ, I is unique
(d) Both (a) and (b) are correct

Ans: (d)

Sol: As shown in graph, the curve is unsymeertrical about i_0 and the range of deviation for which 2 angles of incidence ar possible for same deviation is $\delta_0 - \delta_m$.

Q.6 If electric potential due to some charge distribution is given by $V = 3/r^2$, when r is radial distance, then find electric field at (1,1,1)

(a) $\frac{2}{\sqrt{3}}$ **(b)** $\frac{2(\hat{i}+\hat{j}+\hat{k})}{3}$ **(c)** $\frac{2}{8(\hat{i}+\hat{j}+\hat{k})}$ **(d)** $\frac{3}{2(\hat{i}+\hat{j}+\hat{k})}$

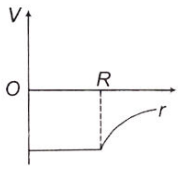
Ans: (b)

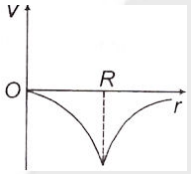
Sol: Given $V = \frac{3}{r^2}$

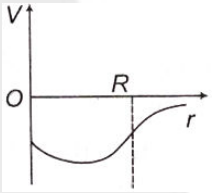
$$\Rightarrow \quad E = -\left(\frac{dV}{dr}\right)r = -\frac{\partial}{\partial r}\left[\frac{3}{r^2}\right]r = \frac{6}{r^4}r$$

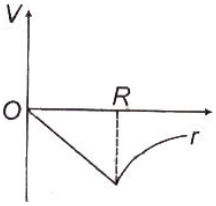
or $$E = 6\frac{(\hat{i}+\hat{j}+\hat{k})}{(\sqrt{3})^4} = \left(\frac{2}{3}\right)(\hat{i}+\hat{j}+\hat{k}) \qquad [\because r = \hat{i}+\hat{j}+\hat{k}]$$

Q.7 The diagram showing the variation of gravitational potential of earth with distance from the centre of earth is

(a) 

(b) 

(c) 

(d) 

Ans: (c)

Sol: The potential inside the surface,

$$V_{in} = \frac{-Gm}{2R}\left[3-\left(\frac{r}{R}\right)^2\right]$$

$$V_{surface} = -\frac{Gm}{R} \text{ and } V_{out} = -\frac{Gm}{r}$$

Q.8 Which is untrue regarding molar heat capacity ?

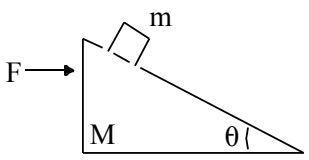
(a) it is path dependent **(b) it is verying with phase**

(c) it is path independent **(d) None of the above**

Ans: (c)

Sol: Different processes have different molar heat capacities.
Substance has different heat capacities in different phases.

Q.9 Find the force F to be applied on M, so that m remains stationary with respect to M ?

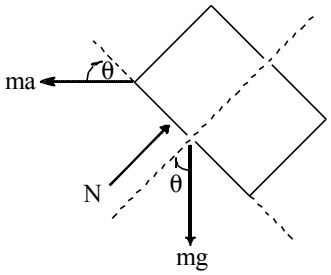


(a) (M + m) g **(b) (M + m) g sin θ**

(c) (M + m) g cos θ **(d) (M + m) g tan θ**

Ans: (d)

Sol: Let us say that the whole system (M + m) is moving with acceleration a. $F(M+m)a$ draw the FBD for m. For equilibrium of m,



$$ma = \cos\theta = mg\sin\theta$$

$$\Rightarrow \quad a = g\tan\theta$$

$$\Rightarrow \quad \frac{F}{(M+m)} = g\tan\theta$$

$$\Rightarrow \quad F = (M+m)\, g\, \tan\theta$$

Q.10 The diffusion current in a *p-n* junction is
(a) from the n-side to the p-side
(b) from the p-side to the n-side
(c) from the n-side to the p-side, if the junction is forward-biased and in the opposite direction, if it is reverse -biased
(d) from the p-side to the n-side, if the junction is forward -biased and in the opposite direction, if it is reverse-biased.

Ans: (b)

Sol: The diffusion current in semiconductor is due to transition of charge carriers across the deplation layer due to concentration difference. As a result, holes try to diffuse from p-side to n-side while electron try to diffuse from n-side to p-side resulting in a net current from p-side to n-side termed as diffusion current.

Q.11 Consider two observers moving with respect to each other at a speed v along a straight line. They observe a block of mass m moving a distance I on a rough surface. The following quantities will be same as observed by the two observers
(a) kinetic energy of the block at time **(b) work done be friction**
(c) total work done on the block **(d) acceleration of the block**

Ans: (d)

Sol: Since, relative acceleration between the observeres is zero, so acceleration of both the observers are same, as a result acceleration of block with respect to observers is same. Kinetic energy, work done by friction and total work done are depending on displacement and velocity of the block, which may be different for both.

Q.12 Why does a glass sometimess break, if we quickly pour boiling water into it ?
(a) Hot water expands, pushing the glass out
(b) The hot water cools when it touches the glass, shrinking and pulling the glass in
(c) The glass becomes hot and expands causing the molecules to break
(d) The inside of the glass expands faster than the outside of the glass, causing the glass to break

Ans: (d)

Sol: When hot water is poured into the glass, the inner portion of the glass comes into the contact of hot water immediately while of outer portion after sometimes as heat is conduced through glass. As a result the inside of the glass expand faster than the outside of the glass and hence causes the glass to break (due to uneven expansion of glass).

Q.13 All the photons emitted by a source of light do not have the same energy. Now Mark the correct option

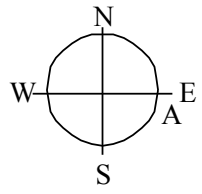
(a) it is not possible at all **(b) Source is monochromatic**

(c) Source is not monochromatic **(d) Source must emit white light**

Ans: (c)

Sol: Source must have at least two wavelengths . only then all photons do not have the same energy.

Q.14 A body is moving with uniform speed v in a horizontal circle in anti-clockwise direction as shown in figure. The motion starts from point A, find the change in velocity in second quarter of revolution.



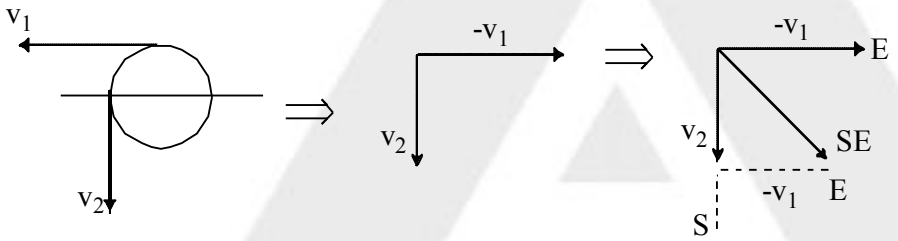
(a) $\sqrt{2}$ **v N - W** **(b)** $\sqrt{2}$ **v N - E**

(c) $\sqrt{2}$ **v S - W** **(d)** $\sqrt{2}$ **v S - E**

Ans: (d)

Sol: The diagram is clearly showing the change in velocity vector

$\Delta V = V_2 - V$ $V = V_2 - V_1$



So, direction will be SE.

Q.15 A parallel plate capacitor is made up of two plates having are 10cm² and are separeted by 2 mm. There is a vacuum in between the plates. If we double the amount of initial charge on the capacitor, then what would be the final capacitance ?

(a) 8.85×10^{-12} **F** **(b)** 4.425×10^{-12} **F** **(c)** $17.7.\times10^{-12}$ **F** **(d) None of these**

Ans: (b)

Sol: Capacitance of a capacitor is independent of charge and potential of capacitor.
Capacitance of a parallel plate capacitor with

vacuum, $C_0 = \dfrac{\varepsilon_0 A}{d}$

$$= \frac{8.85\times10^{-12}\times10\times10^{-4}}{2\times10^{-3}}$$

$$= 4.425\times10^{-9}\times10^{-3}$$

$$= 4.425\times10^{-12}\,\text{F}$$

Q.16 Find the maximum potential difference which may be applied across an X-ray tube with tungsten target without emitting any characteristc K or L X-ray. The energy

levels of tungsten atoms are as follows (K shell - 69.5 ke V, L shell - 11.3 kev , M shell - 2.3 kev)

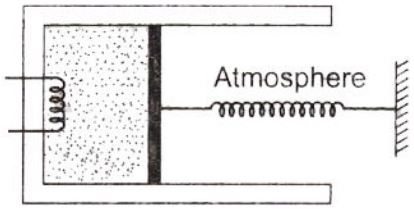
(a) 69.5 keV (b) 11.3 keV (c) 2.3 keV (d) 13.6 keV

Ans: (b)

Sol: To knock out an electron from L shell , $E > 11.3\,keV$. So for no emission of electron form L shell E < 11.3 keV, So

$$E_{max} = 11.3 \text{ keV}$$

Q.17 An ideal monatomic gas is confiend in a cylinder, fitted with piston, which is connected to spring as shown in figure,



The gas is heated by a small electric heater until the piston moves out slowly by 0.1 m. Find the work done by the gas. (Spring constat = 8000 N/m, piston area = $8\times10^{-3}m^2$, atmospheric pressure = 10^5 Pa)

(a) 40 J (b) 80 J (c) 120 J (d) 60 J

Ans: (c)

Sol: Work done $\int_0^{x_0} p_0 A dx + \frac{kx_0^2}{2} = p_0 A x_0 + \frac{kx_0^2}{2}$

$= 80 + 40 = 120\,J$

Where, p= pressure of gas.

p_0 = atmospheric pressure

k = spring constant

x_0 = compression in spring

Q.18 A mass of 250 g hangs on a spring and oscillates vertically with a period of 1.1 s. To double the period, what mass must be added to the 250 g ? (Ignore the mass of the spring)

(a) 250 g (b) 450 g (c) 750 g (d) 550 g

Ans: (c)

Sol: As, $T \propto \sqrt{m}$

$\Rightarrow \quad 1.1 \propto \sqrt{250}$

and $\quad 2.2 \propto \sqrt{250+x}$

$\therefore \quad \frac{1}{2} = \sqrt{\frac{250}{250+x}}$

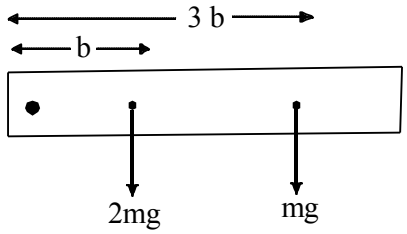
$\Rightarrow \quad x = 750\,g$

Q.19 A nearby massless rod is pivoted at one end so that it can swing freely as a pendulum. Two masses, 2 m and m, are attached to at it distances b and 3b, resepectively, from the pivot. The rod is held horizontal and then released. Find its angular acceleration at the instant it released.

(a) 2 g/ 11*b* **(b) 4 g/17*b*** **(c) 5 g/11*b*** **(d)5g/12*b***

Ans: (c)

Sol: First of all calculate the torque experienced by rod about the pivot given and moment of inertia about this pivot.

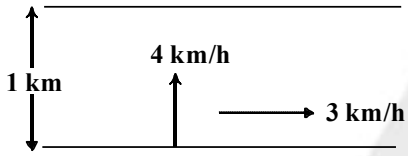


$$\tau = g(2mb + 3mb) = 5mgb \qquad [\because \tau = \text{force} \times \text{distance}]$$

and $$I = 2mb^2 + m(3b)^2 = 11mb^2$$

Since, $\tau = I\alpha$, we have $\alpha = 5g/11b$

Q.20 A man can swim with a speed of 4 km/h in still water. How long does it take to cross a river 1 km wide, if the river flows steadily at 3 km/h and he makes his strokes normal to the river current ? How far down the river does he go when he reaches the other bank ?



(a) 1/3 h, 1 km **(b) 1/4 h, 4/3 km** **(c) 1/3 h, 3/4 km** **(d) 1/4 h, 3/4 km**

Ans: (d)

Sol: Along the normal to river current, time taken to cross the river

$$t = \frac{1}{4}h = 15\,\text{min}$$

Distance covered along the current

$$x = 3 \times \frac{1}{4}\text{km} = 750\,\text{m}$$

Q.21 A ball has been dropped from a height of 64 m above the ground level. Find the distance travelled by the ball in 4th second of its flight. Assume ball comes to rest after collision with ground [Take g = 9.8 m/s²]

Sol: In this quetion, time of flight is coming out to be less than 4s, So if we use the formula directly, then distance travelled – in 4th second $= \frac{1}{2}g \times 4^2 - \frac{1}{2}g \times 3^2$ we will get wrong answer. So, we will use distance travelled in 4th second = total distance travelled- Distance travelled in 3rd second.

$$= 64 - \frac{1}{2} \times 9.8 \times 3^2 = 19.9\,\text{m}$$

Q.22 Consider a parallel beam of light of wavelength 800 nm and intensity 200 W /m², determine the number of photons crossing 1cm² perpendicular to the beam in 2 s.

Sol: Energy of light, $E = \frac{1242}{800}\text{eV}$ $\left[\because E + \frac{hc}{\lambda}\right]$

$= 1.55$ eV

$= 1.55 \times 1.6 \times 10^{-19}$ J

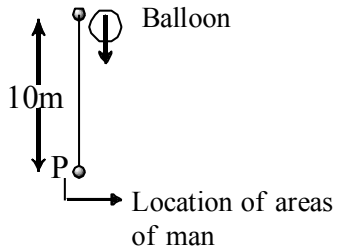
Let n photons cross the area in 2 s, then $P\Delta t$ = energy of n photon

$\therefore \quad 200 \times 10^{-4} \times 2 = n \times 1.55 \times 1.6 \times 10^{-19}$

$\Rightarrow \quad n = 1.61 \times 10^{17}$

Q.23 As a prank, someone drops a water- filled balloon out of a window. The balloon is released from rest at a height of 10 m above the ears of a man who is the target. Beacause of guilty conscience, the prankster shouts a warning after the balloon is released. The warning will do no good, if he shouted after the balloon reaches a certain point, even if the man could react infinitely quickly, Assuming that velocity of sound in air is 343 m/s, Find out how far above the man's ears this point is ?

Sol: Let t_1 is the time taken by the sound to travel 10 . Then $vt_1 = 10$. Let t is the time taken by the balloon to travel 10 m.



Then $\frac{gt^2}{2} = 10$ $[\because u = 0]$

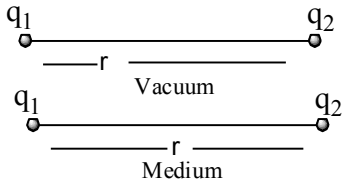
Let the critical height required is h, then

$$10 - h = \frac{g(t - t_1)^2}{2}$$

This gives $h = 0.405\,m$.

Q.24 Two charge particles $+3\mu C$ and $-2\mu C$ are placed 2 m apart. Half of the region between them is filled with a medium having relative permittivity 9. Find the electric force between two charges.

Sol: When a medium has been filled in between the charges, the electric force of intraction between them changes, according to following expression.



$$F_1 = \frac{kq_1q_2}{r^2} \qquad \text{[for vaccum]}$$

$$F_2 = \frac{kq_1q_2}{r_1^2} \qquad \text{[for medium]}$$

For calculating vacuum equivalent separation, the two force has to be equal.

i.e. $F_1 = F_2$

$\Rightarrow r = (\sqrt{\varepsilon_r})r_1$ *i.e.* in medium, two charge paritcels are kept at a distance of r_1, then they will exert the same force when placed in vacuum at a separation of $(\sqrt{\varepsilon_r})r_1$

So, air equivalent distance of medium

$= \sqrt{9} \times 1 = 3m$

So, $r_{eff} = 1 + 3 = 4\,m$

$\therefore$ $$F = \frac{\frac{1}{4\pi\varepsilon_0} q_1 q_2}{r^2}$$

$$= \frac{9\times10^9 \times 2\times3\times10^{-12}}{4^2}$$

$$= 3.375\times10^{-3}\,N$$

Q.25 1 kg of water at 0^0C is heated to 100^0 C, calculate its change in entropy [Swater = 4190 J/ kg-K]

Sol: Change in entropy,

$$dS = \int \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{msdT}{T} = m\sin\left(\frac{T_2}{T_1}\right)$$

$$= 1\times 4190 \text{ in } \left(\frac{373}{273}\right)$$

$$= 1.31 \times 10^3 \text{ J/K}$$

Part - B - CHEMISTRY

Q.26 The tendency to form complexes is maximum for
(a) normal elements
(b) transition elements
(c) inner-transition elements
(d) metals containing fully filled d-orbitals

Ans: (b)

Sol: Transition elements because of having vacant d-orbitals and high charge density have maximum tendency to form complexes.

Q.27 $TiH_{1.73}$ is an example of which type of the hydride ?
(a) Metalic (b) Ionic (c) Covalent (d) Polymeric

Ans: (a)

Sol: $TiH_{1.73}$ is a non-stoichiometric metallic or interstitial hydride.

Q.28 Which of the following is incorrect about vulcanisation of rubber ?
(a) Vulcanised rubber has excellent elasticity
(b) Vulcanised rubber increases resistance to oxidation and organic solvent
(c) Vulcanisation usually establishes cross links at allylic positon type reactive sites
(d) Vulcanised rubber has high-water absorption tendency

Ans: (d)

Sol: Vulcanised rubber has low-water absorption tendency.

Q.29 The vapour density of a chloride of a metal is 81.5 (H = 1) and the chloride contains 34.46 % metal. The specific heat of the metal is 0.115. The molecular formula of chloride is

(a) MCl_2 (b) M_2Cl_3 (c) MCl_3 (d) MCl_4

Ans: (c)

Sol: Vapour density of chloride = 81.5

Molecular mass $= 2 \times 81.5 = 163$

Metal weight = 34.46% of 163 = 56.2

Chlorine weight $= 65.54\%$ of $163 = 106.8$

Number of chlorine atom $= \frac{106.8}{35.5} \approx 3$

Hence, formula $= MCl_3$

Q.30 Which of the following doesn't cause origin of charge on colloid ?

(a) Preferential adsorption (b) Bredig's arc method

(c) Micell's formation (d) Brownian movement

Ans: (d)

Sol: Brownian movement that's simply zig-zag motion of colloids driven by kinetic motion of dispersion medium, basically.

Q.31 Transition metals have the electronic configuration $(n-1)d^{1-10}\ ns^{1-2}$. The d-orbitals are degenerate, Colour of transtion metal ions is due to absorption of some wavelength. This result in

(a) d-s transition (b) s-d transition

(c) s-s transition (d) d-d transition

Ans: (d)

Sol: When a transition metal compound is formed. the degeneracy of d-orbitals is resolved giving two sets of orbitals slightly differing in energy. Absorption of light causes the promotion of electrons form lower energy set of orbitals to higher energy set within the same d-subshell. This is called d-d-transition.

Q.32 In the commercial electrochemical process for aluminium extraction, the electrolyte used is

(a) $Al(OH)_3$ in NaOH solution

(b) An aqueous solution of $Al_2(SO_4)_3$

(c) A molten mixture of Al_2O_3 and Na_3AlF_6

(d) A molten mixture of Al_2O_3 and $Al(OH)_3$

Ans: (c)

Sol: A molten mixture of Al_2O_3 and Na_3AlF_6 is used as an electrolyte in the commercial electrochemical process for extraction of Al.

Q.33 When CS_2 layer containing both Br_2 and I_2 is shaken with excess of Cl_2 water, the violet colour due to I_2 disappears and orange colour due to Br_2 appears. The disappearance of violet colour is due to the formation of,

(a) I_3^- (b) HIO_3 (c) ICl_2 (d) I^-

Ans: (b)

Sol: $\underset{\text{excess}}{5Cl_2} + I_2 + 6H_2O \longrightarrow \underset{\text{Colourless}}{2HIO_3} + 10HCl$

Q.34 Which of the following reaction doesn't support the acidic nature of alkyne ?

(a) Reaction with HBr

(b) Reaction with Grignard reagent
(c) Reaction with ammoniacal silver salt
(d) Reaction with metallic sodium

Ans: (a)

Sol: HBr gives addition reaction on alkyne.

$$HC \equiv CH + HBr \longrightarrow CH_2 = CHBr \xrightarrow{HBr} CH_3\ CHBr_2$$

Q.35 Which of the following is known as pseudo alum ?
(a) $KMn(SO_4)_2 .12H_2O$ **(b) $KCr(SO_4)_2.12H_2O$**
(c) $NH_4Fe(SO_4)_2.12H_2O$ **(d) $FeSO_4.Al_2(SO_4)_3.24H_2O$**

Ans: (d)

Sol: Pseudo alum has the general formula

$M^{II}SO_4 \cdot M^{III}_2(SO_4)_3 \cdot 24H_2O.$

Hence, $FeSO_4 \cdot Al_2(SO_4)_3 \cdot 24H_2O$ is a pseudo alum.

Q.36 A pale yellow precipitate and a gas with pungent odour are formed on warming dilute hydrochloric acid with an aqueous solution containing
(a) Sulphate ion **(b) Sulphide ion**
(c) thiosulphate ion **(d) Sulphate ion**

Ans: (c)

Sol: Sulphide and sulphite ions both react with dil. HCl on warming their aqeous solutions giving H_2S and SO_2 gases respectively but no yellow precipitate are formed. Sulphate ion does not react. The only ion left is thiosulphate, which gives SO_2 gas with a pungent odour and pale yellow precipitate of colloidal sulphur, are formed.

$$S_2O_3^{2-} + 2H^+ \longrightarrow \underset{\substack{\text{gas}\\ \text{pungent}\\ \text{odour}}}{SO_2} + \underset{\substack{\text{pale yellow}\\ \text{preceipitate}}}{S} + H_2O$$

Q.37 Schottky defect in crystals is observed when
(a) unequal number of cations and anions are missing from the lattice
(b) equal number of cations and anions are missing from the lattice
(c) an ion leaves its normal site and occupies an interstitial site.
(d) density of the crystal is increased.

Ans: (b)

Sol: Schottky defect, when equal number of cations and anions are missing from the lattice.

Q.38 The vapour density of PCl_5 at 250° C is found to be 57.9. Percentage dissociation at this temperature is
(a) 80 % **(b) 22%** **(c) 40%** **(d) 60%**

Ans: (a)

Sol: $\alpha = \dfrac{V_\infty - V_0}{(n-1)V_0} = \dfrac{104.25 - 57.9}{(2-1)\times 57.9} = 0.80$

$V_\infty = \dfrac{208.5}{2} V_0 \times 2 =$ molar mass Hence, degree of dissoction = 80%

Q.39 Which of the following statements is not corect ?
(a) Helium has the lowest boiling point among the noble gases
(b) Argon is used in electric bulbs
(c) Krypton is obtained during radioactive disintegration
(d) Xe forms XeF_6

Ans: (c)

Sol: Kr is not obtained during radioactive disintegration.

Q.40 Which of the following has zero electron density in xy plane ?

(a) d_Z^2 **(b)** $d_{x^2-y^2}$ **(c)** p_z **(d)** d_{xy}

Ans: (c)

Sol: p_z orbital has zero electron density in the xy plane.

Q.41 Which of the following alkali metal carbonates gives CO_2 on heating as well as on treating with acids ?

(a) Na_2CO_3 **(b)** K_2CO_3 **(c)** Rb_2CO_3 **(d)** Li_2CO_3

Ans: (d)

Sol: All caronates evolve CO_2 on treatment with a dilute acid but amongst alkali metal carbonates only Li_2CO_3 being thermally unstable decomposes to give CO_2 gas

$$Li_2CO_3 \xrightarrow{\Delta} Li_2O + CO_2$$

Q.42 Which of the following is incorrect about thermosetting polymers ?

(a) They soften on heating and harden on cooling irreversibly

(b) By heating polymer can be reshaped and reused

(c) They possess three dimensional network structure containing cross links

(d) They are strong-hard and more brittle

Ans: (b)

Sol: Thermo-setting polymers cannot be reshped or reused.

Q.43 The process of zone refining is based upon

(a) fractional crystallisation

(b) fractional distillation

(c) magnetic properties of impurities

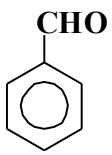
(d) impurities are less fusible than metals

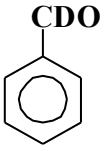
Ans: (a)

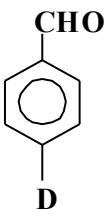
Sol: Zone refining is based on fractional crystallisation or difference in solubility of impurities in molten and solid state of metal.

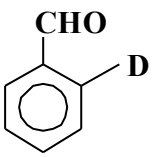
Q.44 In Gattermann-Koch reaction  **+ CO + DCl** $\xrightarrow{AlCl_3}$ **?**

The product formed is

(a) 

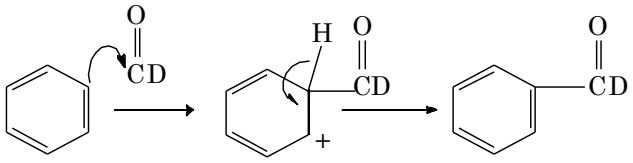
(b) 

(c) 

(d) 

Ans: (b)

Sol: $C \overset{\leftarrow}{=} O + DCl \xrightarrow{AlCl_3} D\overset{+}{C} = O + AlCl_4^-$



Q.45 The non existence of PbI_4 is due to
(a) small size of Pb^{4+} ions and large size of I^- ions
(b) Highly oxidising power of Pb^{4+} ions
(c) Highly reducing power of I^- ions
(d) Both (b) and (c)

Ans: (d)

Sol: The non existance of PbI_4 is due to stronger oxidising power of Pb^{4+} and reducing power of I^- ions.

Q.46 A metal electrode has a reduction potential of 0.136 V when measured against a standard calomel electrode (E0 calomel (oxidn) = -0.244 V). The potential of metal electrode against SHE is

Sol: $M^{n+} + ne^- \longrightarrow M \; E^0 = X$

$2Hg + 2Cl^- \longrightarrow Hg_2Cl_2 + 2e^-, E^0 = -0.244\,V$

$E^0_{M^{n+}|M} + E^0_{Cal.} = 0.136$ V $\qquad E^0_{M^{n+}|M} + (-0.244\ V) = 0.136$

$E^0_{M^{n+}|M} = 0.380$ V $\qquad E^0_{SHE} = 0.000\,V$

$E^0_{cell\ (againstSHE)} = E^0_{SHE} + E^0_{M^{n+}|M} \qquad = 0.000 + 0.380$ V $= 0.380$ V

Q.47 5.39 g of a mixture of $FeSO_4 \cdot 7H_2O$ and anhydrous ferric sulphate requires 80 mL = of 0.125 N permanganate solution for complete conversion to the ferric suphate.the individual weight of ferric sulphate in the original mixture is

Sol: Equivalent weight of $FeSO_4 \cdot 7H_2O = Mol.Wt = 278$

80 mL 0.125 (N) permanganate solution

$\equiv (80 \times 0.125)$ N solution

$= $ m eq of $FeSO_4 \cdot 7H_2O = 10$

$$\text{Weight} = \frac{10 \times 278}{1000} = 2.78\,g$$

Weight of anydrous $Fe_2(SO_4)_3 = 5.39 - 2.78 = 2.61\,g$

Q.48 A viral preparation was inactivated in a chemical bath. The inactivation process was found to be first order in virus concentration and at the begnining of the experiment 2.0% of the virus was found to be inactivated per minute. The value of k for the inactivatiion process is

Sol: $k = \frac{0.02}{60} = 3.3 \times 10^{-4}\ s^{-1}$

Q.49 At which pressure and temperature conditions is the behaviour of a real gas closest to that of an ideal gas ?

Sol: A real gas at high temperature and low pressure tends to approach ideal behaviour.

Q.50 The ionic product of water at 25⁰ C is 1×10^{-14}. The dissociation constant of water at 25°C is

Sol: $K_w = [H^+][OH^-]$; $K_d = \dfrac{[H^+][OH^-]}{[H_2O]}$

Dissociation constant of water

K_d can be expressed as $K_d = \dfrac{[H^+][OH^-]}{[H_2O]}$

H_2O can be taken as constant
constant

$K_d = [H^+][OH^-]$

$K_w = 1\times10^{-14}$

Part - C - MATHEMATICS

Q.51 If x > 1, y >1, z >1 are in GP, then $\dfrac{1}{1+\text{In x}}, \dfrac{1}{1+\text{In y}}, \dfrac{1}{1+\text{In z}}$ are in

(a) AP (b) HP (c) GP (d) None of these

Ans: (b)

Sol: Let the common ratio of the GP be r, then, $y = xr$ and $z = xr^2$

$\Rightarrow$ $\text{In y} = \text{In x} + \text{In r}$ and $\text{in z} = \text{In x} + 2\text{In r}$

Again, let $A = 1 + \text{In x}$, $D = \text{In r}$, then

$$\frac{1}{1+\text{In x}} = \frac{1}{A},$$

$$\frac{1}{1+\text{In y}} = \frac{1}{1+\text{In x}+\text{In r}} = \frac{1}{A+D}$$

and $$\frac{1}{1+\text{In z}} = \frac{1}{1+\text{In x}+2\text{In r}} = \frac{1}{A+2D}$$

Hence $\dfrac{1}{1+\text{In x}}, \dfrac{1}{1+\text{In y}}, \dfrac{1}{1+\text{In z}}$ are in HP.

Q.52 If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the intervel

(a) $(2,\infty)$ (b) (1,2) (c) (-2,-1) (d) None of the above

Ans: (a)

Sol: $\log_{0.3}(x-1) < \log_{0.09}(x-1)$

$\Rightarrow$ $x-1 > 0$

and $\log_{(0.3)}(x-1) < \log_{(0.3)^2}(x-1)$

$\Rightarrow$ $x > 1$

and $\log_{0.3}(x-1) < \dfrac{1}{2}\log_{0.3}(x-1)$

$\Rightarrow$ $x > 1$ and $\log_{0.3}(x-1) < 0$

$\Rightarrow \quad x > \text{ and } x-1 > 1$

$\Rightarrow \quad x > 1 \text{ and } x > 2$

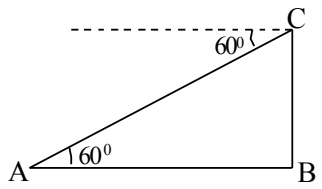
$\Rightarrow \quad x \in (2, \infty)$

Q.53 From the top of a tower, the angle of depression of a point on the ground is 60^0. If the distance of this point from the tower is $\frac{1}{\sqrt{3}+1}$ m, then the height of the tower is_____ .

(a) $\frac{4\sqrt{3}}{2}$ m **(b) $\frac{\sqrt{3}+3}{2}$ m** **(c) $\frac{3-\sqrt{3}}{2}$ m** **(d) $\frac{\sqrt{3}}{2}$ m**

Ans: (c)

Sol: Let h be the height of the tower.



$\frac{1}{\sqrt{3}+1}$

In ΔABC, $\tan 60^0 = \frac{h}{\frac{1}{(\sqrt{3}+1)}}$

$\frac{\sqrt{3}}{(\sqrt{3}+1)} = \frac{h}{1}$

$h = \frac{\sqrt{3}(\sqrt{3}-1)}{3-1}$

$= \frac{3-\sqrt{3}}{2}$ m

Q.54 If the normals from any point to the parabola $x^2 = 4y$ cuts the line y = 2 in points whose abscissae are in AP, then the slopes of the tangents at the 3 conormal points are in

(a) AP **(b) GP** **(c) HP** **(d) None of these**

Ans: (b)

Sol: The equation of the normal at any point $(2t, t^2)$ to tha parabola $x^2 = 4y$ is,

$$x + ty = 2t + t^3 \qquad \text{....(i)}$$

If it passes through $p(x_1, y_1)$ then

$$x_1 + ty_1 = 2t + t^3$$

$\Rightarrow \quad t^3 + t(2 - y_1) - x_1 = 0$

This is a cubic equation in t. so it gives three values of t say t_1, t_2, t_3.

$\therefore \quad t_1 + t_2 + t_3 = -\dfrac{\text{Coefficient of } t^2}{\text{Coefficient of } t^3} = 0$

$\Rightarrow \quad t_1^3 + t_2^3 + t_3^3 = 3t_1t_2t_3$ ....(iii)

The coordiantes of the foot of the normals are $(2t_1, t_1^2), (2t_2, t_2^2)$, and $(2t_3, t_3^3)$,

The normal in eq (i) meets $y = 2at$ $x = t^3$.

Thus, three normals form $P(x_1, y_1)$ cut the line $y = 2$ at t_1^3, t_2^3, t_3^3.

It is given that t_1^3, t_2^3 and t_3^3. are in AP.

$\therefore \quad 2t_2^3 = t_1^3 + t_3^3$

$\Rightarrow \quad 3t_2^3 = t_1^3 + t_2^3 + t_3^3$

$\Rightarrow \quad 3t_2^3 = 3t_1 \ t_2t_3$ [form Eq (ii)]

$\Rightarrow \quad t_2^2 = t_1t_3$

$\Rightarrow \quad \dfrac{1}{t_1} \times \dfrac{1}{t_3} = \left(\dfrac{1}{t_2}\right)^2$

$\Rightarrow \quad \dfrac{1}{t_1}, \dfrac{1}{t_2}, \dfrac{1}{t_3},$ are in GP.

Hence, slopes of the tangents at 3 conormal point are in GP.

Q.55 The equation $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ represents a hyperbola

(a) The length of whose transverse axis is $4\sqrt{3}$

(b) The length of whose conjugate axis is 4

(c) whose centre is (–1, 2)

(d) Whose eccentricity is $\sqrt{\dfrac{19}{3}}$.

Ans: (d)

Sol: Given that, $16(x^2 - 2x) - 3(y^2 - 4y) = 44$

$\Rightarrow \quad 16(x-1)^2 - 3(y-2)^2 = 48$

$\Rightarrow \quad \dfrac{(x-1)^2}{3} - \dfrac{(y-2)^2}{16} = 1$

This equation represents a hyperbola with eccentricity, so eccentricity is

$$e = \sqrt{1 + \left(\frac{\text{Length of conjugate axis}}{\text{Length of transverse axis}}\right)^2}$$

$$= \sqrt{1 + \left(\frac{4}{\sqrt{3}}\right)^2} = \sqrt{\frac{19}{3}}$$

Q.56 The point on the curve $3y = 6x - 5x^3$, the normal at which passes through the origin is.....?

(a) $\left(1, \frac{1}{3}\right)$ **(b)** $\left(\frac{1}{3}, 1\right)$ **(c)** $\left(2, -\frac{28}{3}\right)$ **(d)** $\left(-1, \frac{1}{3}\right)$

Ans: (a)

Sol: Let the required point be (x_1, y_1)

now $3y = 6x - 5x^3$

$\Rightarrow \quad 3\frac{dy}{dx} = 6 - 15\,x^2 \quad \Rightarrow \quad \frac{dy}{dx} = 2 - 5x^2$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 2 - 5x_1^2$

The equation of the normal at (x_1, y_1) is

$$y - y_1 = \frac{-1}{2 - 5x_1^2}(x - x_1)$$

If it passes through the origin, then

$$0 - y_1 = \frac{-1}{2 - 5x_1^2}(0 - x_1)$$

$\Rightarrow \quad y_1 = \frac{-x_1}{2 - 5x_1^2}$ ....(i)

Since, (x_1, y_1) lies on the given curve. Therefore

$$3y_1 = 6x_1 - 5x_1^3 \quad \text{....(ii)}$$

On solving Eqs (i) and (ii), we get

$$x_1 = 1 \text{ and } y_1 = \frac{1}{3}$$

Hence, the required point is $\left(1, \frac{1}{3}\right)$.

Q.57 The function $f(x) = \sqrt{\cos(\sin x)} + \sin^{-1}\left(\frac{1 + x^2}{2x}\right)$ is defined for?

(a) $x \in \{-1, 1\}$ **(b)** $x \in [-1, 1]$ **(c)** $x \in R$ **(d)** $x \in (-1, 1)$

Ans: (a)

Sol: Since, $f(x) = \sqrt{\cos(\sin x)} + \sin^{-1}\left(\frac{1 + x^2}{2x}\right)$

We know that, $\sqrt{\cos(\sin x)}$ is defined $\forall x \in R$ and $\sin^{-1}\left(\frac{1 + x^2}{2x}\right)$ is defined only for $x \in \{-1, 1\}$.

$\therefore \quad f(x)$ is defined for $x \in \{-1, 1\}$.

Q.58 Let $F(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$, where $\alpha\in R$. Then, $[F(\alpha)]^{-1}$ is equal to .....?

(a) F (2 α) (b) F (α^{-1}) (c) F ($-\alpha$) (d) None of these

Ans: (c)

Sol: Now, $F(\alpha)F(-\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\times\begin{bmatrix}\cos\alpha & +\sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$

$$=\begin{bmatrix}\cos^2\alpha+\sin\alpha & (\cos\alpha\sin\alpha-\sin\alpha\cos\alpha) & 0\\ \sin\alpha\cos\alpha-\cos\alpha\sin\alpha & \sin^2+\alpha+\cos^2\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I \qquad [\because \sin^2\theta+\cos^2\theta=1]$$

$\therefore$ $F(-\alpha)=[F(\alpha)]^{-1}$

Q.59 Ratio of the area cut off a parabola by any double ordinates is that of the corresponding rectangle contained by that double ordinate and tangent Qt vertex is

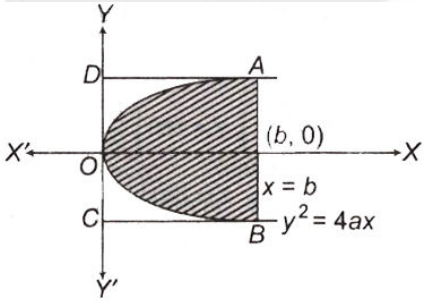
(a) 1/2 (b) 1/3 (c) 2/3 (d) 1

Ans: (c)

Sol: Let $y^2=4ax$ be a parabola and let $x=b$ be a double ordinate. Then

Let A_1 = Area enclosed by the parabola

$y^2=4ax$ and the double ordinate $x=b$



$$=2\int_0^b y\,dx=2\int_0^b\sqrt{4ax}\,dx$$

$$=4\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^b=4\sqrt{a}\times\frac{2}{3}b^{3/2}$$

$$=\frac{8}{3}a^{1/2}b^{3/2}$$

And A_2 = Area of the rectangle ABCD

$$=AB\times AD$$

$= 2\sqrt{4ab} \times b$

$= 4a^{1/2}b^{3/2}$

$\therefore \quad \frac{A_1}{A_2} = \frac{\frac{8}{3}a^{1/2}b^{3/2}}{4a^{1/2}b^{3/2}} = \frac{2}{3}$

Q.60 The interval in which the function $f(x) = xe^{2-x}$ increases is

(a) $(2, \infty)$ **(b) $(-\infty, 1)$** **(c) (0, 2)** **(d) None of thses**

Ans: (b)

Sol: Given that, $f(x) = xe^{2-x}$

$\Rightarrow \quad f'(x) = e^{2-x}(1-x)$

For $f(x)$ to be increasing, we have $f'(x) > 0$

$\Rightarrow \quad e^{2-x}(1-x) > 0$

$\Rightarrow \quad x < 1$ Hence, option (b) is correct.

Q.61 If z_1 and z_2 are two non-zero complex number such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to

(a) $-\pi$ **(b) $-\frac{\pi}{2}$** **(c) 0** **(d) $\frac{\pi}{2}$**

Ans: (c)

Sol: Since, $|z_1 + z_2| = |z_1| + |z_2|$ On squaring both sides, we get

$|z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\arg z_1 - \arg z_2)$

$= |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$

$\Rightarrow \quad 2|z_1||z_2|\cos(\arg z_1 - \arg z_2) = 2|z_1||z_2|$

$\Rightarrow \quad \cos(\arg z_1 - \arg z_2) = 1$

$\Rightarrow \quad \arg z_1 - \arg z_2 = 0$

Q.62 If B, C are square matrices of order n and if $A = B + C$, $= BC = CB$, $C^2 = 0$ then for any positive integer p, $A^{p+1} = B^k [B + (p+1)C]$, k is

(a) p +1 **(b) p** **(c) p - 1** **(d) p + 2**

Ans: (b)

Sol: We have $BC = CB$

and $A = B + C$

$\Rightarrow \quad A^{p+1} = (B+C)^{p+1} = {}^{p+1}C_0 B^{p+1} + {}^{p+1}C_1 B^p C + {}^{p+1}C_2 B^{p-1}C^2 + {}^{p+1}C_r B^{p+1-r}C^r + ...$

But $C^2 = 0 \Rightarrow C^3 = C^4 = .. = C^r = 0.$

$\therefore \quad A^{p+1} = {}^{p+1}C_0 B^{p+1} + {}^{p+1}C_1 B^p C_1$

$= B^{p+1} + (p+1)B^p C = B^p [B + (p+1)C]$

Thus, k = p

Q.63 If $\alpha+i\beta$ is one of the roots of the equation $x^3+qx+r=0$, then 2α is one of the roots of equation

(a) $x^3+qx+r=0$ (b) $x^3-px-r=0$ (c) $x^3+qx-r=0$ (d) None of these

Ans: (c)

Sol: Since, $\alpha+i\beta$ is a root of the equation $x^3+qx+r=0$, $\alpha-i\beta$ is also its roots. Let the third root be γ, so that

$\alpha+i\beta+(\alpha-i\beta)+\gamma=0$

$\Rightarrow \quad \gamma=-2\alpha$

Also, γ is the root of the given equation, therefore 2α is one of the roots of the equation

$(-x)^3+q(-x)+r=0$

$\Rightarrow \quad x^3+qx-r=0$

Q.64 Total number of solutions of $|\cot x|=\cot x+\frac{1}{\sin x}, x\in[0, 3\pi]$ is equal to

(a) 1 (b) 3 (c) 2 (d) zero

Ans: (b)

Sol: $|\cot x|=\cot x+\frac{1}{\sin x}$

Let $\cot x>0$ $\cot x=\cot x+\frac{1}{\sin x}=0$

$\Rightarrow \frac{1}{\sin x}=0$, which is not possible

Let $\cot x\le 0$

$\Rightarrow \quad -\cot x=\cot x+\frac{1}{\sin x}$

$\Rightarrow \quad -2\cot x=\frac{1}{\sin x}$

$\Rightarrow \quad \cos x=-\frac{1}{2}$

$\Rightarrow \quad x=\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$

$\therefore$ The number of solutions are 2.

Q.65 The equation of the normal to the curve $y(x-2)(x-3)-x+7=0$ at the point, where it cuts the x-axis is

(a) $20x+y=140$ (b) $20x-y=7$ (c) $x-20y=7$ (d) $20x-y=14$

Ans: (a)

Sol: **Equation of given curve is**

$y(x-2)(x-3)-x+7=0$ .....(i)

This curve cuts the x-axis, where y = 0

$\therefore \quad 0(x-2)(x-3)-x+7=0$

$\Rightarrow \quad x = 7$

The point is (7, 0)

On differentiating Eq, (i) we get

$$\frac{dy}{dx}(x-2)(x-3)+y(x-3+x-2)-1=0$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{1-y(2x-5)}{(x-2)(x-3)}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(7,\,0)}=\frac{1-0(14-5)}{(7-2)(7-3)}=\frac{1}{20}$$

$\therefore$ The equation of the normal is $y-0=-20(x-7)$ or $y+20x=140$

Q.66 **If $\int \frac{\cos 4x+1}{\cot x-\tan x}dx = A\cos 4x + B$, then**

(a) $A=-\frac{1}{2}$ **(b)** $A=-\frac{1}{8}$ **(c)** $A=-\frac{1}{4}$ **(d) None of these**

Ans: (b)

Sol: Given that,

$$\int \frac{\cos 4x+1}{\cot x-\tan x}dx = A\cos 4x + B \qquad \text{....(i)}$$

$$\text{Let } I=\int \frac{\cos 4x+1}{\cot x-\tan x}dx$$

$$=\int \frac{2\cos^2 2x}{\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}}dx=\int \frac{2\cos^2 2x}{\frac{\cos 2x}{\sin x\cos x}}dx$$

$$=\int \sin 2x\cos 2x\,dx$$

$$=\frac{1}{2}\int \sin 4x\,dx \qquad =-\frac{1}{8}\cos 4x+B$$

$\Rightarrow \quad A\cos 4x+B=-\frac{1}{8}\cos 4x+B$ [form Eq. (i)]

$\Rightarrow \quad A=-\frac{1}{8}$

Q.67 **$\cos^{-1}[\cos\{2\cot^{-1}(\sqrt{2}-1)\}]$ is equal to**

(a) $\sqrt{2}-1$ **(b)** $\pi/4$ **(c)** $3\pi/4$ **(d) None of these**

Ans: (c)

Sol: $\cos^{-1}[\cos\{2\cot^{-1}(\sqrt{+2}-1)\}]$

$$=\cos^{-1}\left[\cos\left\{2\cos^{-1}\left(\frac{\sqrt{2}-1}{\sqrt{4-2\sqrt{2}}}\right)\right\}\right]$$

$$= \cos^{-1}\left[\cos\left\{\cos^{-1}\left(\frac{2(\sqrt{2}-1)^2}{4-2\sqrt{2}}-1\right)\right\}\right]$$

$$= \cos^{-1}\left[\cos\left\{\cos^{-1}\left(\frac{6-4\sqrt{2}-4+2\sqrt{2}}{4-2\sqrt{2}}\right)\right\}\right]$$

$$= \cos^{-1}\left[\cos\left\{\cos^{-1}\left(\frac{2-2\sqrt{2}}{4-2\sqrt{2}}\right)\right\}\right]$$

$$= \cos^{-1}\left[\cos\left\{\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right\}\right]$$

$$= \cos^{-1}\left[\cos\left\{\pi-\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right\}\right]$$

$$= \cos^{-1}\left[-\cos\left\{\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right\}\right]$$

$$= \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$$

Q.68 Which statement is correct ?
(a) ac < 0 **(b) ac > 0** **(c) ab > 0** **(d) None of these**

Ans: (a)

Sol: Since, $1+e^{x^2} > 0$ and $ax^3+bx^2+cx+d=0$

For some volues of x between 0 and 1 and 1 and 2.

Now, let $a^2x^3+abx^2+acx+ad=0$ has three real roots.

Now, $3a^2x^2+2abx+ac=0$ has two real roots.

Its roots $=\dfrac{-2ab\pm\sqrt{4a^2b^2-12a^2(ac)}}{6a^2}$

Obviously both roots are real, when ac < 0 and one of them is negative.

Q.69 The equation of the common tangent to the curves $y^2=8x$ and $xy=-1$ is
(a) $3y=9x+2$ **(b) $y=2x+1$** **(c) $2y=x+8$** **(d) $y=x+2$**

Ans: (d)

Sol: Tangent to the curve $y^2=8x$ is $y=mx+\dfrac{2}{m}$.

So, it must satisfy $xy=-1$

$\therefore \quad x\left(mx+\dfrac{2}{m}\right)=-1$

$\Rightarrow \quad mx^2+\dfrac{2}{m}x+1=0$

Since, it has equal roots.

$\therefore \quad D = 0$

$\Rightarrow \quad \frac{4}{m^2} - 4m = 0$

$\Rightarrow \quad m^3 = 1 \Rightarrow m = 1$

So, equation of common tangent is $y = x + 2$.

Q.70 If the tangent from a point P to the circle $x^2 + y^2 = 1$ is perpendicular to the tangent from P to the circle $x^2 + y^2 = 3$, then the locus of P is

(a) a circle of radius 2 **(b) a circle of radius 4**
(c) a circle of radius 3 **(d) None of these**

Ans: (a)

Sol: The equation of a tangent at any point $(\cos\alpha, \sin\alpha)$ to $x^2 + y^2 - 1$

$$x\cos\alpha + y\sin\alpha = 1 \qquad \text{....(i)}$$

The equation of the tangent to $x^2 + y^2 = 3$

Perpendicular to Eq (i) is $x\sin\alpha - y\cos\alpha = \sqrt{3}$ .....(ii)

Let the coordinate of P be (h, k) then

$h\cos\alpha + k\sin\alpha = 1$ and $h\sin\alpha - k\cos\alpha = \sqrt{3}$

Eliminating (h, k) from the these two equation we gets

$$h^2 + k^2 = 4$$

So, Locus of a point (h , k) is $x^2 + y^2 = 4$ which is is a circle of radius 2.

Q.71 If n is an even natural number, then $\sum_{r=0}^{n} \frac{(-1)^r}{{}^nC_r}$ equals to_____ .

Sol: Now, $\sum_{r=0}^{n} \frac{(-1)^r}{{}^nC_r} = \sum_{r=0}^{\frac{n}{2}-1} \left[\frac{(-1)^r}{{}^nC_r} + \frac{(-1)^{n-r}}{{}^nC_{n-r}} \right] + \frac{(-1)^{n/2}}{{}^nC_{n/2}}$

$$= \sum_{r=0}^{\frac{n}{2}-1} (-1)^r \left[\frac{1}{{}^nC_r} + \frac{(-1)^n}{{}^nC_r} \right] + \frac{(-1)^{n/2}}{{}^nC_{n/2}}$$

$$= \left(\sum_{r=0}^{\frac{n}{2}-1} (-1)^r \frac{2}{{}^nC_r} \right) + \frac{(-1)^{n/2}}{{}^nC_{n/2}} = \frac{(-1)^{n/2}}{{}^nC_{n/2}}$$

Q.72 If P(x,y,z) is a point on the line segment joining Q(2,2,4) and R(3,5,6) such that the projections of OP on the axes are $\frac{13}{5}, \frac{19}{5}, \frac{26}{5}$ respectively, then P divides QR in the ratio ?

Sol: Since, OP has projections $\frac{13}{5}, \frac{19}{5}$ and $\frac{26}{5}$ on the coordinate axes, therefore

$$OP = \frac{13}{5}\hat{i} + \frac{19}{5}\hat{j} + \frac{26}{5}\hat{k},$$

Suppose, P divides the join of Q (2 ,2, 4) and R (3, 5, 6) in the ratio $\lambda : 1$, then

$$OP = \left(\frac{3\lambda+2}{\lambda+1}\right)\hat{i} + \left(\frac{5\lambda+2}{\lambda+1}\right)\hat{j} + \left(\frac{6\lambda+4}{\lambda+1}\right)\hat{k}$$

$$\therefore \quad \frac{13}{5}\hat{i} + \frac{19}{5}\hat{j} + \frac{26}{5}\hat{k} = \left(\frac{3\lambda+2}{\lambda+1}\right)\hat{i} + \left(\frac{5\lambda+2}{\lambda+1}\right)\hat{j} + \left(\frac{6\lambda+4}{\lambda+1}\right)\hat{k}$$

$$\Rightarrow \quad \frac{3\lambda+2}{\lambda+1} = \frac{13}{5}, \frac{5\lambda+2}{\lambda+1} = \frac{19}{5} \text{ and } \frac{6\lambda+4}{\lambda+1} = \frac{26}{5}$$

$$5(3\lambda+2) = 13(\lambda+1)$$

$$\Rightarrow \quad \lambda = \frac{3}{2}$$

Thus, required ratio is 3 : 2.

Q.73 Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$, **then y (1) equals**

Sol: $x^{2x} - 2x^x \cot y - 1 = 0$

Now $x = 1$

$1 - 2\cot y - 1 = 0 \Rightarrow \cot y = 0$

$$\Rightarrow y = \frac{\pi}{2}$$

Now differentiating (i) w.r.t. "x", we get

$$2x^{2x}(1+\log x) - 2$$

$$\left(x^x(-\operatorname{cosec}^2 y)\frac{dy}{dx} + \cot y x^x (1+\log x)\right) = 0$$

$$\text{Now at} \left(1, \frac{\pi}{2}\right)$$

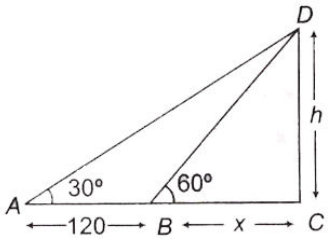
$$2(1+\log 1) - 2\left(1(-1)\left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)^{+0}}\right) = 0$$

$$\Rightarrow \ 2 + 2\left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)} = 0$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)} = -1$$

Q.74 The elevation of an object on a hill is observed from a certain point in the horizontal plane through its base, to be 30^0, After walking 120 m towards it on level ground, the elevation is found to be 60^0, Then, the height of the object (in metres) is.....

Sol: In ΔCAD, $\tan 30^0 = \dfrac{CD}{AC}$



$$\frac{1}{\sqrt{3}} = \frac{h}{120 + x}$$

$$\sqrt{3}h = 120 + x \qquad \text{.....(i)}$$

And in ΔCBD, $\tan 60^0 = \dfrac{CD}{BC}$

$$\sqrt{3} = \frac{h}{x}$$

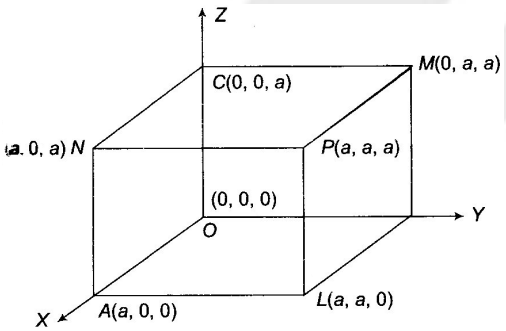
$$\Rightarrow \quad h = \sqrt{3}x \qquad \text{.....(ii)}$$

From Eq (i) and (ii) we get $x = 60\,\text{m}$ On putting $x = 60$ in Eq (i) we get

$$h = 60\sqrt{3}\,\text{m}$$

Q.75 The angle between two diagonals of a cube is.....

Sol: DC of OP are $\left(\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right)$



DC of CL are $\left(\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{-1}{\sqrt{3}}\right)$

$$\therefore \cos\theta = \frac{1+1-1}{3} = \frac{1}{3}$$

$$\Rightarrow \tan\theta = 2\sqrt{2} \Rightarrow \theta = \tan^{-1}(2\sqrt{2})$$

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