PHYSICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

01. The pitch of the screw gauge is 1 mm and there are 100 divisions on the circular scale. When nothing is put in between the jaws, the zero of the circular scale lies 8 divisions below the reference line. When a wire is placed between the jaws, the first linear scale divisions clearly visible while 72^{nd} division on circular coincides with the reference line. The radius of the wire is:

1) 0.90 mm **2)** 1.64 mm **3)** 0.82 mm **4)** 1.80 mm

Key:3

Solution: Diameter $= MSR + LC \times$ corrected HSR

$= 1 + 0.01(72 - 8)$

$= 1.64\,mm$ $\therefore$ radius, $r = 0.82\,mm$

02. An engine of a train, moving with uniform acceleration, passes the signal post with velocity u and the last compartment with velocity υ. The velocity with which middle point of the train passes the signal post is:

1) $\frac{\upsilon - u}{2}$ **2)** $\frac{u + \upsilon}{2}$ **3)** $\sqrt{\frac{\upsilon^2 - u^2}{2}}$ **4)** $\sqrt{\frac{\upsilon^2 + u^2}{2}}$

Key:4

Solution: $v^2 - u^2 = 2as \Rightarrow v^2 - u^2 \propto s$

$$\frac{\ell}{\ell / 2} = \frac{v^2 - u^2}{v_1^2 - u^2} \Rightarrow v_1 = \sqrt{\frac{v^2 + u^2}{2}}$$

03. An α particle and proton are accelerated from rest by a potential difference of 200 V. after this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$ is:

1) 2.8 **2)** 7.8 **3)** 8 **4)** 3.8

Key:1

Solution:

Kinetic energy $K = \frac{\rho^2}{2m} = vq$, $p = \sqrt{2mvq}$, $p\alpha\sqrt{mq}$

$\lambda = \frac{h}{p} \Rightarrow \lambda \infty \frac{1}{p} \Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \frac{p_\alpha}{p_p} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4\times 2}{1\times 1}}$ $\frac{\lambda_p}{\lambda_\alpha} = 2.8$

04. Two coherent light sources having intensity in the ratio $2x$ produce an interference pattern. The ratio $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ will be:

1) $\frac{\sqrt{2x}}{x+1}$ **2)** $\frac{2\sqrt{2x}}{x+1}$ **3)** $\frac{\sqrt{2x}}{2x+1}$ **4)** $\frac{2\sqrt{2x}}{2x+1}$

Key:4

Solution: $\frac{I_1}{I_2} = 2x, I_1 = 2xI_2$

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ $\therefore \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{4\sqrt{I_1 I_2}}{2(I_1 + I_2)} = \frac{2\sqrt{2x}}{(2x+1)}$

05. Match list – I with list – II :

Column-I		Column-II	
A)	h(Planck's constant)	i)	$\left[M\, L\, T^{-1}\right]$
B)	E (kinetic energy)	Ii)	$\left[M\, L^2\, T^{-1}\right]$
C)	V (electric potential)	iii)	$\left[M\, L^2\, T^{-2}\right]$
D)	P (linear momentum)	iv)	$\left[M\, L^2 I^{-1}\, T^{-3}\right]$

Choose the correct answer from the options given below:

1) A-iii;B-iv;C-ii;D-i **2)** A-i;B-ii;C-iv;D-iii

3) A-ii; B-iii;C-iv;D-i **4)** A-iii;B-ii;C-iv;D-i

Key:3

Solution: (a)Planck's constant $E = h\upsilon$

$$\upsilon = \frac{1}{T} = T^{-1}$$

$$h = \frac{E}{\upsilon} = \frac{ML^2T^{-2}}{T^{-1}}$$

(b) Kinetic energy $E=\frac{1}{2}mV^2 \Rightarrow \left[ML^2T^{-2}\right]$

(c) Electric potential $r=\frac{\omega}{q}=\frac{ML^2T^{-2}}{AT}=\left[ML^2T^{-3}A^{-1}\right]$

$i=\frac{q}{t} \Rightarrow q=it$

$q=[AT]$ $\qquad \left[ML^2I^{-1}T^{-3}\right]$

(d) linear momentum

$p=mv \Rightarrow \left[MLT^{-1}\right]$

06. Given below are two statements : one is labelled as Assention A and the other is labelled as Reason R.

Assention A: when a rod lying freely is heated, no thermal stress is developed in it

Reason R : On heating, the length of the rod increases

In the light of the above statements, choose the correct answer from the options given below:

1) A is true but R is false

2) Bothe A and B are true but R is NOT the correct explanation of A

3) Both A and R are true and R is the correct explanation of A

4) A is false but R is true

Key:2

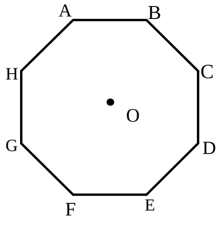
Solution:

Stress is developed only if the expansion is hindered both A and R are true but Reason not the correct explanation of A

07. In an octagon ABCDEFGH of equal side, what is the sum of

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}$,

If, $\overrightarrow{AO}=2\hat{i}+3\hat{j}-4\hat{k}$



1) $16\hat{i}+24\hat{j}-32\hat{k}$ **2)** $16\hat{i}+24\hat{j}+32\hat{k}$

3) $16\hat{i}-24\hat{j}+32\hat{k}$ **4)** $-16\hat{i}-24\hat{j}+32\hat{k}$

Key:4

Solution:

$$\frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}+\vec{e}+\vec{f}+\vec{g}+\vec{h}}{8}=0$$

$$\vec{b}+\vec{c}+\vec{d}+\vec{c}+\vec{f}+\vec{g}+\vec{h}=-\vec{a}$$

$$\overline{AB}+\overline{AC}+\overline{AD}+\overline{AE}+\overline{AF}+\overline{AG}+\overline{AH}$$

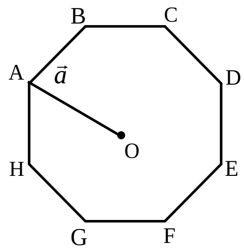
$$\vec{b}-\vec{a}+\vec{c}-\vec{a}+\vec{d}-\vec{a}+\vec{e}-\vec{a}+\vec{f}-\vec{a}+\vec{g}-\vec{a}+\vec{h}-\vec{a}$$

$$\vec{b}+\vec{c}+\vec{d}+\vec{e}+\vec{f}+\vec{g}+\vec{h}-7\vec{a}$$

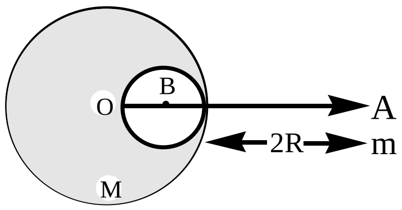
$$-\vec{a}-7\vec{a}=-8\vec{a}$$

$$=-8\left(\overrightarrow{OA}\right)=-8\times 2\hat{i}+3\hat{j}-4\hat{k}$$

$$=-16\hat{i}-24\hat{j}+32\hat{k}$$



08. A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is:



1) 50:41 **2)** 25:36 **3)** 41:50 **4)** 36:25

Key:1

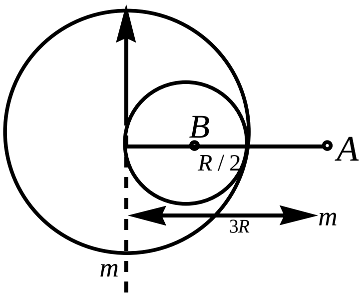
Solution:

$$F_1=\frac{GMm}{(3R)^2}=\frac{GMm}{9R^2} \qquad (1)$$

$$F_2=\frac{GMm}{9R^2}=\frac{G\left(\frac{m}{8}\right)m}{\left(\frac{5R}{2}\right)^2}$$

$$F_2=\frac{GMm}{9R^2}-\frac{GMm}{50R^2}\Rightarrow\frac{GMm}{R^2}\left(\frac{1}{a}-\frac{1}{50}\right)=\frac{41}{50}\times\frac{GMm}{R^2} \qquad (2)$$

(1) & (2) $\frac{F_1}{F_2} = \frac{GMm}{9R^2}$ $\frac{41}{50}\frac{GMm}{9R^2}$. $= \frac{50}{41}$



Let the particle of mass m be place θ on A $F_1 = \frac{Gmm}{(2R)^2} = \frac{GMm}{4R^2}$

when a spherical part of radius $\frac{R}{2}$ is taken then the mass of remaining spheric becomes

$$\left(\frac{4\pi R^3}{3} - \frac{4\pi}{3}\left(\frac{R}{2}\right)^3\right)d = \frac{4\pi R^3}{3}\left(1 - \frac{1}{8}\right) = \frac{7}{8}\frac{4\pi R^3}{3}$$

Now force on m place at A $F_2 = -\frac{GMm}{4R^2}$

09. If the time period of a two meter long simple pendulum is 2s, the acceleration due to gravity at the place where pendulum is executing S.H.M. is:

1) $2\pi^2 ms^{-2}$ **2)** $\pi^2 ms^{-2}$ **3)** $16m/s^2$ **4)** $9.8ms^{-2}$

Key:1

Solution: $T = 2\pi\sqrt{\frac{\ell}{g_{pla}}}$ $2 = 2\pi\sqrt{\frac{\ell}{g_{pla}}}$ $s.q.s$

$g = \pi^2\ell$ $g \Rightarrow 2\pi^2 m/\sec^2$

10. Given below are two statements : one is labeled as Assertion A and the other is labeled as Reason R.

Assertion A: The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason R: The product of their mass and radius must be same. $M_1R_1 = M_2R_2$

In the light of the above statements, choose the most appropriate answer from the options given below:

1) A is not correct but R is correct

2) Both A and R are correct and R is correct explanation of A

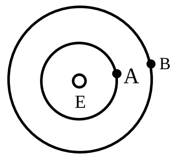
3) Both A and R are correct but R is NOT the correct explanation of A

4) A is correct but R is not correct.

Key:2

Solution: According to Kepler's law

11. Two satellites A and B of masses 200 kg and 400 kg are revolving round the earth at height of 600 km and 1600 km respectively.
If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$:



[Given : radius of earth = 6400 km, mass of earth $= 6\times10^{24} kg$]

1) $4.24\times10^{2} s$ **2)** $1.33\times10^{3} s$ **3)** $3.33\times10^{2} s$ **4)** $4.24\times10^{3} s$

Key:2

Solution: $V = \sqrt{\dfrac{2GMe}{r}}$

$$T = \frac{2\pi r}{\sqrt{\frac{2GMe}{r}}} = 2\pi r\sqrt{\frac{r}{2GMe}} \qquad T = \sqrt{\frac{4\pi^2 r^3}{2GMe}} = \sqrt{\frac{2\pi^2 r^3}{GMe}}$$

$$T_B - T_A = \sqrt{\frac{2\pi^2\times\left[8000\times10^3\right]^3}{6.67\times10^{-11}\times6\times10^{24}}} - \sqrt{\frac{2\pi^2\left(7000\times10^3\right)^3}{6.67\times10^{-11}\times6\times10^{24}}}$$

$$= \sqrt{\frac{19.7192\times512\times10}{40\times10^{13}}} - \sqrt{\frac{19.71\times343\times10}{40\times10^{13}}}$$

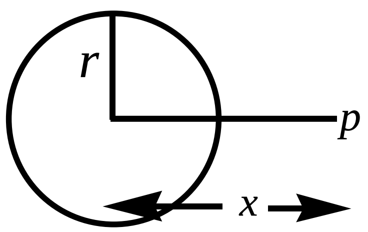
$$= \sqrt{256\times10^5} - \sqrt{171.5\times10^5} \qquad = \sqrt{25.6\times10^6} - \sqrt{17.15\times10^6}$$

$$= 5.0596\times10^3 - 4.1412\times10^3 \qquad = 0.6476\times10^3 = 6.47\times10^2$$

12. Magnetic fields at two points on the axis of a circular coil at a distance of 0.05 m and 0.2 m from the centre are in the ratio 8:1. The radius of coil is________

1) 0.2 m **2)** 0.15 m **3)** 1.0 m **4)** 0.1 m

Key:4



Solution:

$$B_p = \frac{\mu_0 i}{2}\frac{r^2}{\left(r^2+x^2\right)^{\frac{3}{2}}}$$

$$B_{0.05} = \frac{\mu_0 i}{2} \times \frac{r^2}{\left(r^2 + (0.05)^2\right)^{\frac{3}{2}}} \qquad (1)$$

$$B_{0.2} = \frac{\mu_0 i}{2} \times \frac{r^2}{\left[r^2 + (0.2)^2\right]^{\frac{3}{2}}} \qquad (2)$$

$$\frac{(1)}{(2)} \; \frac{B_{0.05}}{B_{0.2}} = \left[\frac{r^2 + (0.2)^2}{r^2 + (0.05)^2}\right]^{\frac{3}{2}} \qquad \left(\frac{8}{1}\right)^{\frac{2}{3}} = \frac{r^2 + (0.2)^2}{r^2 + (0.05)^2}$$

$$4\left(r^2 + (0.05)^2\right) = r^2 + (0.2)^2 \quad 3r^2 = (0.2)^2 - 4 \times (0.05)^2 = (0.2)^2 - (2 \times 0.05)^2$$

$$3r^2 = (0.02)^2 - (0.1)^2 = 0.04 - 0.01 \quad r^2 = \frac{0.03}{3} = 0.01 \quad r = 0.1\,m$$

13. Two radioactive substances X and Y originally have N_1 and N_2 nuclei respectively. Half life of X is half of the half life of Y. After there half lives of Y, number of nuclei of both are equal. The ratio $\frac{N_1}{N_2}$ will be equal to:

1) $\frac{1}{3}$ **2)** $\frac{8}{1}$ **3)** $\frac{3}{1}$ **4)** $\frac{1}{8}$

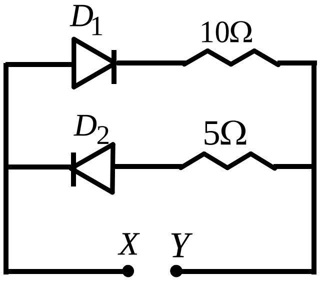
Key:2

Solution: $T_x = \frac{T_y}{2}$ $\quad \frac{1}{\lambda_x} = \frac{1}{2\lambda_y}$ $\quad \lambda_x = 2\lambda_y$ $\quad t = 3T_y$ $\quad N_x = N_1 e^{-\lambda_x 3T_y}$

$N_y = N_2 e^{-\lambda_y 3T_y}$ $\quad N_x = N_y$ $\quad N_1 e^{-\lambda_x 3T_y} = N_2 e^{-\lambda_y 3T_y}$

$N_1 e^{-\lambda_x \times 3 . \frac{\ln(2)}{\lambda_y}} = N_2 e^{-\lambda_y \frac{3\ell n(2)}{\lambda_y}}$ $\quad N_1 e^{-6\ell n(2)} = N_2 e^{-3\ell n(2)}$ $\quad \frac{N_1}{N_2} = e^{3\ell n(2)} = 8$

14. A 5 V battery is connected across the points X and Y. Assume D_1 and D_2 to be normal silicon diodes. Find the current supplied by the battery if the $+ve$ terminal of the battery is connected to point X.



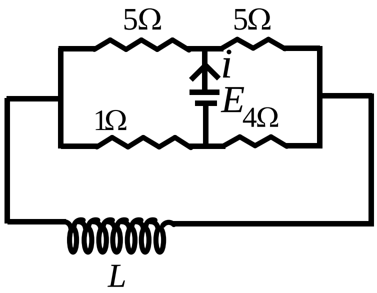
1) $\sim 0.43\,A$ **2)** $\sim 0.5A$ **3)** $\sim 1.5A$ **4)** $\sim 0.86A$

Key:1

Solution: Diode $'D_2'$ is in reverse bias; $Si-$ potential barier $+0.7V$

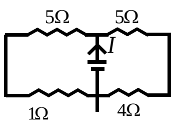
$$i=\frac{(V-V_2)}{R}=\frac{5-0.7}{10}=0.43A$$

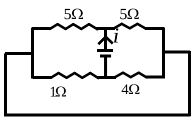
15. The current (i) at time $t=0$ and $t=\infty$ respectively for the given circuit is:



1) $\frac{5E}{18},\frac{18E}{55}$ **2)** $\frac{5E}{18},\frac{10E}{33}$ **3)** $\frac{18E}{55},\frac{5E}{18}$ **4)** $\frac{10E}{33},\frac{5E}{18}$

Key:2

Solution: At $t=0$  $I(t=0)=\frac{\in\times 15}{6\times 9}=\frac{5E}{18}$

At $t=\infty$  $I(t=\infty)=\frac{E}{\frac{5}{2}+\frac{y}{5}}=\frac{10E}{33}$

16. A student is performing the experiment of resonance column. The diameter of the column tube is 6 cm. The frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336m/s. The zero of the metre scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is:

1) 18.4 cm **2)** 13 cm **3)** 16.6 cm **4)** 14.8 cm

Key:4

Solution: $\lambda=\frac{v}{n}=\frac{336}{504}=66.66\ cm$, $\frac{\lambda}{4}=\ell+e=\ell+0.3d=\ell+1.8$

$16.66=\ell+1.8 \qquad \ell=14.86\ cm$

17. A proton, a deuteron and an α particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces acting on them is______ and their speed is _____ in the ratio.

1) 1:2:4 and 2:1:1 **2)** 1:2:4 and 1:1:2

3) 4:2:1 and 2:1:1 **4)** 2:1:1 and 4:2:1

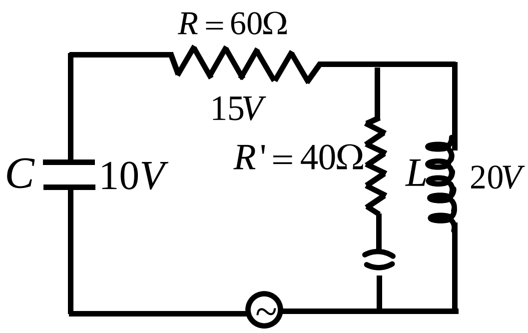
Key:4

Solution: $F = qVB = \frac{qPB}{m}$ $\quad V = \frac{p}{m}$

$v_1, v_2, v_3 = \frac{q_1}{m_1} : \frac{q_2}{m_2} : \frac{q_3}{m_3}$ $\quad \frac{q}{m} : \frac{q}{2m} : \frac{2q}{4m}$

$F_1 : F_2 : F_3 \Rightarrow 2:1:1$ $\quad v_1 : v_2 : v_3 = 4:2:1$

18. The angular frequency of alternating current in a L.C.R circuit is 100 rad/s. The components connected are shown in the figure. Find the value of inductance of the coil and capacity of condenser.



1) $0.8H \; and \; 250\,\mu F$ **2)** $1.33H \; and \; 250\,\mu F$

3) $1.33H \; and 150\,\mu F$ **4)** $0.8H \; and \; 150\mu F$

Key:1

Solution: Since, key is open

$15 = i_{rms}(60) \quad (v = iR)$ $\quad i_{rms} = \frac{15}{60}$

$i_{rms} = \frac{1}{4}A$ $\quad$ Now $20 = \frac{1}{4}(X_L)$ $[v = i\,X_L]$

$20 = \frac{1}{4}(\omega L)$ $\quad 20 = \frac{1}{4}(100L)$ $\quad L = \frac{20}{25}$

$L = \frac{4}{5}$ $\quad L = 0.8\ H$ $\quad$ And $10 = \frac{1}{4}(X_C)$ $[v = iX_C]$

$10 = \frac{1}{4}\left(\frac{1}{\omega C}\right)$ $\quad 10 = \frac{1}{4}\left(\frac{1}{100C}\right)$ $\quad C = \frac{1}{4 \times 10^3}$

$C = 0.25 \times 10^{-3} F$ $\quad C = 250 \times 10^{-6} F$ $\quad C = 250\,\mu F$

19. Given below are two statements:

Statement I: A speech signal of 2 kHz is used to modulate a carrier signal of 1 MHz. The bandwidth requirement for the signal is 4 kHz.

Statement II: The side band frequencies are 1002 kHz and 998 kHz.

In the light of the above statements, choose the correct answer from the options given below:

1) Statement I is false but statement II is true

2) Both statement I and statement II is true

3) Statement I is true but statement II is false

4) Both statement I and statement II are false

Key:2

Solution: $V.S.B = f_C + f_m$

$L.S.B = f_C - f_m$

$B.w = f_c + f_m - (f_c - f_m)$

$B.w = f_c + f_m - f_c + f_m$

$B.w = 2f_m$

$B.w = 4kHz$

$V.S.B = 1000 + 2 = 1002\,\mathbf{kHz}$

$L.S.B = 1000 - 2 = 998\ kHz$

20. A diatomic gas, having $C_p = \frac{7}{2}R$ and $C_V = \frac{5}{2}R$, is heated at constant pressure. The ratio $dU : dQ; dW$:

1) 5:7:2 **2)** 5:7:3 **3)** 3:5:2 **4)** 3:7:2

Key:1

Solution: $dU = nc_V dT = n\left(\frac{5}{2}\right)R\Delta T$

$dQ = nC_p dT = n\left(\frac{7}{2}\right)R\Delta T$

$dW = nR\Delta T = nR\Delta T$

$dU : dQ : dw = n\left(\frac{5}{2}\right)R\Delta T : n\left(\frac{7}{2}\right)RDT : nR\Delta T$

$= \frac{5}{2} : \frac{7}{2} : 1$

$= 5 : 7 : 2$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. A monoatomic gas of mass 4.0 u is kept in an insulated container. Container is moving with velocity 30 m/s. If container is suddenly stopped then change in temperature of the gas ($R =$ gas constant) is $\frac{x}{3R}$. Value of x is_________

Key: 3600.00

Solution: $KE = \frac{1}{2}mv_0^2 \quad \frac{3}{2}KT = \frac{1}{2}nmv_0^2$

$\frac{3}{2}nRT = \frac{1}{2}nmv_0^2 \quad \Delta T = \frac{mv_0^2}{3R}$

$\Delta T = \frac{4(900)}{3R} = \frac{1}{3R} \qquad x = 3600$

22. In a certain thermodynamical process, the pressure of a gas depends on its volume as kV^3. The work done when the temperature changes from 100^0C to 300^0C will be_______ nR.

Key:50

Solution: $pv^{-3} = k$

Polytropic process

$x = -3 \qquad w = \frac{-nR(\Delta T)}{x-1} = \frac{-nR(200)}{-3-1} \qquad w = 50nR$

23. The electric field in a region is given by $\vec{E} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right)\frac{N}{C}$. The ratio of flux of reported field through the rectangular surface of area $0.2m^2$ (parallel to $y-z$ plane) to that of the surface of area $0.3m^2$ (parallel to $x-z$ plane) is $a:b$, where $a =$_____.
[Here $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and $z-$axis respectively)

Key:1

Solution: $\phi_1 = \frac{3}{5}(0.2)\in_0 \quad \phi_2 = \frac{4}{5}(0.3)\in_0 \qquad \frac{\phi_1}{\phi_2} = \frac{0.6}{1.2} = \frac{1}{2} \qquad \frac{a}{b} = \frac{1}{2} \qquad a = 1$

24. A small bob tied at one end of a thin string of length 1 m is describing a vertical circle so that the maximum and minimum tension in the string are in the ratio 5:1. The velocity of the bob at the highest position is_________ m/s. (Take $g = 10m/s^2$)

Key:5

Solution: $T_{\max}=\frac{mv^2}{\ell}+mg$, $T_{\min}=\frac{m}{\ell}\left(v^2-4g\ell\right)-mg$, $\frac{5}{1}=\frac{\frac{v^2}{\ell}+g}{\frac{v^2}{\ell}-5g}$ $v^2=\frac{13g\ell}{2}$

$v_H^2=\frac{13g\ell}{2}-4gl$, $v_H^2=5g\ell/2$ $\quad v=5$

25. The potential energy (U) of a diatomic molecule is a function dependent on r (inter atomic distance) as

$$U=\frac{\alpha}{r^{10}}-\frac{\beta}{r5}-3$$

Where, α and β are positive constants. The equilibrium distance between two atoms will be $\left(\frac{2\alpha}{\beta}\right)^{\frac{a}{b}}$, where $a=$ ________ .

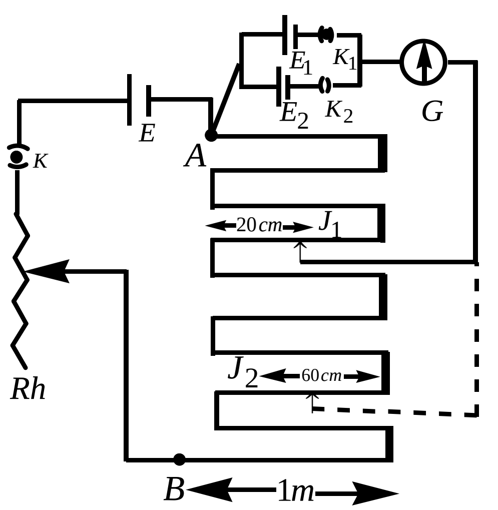
Key: 1

Solution: $u=\frac{\alpha}{r^{10}}-\frac{\beta}{r^5}-3$. At equilibrium $F=0$

$F=\frac{-du}{dr}=\frac{10\alpha}{r^{11}}-\frac{5\beta}{r^6}-0$, $D=\frac{10\alpha}{r^{11}}-\frac{5\beta}{r^6}$, $\frac{10\alpha}{r^{11}}=\frac{5\beta}{r^6}$ $\quad \alpha=\frac{\beta}{2}r^5$

$\frac{2\alpha}{\beta}=r^5$ $\quad r=\left(\frac{2\alpha}{\beta}\right)^{1/5}$

26. In the given circuit of potentiometer, the potential difference E across AB (10 m length) is larger than E_1 and E_2 as well. For key K_1 (closed), the jockey is adjusted to touch the wire at point J_1 so that there is no reflection in the galvanometer. Now the first battery (E_1) is replaced by second battery (E_2) for working by making K_1 open and K_2 closed. The galvanometer gives then null deflection at J_2. The value of $\frac{E_1}{E_2}$ is $\frac{a}{b}$, where $a=$ ______ .



Key:2

Solution: $\frac{E_2}{E_1}=\frac{I_2}{I_1} \quad \Rightarrow \quad \frac{760}{380} \quad \Rightarrow 2$

27. A transmitting station releases waves of wavelength 960m. A capacitor of $2.56\mu F$ is used in the resonant circuit. The self inductance of coil necessary for resonance is_____ $\times 10^{-8} H$.

Key:10

Solution: $\omega_r=\frac{1}{\sqrt{LC}}$, $2\pi f=\frac{1}{\sqrt{LC}}$. $4\pi^2\frac{C^2}{\lambda^2}=\frac{1}{LC}$

$$4\pi^2\times\frac{9\times10^8\times10^8}{960\times960}=\frac{1}{L\times2.56\times10^{-6}},\ L=10\times10^{-8}$$

28. 512 identical drops of mercury are charged to a potential of 2V each. The drops are joined to form a single drop. The potential of this drop is______ V.

Key:128

Solution: $V_{big}=512\,V_{small}$, $\frac{4}{3}\pi R^3=8^3\frac{4}{3}\pi r^3$, $R=8r$, $v_{real}=\frac{Kq}{r}$

$$v_{big}=\frac{Kq'}{R}\ q'=512q,=\frac{K\times512q}{8r},\quad v_{big}=\frac{512}{8}\frac{Kq}{r}\quad =64\ v_{small}\ =64\times2$$

$v_{big}=128\ volt$

29. A coil of inductance 2 H having negligible resistance is connected to a source of supply whose voltage is given by $V=3t\,volt$ (where t is in second). If the voltage is applied when $t=0,$ then the energy stored in the coil after 4 s is_____ J.

Key:144

Solution: $V=L\frac{di}{dt}$, $\quad i=\int_0^9\frac{3t}{2}dt=\left(\frac{3t^2}{4}\right)_0^4=\frac{3}{4}\times4\times4$

$$i=12,\ E=\frac{1}{2}Li^2\quad=\frac{1}{2}\times2\times(12)^2=144J$$

30. The same size images are formed by a convex lens when the object is placed at 20 cm or at 10 cm from the lens. The focal length of convex lens is________ cm.

Key: 15

Solution: $-\left|\frac{f}{f+u}\right|=\left|\frac{f}{f+u}\right|-(f+u)=(f+u)-(f-20)=(f-10)-f+20=f-10$

$2f=30,\qquad f=\frac{30}{2}$

$=15\ cm$

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. Which of the glycosidic linkage between galactose and glucose is present in lactose?

1) C-1 of galactose and C-4 of glucose

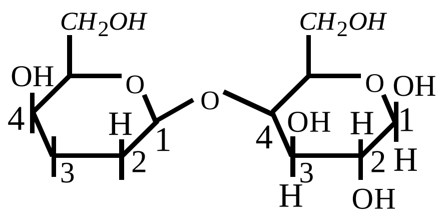
2) C-1 of galactose and C-6 of glucose

3) C-1 of glucose and C-4 of galactose

4) C-1 of glucose and C-6 of galactose

Key: 1

Solution:



galactose glucose

$C_1 -$ galactose $C_4 -$ of glucose

32. Given below are two statements:

Statement I : CeO_2 Canbe used for oxidation of aldehydes and ketones.

Statement II :Aqueous solution of $EuSO_4$ is a strong reducing agent.

In the light of the above statement, choose the correct answer from the options given below :

1) Statement I is false but statement II is true

2) Both Statement I and Statement II are false

3) Statement I is true but Statement II is false

4) Both Statement I and Statement II are true

Key: 4

Solution: $CeO_2 \rightarrow Ce^{+4} \rightarrow Ce^{+3}$ strong oxidizing agent

$EuSO_4 \rightarrow Eu^{+2} \rightarrow Eu^{+3}$ strong reducing agent

Sina Lanthanide $+3$ state is more stable.

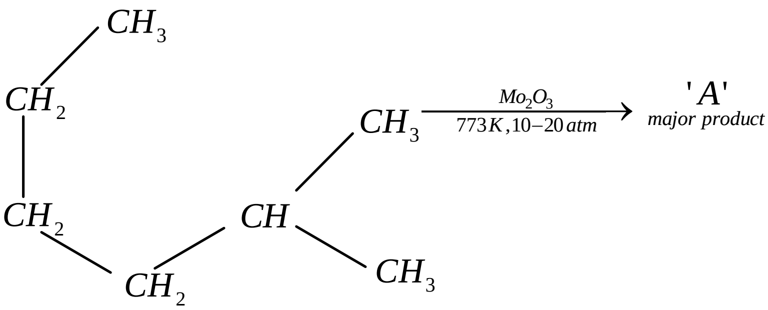
33. In Freundlich isotherm at moderate pressure, the extent of adsorption $\left(\frac{x}{m}\right)$ is directly proportional to p^x. The value of x is :

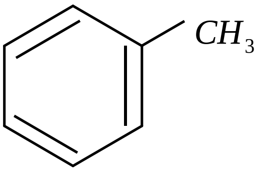
1) α **2)** Zero **3)** 1 **4)** $\frac{1}{n}$

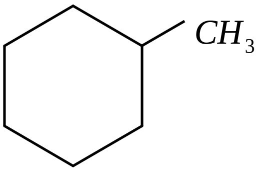
Key: 4

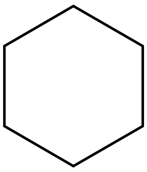
Solution: $\frac{\lambda}{m} = K.P^{\frac{1}{n}}$

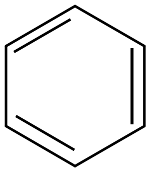
34. Identify A in the given chemical reaction



1) 

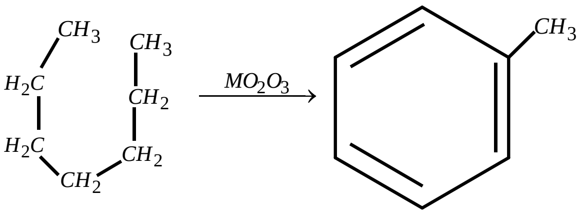
2) 

3) 

4) 

Key:1

Solution: Aromatiston, dehydrogenation & cyclistion



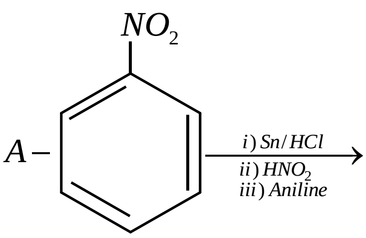
35. Ellingham diagram is a graphical representation of :

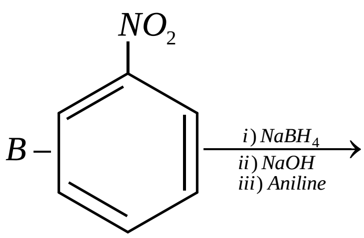
1) $\Delta H\ vs\ T$ **2)** $\Delta G\ vs\ T$ **3)** $(\Delta G - T\Delta S)\ vs\ T$ **4)** $\Delta G\ vs\ P$

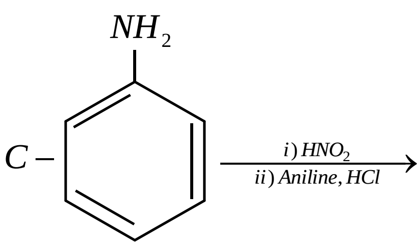
Key:2

Solution: In Elinghan diagram $\Delta G\ vs\ T$

36. Which of the following reaction/s will not give p-aminoazobenzene?





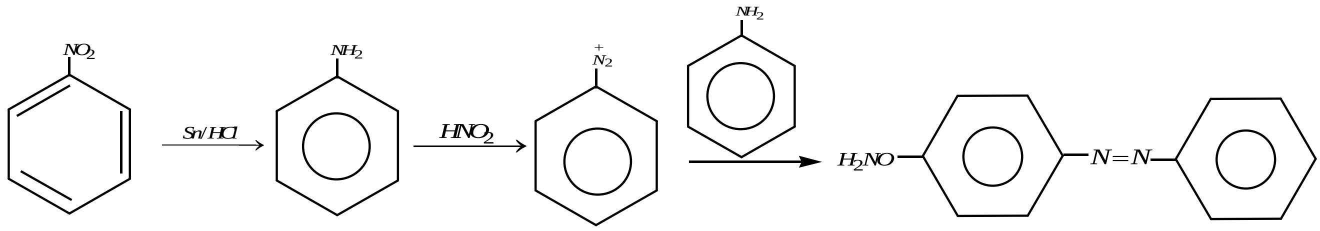


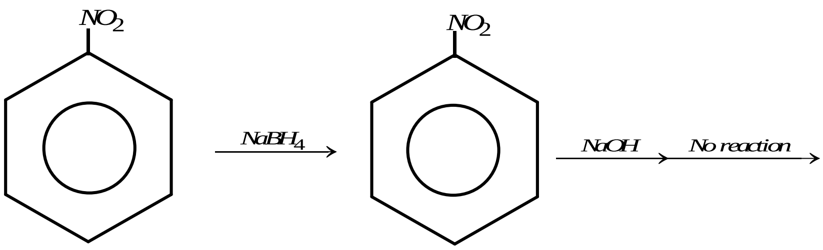
1) A and B **2)** C only **3)** B only **4)** A only

Key:3

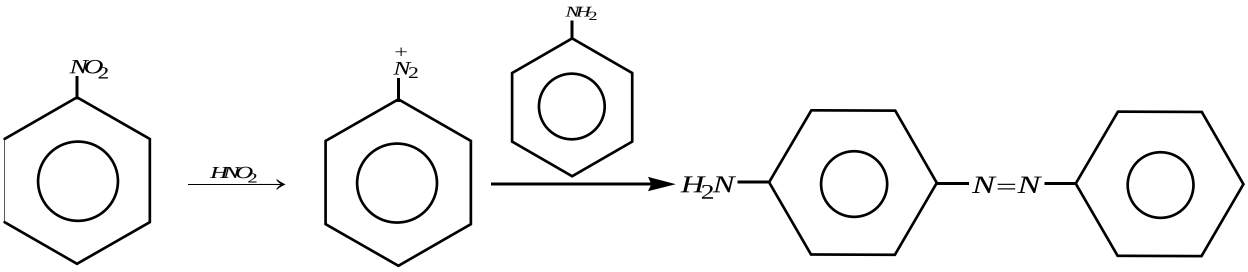
Solution:

A)





B)



C)

37. In which of the following pairs, the outer most electronic configuration will be the same?

1) Fe^{2+} and Co^{+} **2)** Ni^{2+} and Cu^{+} **3)** V^{2+} and Cr^{+} **4)** Cr^{+} and Mn^{2+}

Key:4

Solution: $Cr^{+} \Rightarrow [Ar]3d^{5}, Mn^{+2} \Rightarrow [Ar]3d^{5}$

38. The correct statement about B_2H_6 is :

1) All B – H – B angles are of 120^{o}

2) The two B – H – B bonds are not of same length.

3) Its fragment, BH_3 behaves as a Lewis base.

4) Terminal B – H bonds have less p – character when compared to bridging bonds.

Key:4

Solution: $B-H$ [terminal] having less p character as compared to bridge bond.

$B-H-B$ bridge bond having same bond length.

$B-H-B$ Bond angle $=90^{0}$

BH_3 is acts as lewis acid.

39. Which one of the following reactions will not form acetaldehyde?

1) $CH_2=CH_2+O_2 \xrightarrow[H_2O]{Pd(II)/Cu(II)}$ **2)** $CH_3CN \xrightarrow[ii)H_2O]{i)DIBAL-H}$

3) $CH_3CH_2OH \xrightarrow{CrO_3-H_2SO_4}$ **4)** $CH_3CH_2OH \xrightarrow[573K]{Cu}$

Key:3

Solution: $CH_3-CH_2-OH \xrightarrow[\text{strong oxidising agent}]{CrO_3.H_2SO_4} CH_3-\overset{\overset{O}{\|}}{C}-OH$

$$CH_3CH_2OH \xrightarrow[573K]{Cu} CH_3-\overset{\overset{O}{\|}}{C}-H$$

$$CH_3CN \xrightarrow[(2)510]{(i)DiBAC-H} CH_3-\overset{\overset{O}{\|}}{C}-H$$

$$CH_2=CH_2+O_2 \xrightarrow[H_2O]{Pd(II)\int Cu(II)} CH_3-\overset{\overset{O}{\|}}{C}-H.$$

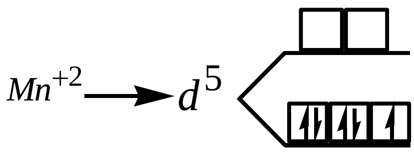
40. The hybridization and magnetic nature of $[Mn(CN)_6]^{4-}$ and $[Fe(CN)_6]^{3-}$ respectively are

1) sp^3d^2 and diamagnetic **2)** d^2sp^3 and paramagnetic

3) sp^3d^2 and paramagnetic **4)** d^2sp^3 and diamagnetic

Key:2

Solution: $\left[Mn(CN)_6\right]^{-4}$

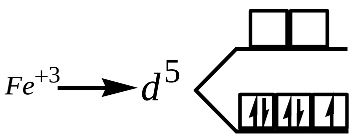


$CN^- - SFL$

$A_0 > P$

Hyb is d^2Sp^3 and paramagnetic.

$\left[Fe(CN)_6\right]^{3-}$



$CN^- - SFL$

$A_0 > P$

Hyb is d^2Sp^3 and paramagnetic

41. Given below are two statements :

Statement I : An allotrope of oxygen is an important intermediate in the formation of reducing smog.

Statement II : Gases such as oxides of nitrogen and sulphur present in troposphere contribute to the formation of photochemical smog. In the light of the above statements, choose the correct answer from the options given below:

1) Both statement I and Statement II are true

2) Statement I is true but Statement II is false

3) Both Statement I and Statement II are false

4) Statement I is false but Statement II is true

Key:3

Solution: Reducing smog in a mixture of smoke, fog and SO_2.

Tropospheric pollutants such as hydrocarbon and Nitrogen oxide contribute to the formation of photo chemical smog.

42. Which of the following equation depicts the oxidizing nature of H_2O_2?

1) $Cl_2 + H_2O_2 \rightarrow 2HCl + O_2$

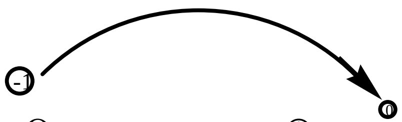
2) $2I^- + H_2O_2 + 2H^+ \rightarrow I_2 + 2H_2O$

3) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

4) $KIO_4 + H_2O_2 \rightarrow KIO_3 + H_2O + O_2$

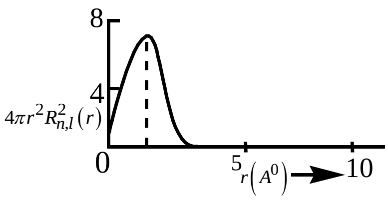
Key:2

Solution:

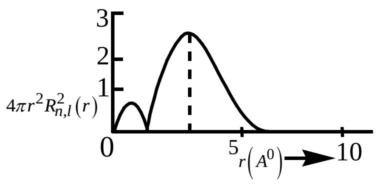


$$2I^{\Theta} + H_2O_2 + 2H^{\oplus} \rightarrow I_2 + 2H_2O$$

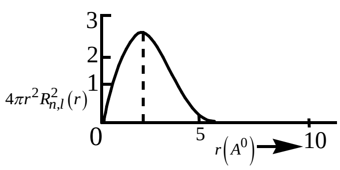
43. The plots of radial distribution functions for various orbitals of hydrogen atom against '*r*' are given below:



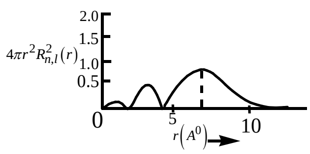
A)



B)



C)



D)

1) (C) **2)** (A) **3)** (B) **4)** (D)

Key:4

Solution:

No. of peaces $n - \ell$

44. Complete combustion of 1.80 g of an oxygen containing compound $\left(C_xH_yO_z\right)$ gave 2.64 g of CO_2 and 1.08 g of H_2O. The percentage of oxygen in the organic compound is :

1) 51.63 **2)** 53.33 **3)** 63.53 **4)** 50.33

Key:2

Solution: $\%C = \frac{12}{44} \times \frac{2.64}{1.8} \times 100 = 40$

$\% H = \frac{2}{18} \times \frac{1.08}{1.80} \times 100 = 6.66$ $\quad\quad$ $\% O = 100 - [40 + 6.66] = 53.34$

45. The solubility of $AgCN$ in a buffer solution of $pH = 3$ is x. The value of x is:

$\left[Assume : No\ cyano\ complex\ is\ formed : K_{sp}(AgCN) = 22x10^{-16}\ and\ K_a(HCN) = 6.2x10^{-10} \right]$

1) 1.6×10^{-6} **2)** 1.9×10^{-5} **3)** 2.2×10^{-16} **4)** 0.625×10^{-6}

Key:2

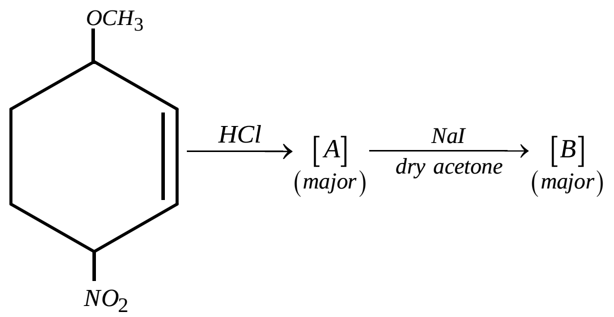
Solution: $A_gCN \rightleftharpoons A_g^{\oplus} + CN^{\Theta}$ $\quad\quad$ $H^{\oplus} + CN^{\Theta} \rightleftharpoons H^+ + CN^{\Theta}$

$$K_{sp} \times \frac{1}{K_a} = \left[A_g^+\right]\left[CN^{\Theta}\right] \times \frac{[HCN]}{\left[H^+\right]\left[CN^{\Theta}\right]}$$

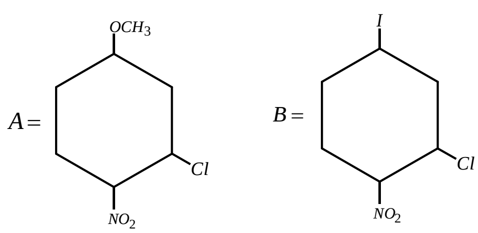
$$2.2 \times 10^{-16} \times \frac{1}{6.6 \times 10^{-10}} = \frac{\Delta.\,\Delta}{10^{-3}}$$

$$\Delta^2 = \frac{10^{-8}}{30} \quad\quad \Delta = 1.9 \times 10^{-5}$$

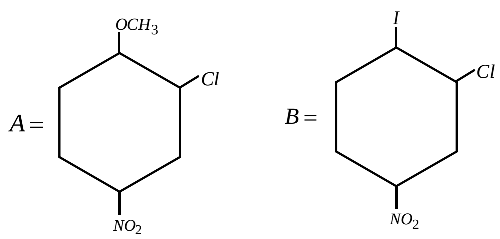
46. Identify A and B in the chemical reaction



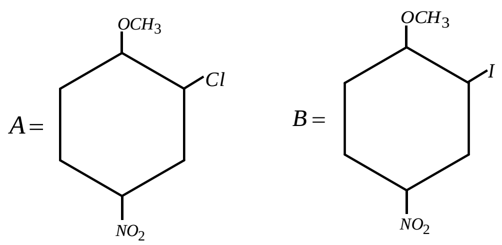
1)



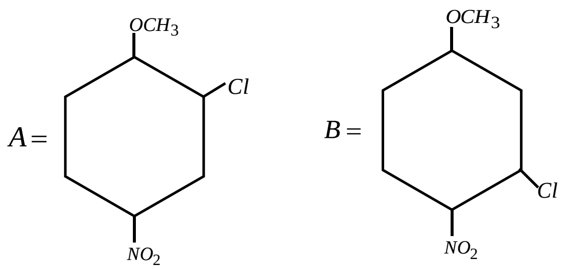
2)



3)

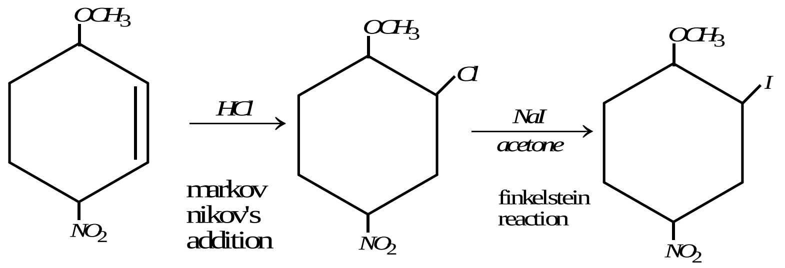


4)



Key:3

Solution:



47. Which statement is correct ?

1) Synthesis of Buna-S needs nascent oxygen.

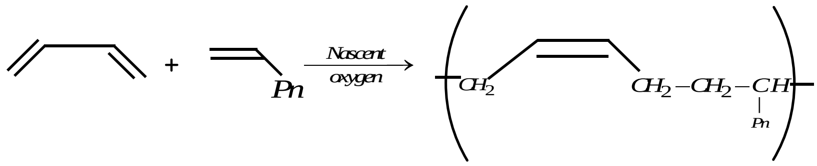
2) Neoprene is an addition copolymer used in plastic manufacturing.

3) Buna-N is a natural polymer.

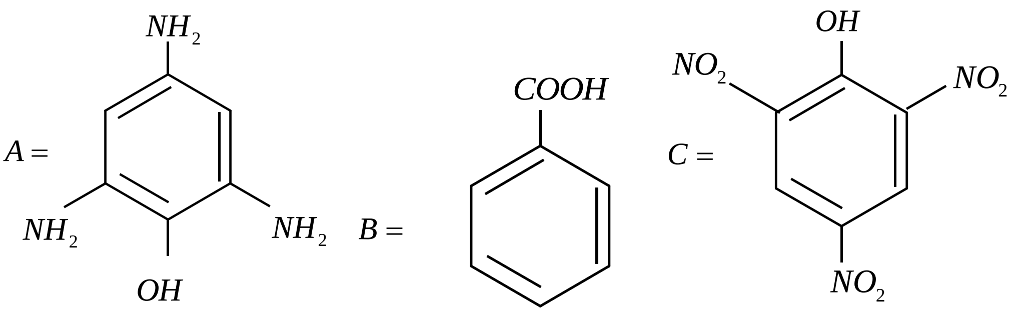
4) Buna-S is a synthetic and linear thermosetting polymer.

Key:1

Solution:



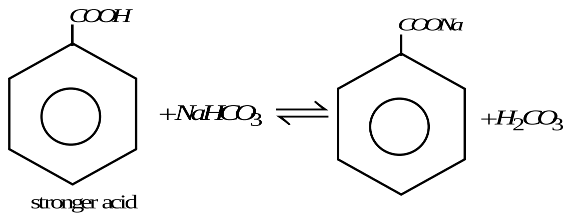
48. Compound (s) which will liberate carbon dioxide with sodium bicarbonate solution is/are:



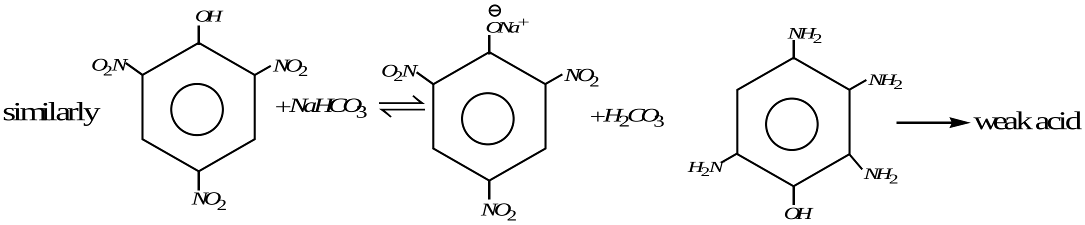
1) B and C only **2)** B only **3)** A and B only **4)** C only

Key:1

Solution:



Equilibrium favours forward and CO_2 is liberated.



49. According to molecular orbital theory, the species among the following that does not exist is:

1) He_2^+ **2)** He_2^- **3)** O_2^{2-} **4)** Be_2

Key:4

Solution: Be_2 bond order zero.

50. The major product of the following chemical reaction is :

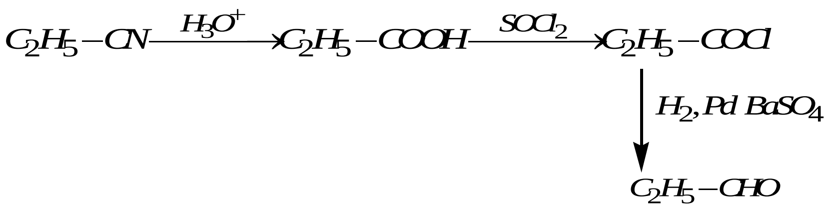
$$CH_3CH_2CN \xrightarrow[3)\, Pd/BaSO_4, H_2]{1)\, H_3O^+, \Delta \quad 2)\, SOCl_2}$$

1) $(CH_3CH_2CO)_2O$ **2)** CH_3CH_2CHO

3) $CH_3CH_2CH_2-OH$ **4)** $CH_3CH_2CH_3$

Key:2

Solution:



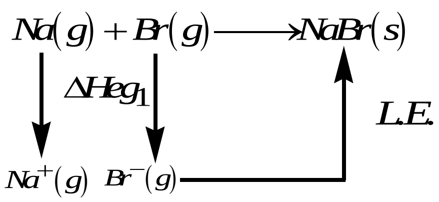
(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

51. The ionization enthalpy of Na^+ formation from $Na_{(g)}$ is $495.8\ kJ\ mol^{-1}$, while the electron gain enthalpy of Br is $-325.0\ kJ\ mol^{-1}$. Given the lattice enthalpy of $NaBr$ is $-728.4\ kJ\ mol^{-1}$. The energy for the formation of $NaBr$ ionic solid is $(-)$_______$\times 10^{-1}\ kJ\ mol^{-1}$.

Key:5576

Solution:



$L.E.\Delta H_{formation} = IE_1 + \Delta Heg_1 + LE$

$= 495.8 + (-32.5) + (-728.4)$

$= -557.6$

$= -5576 \times 10^{-1} KJ/mol$.

Note : The above calculation is not for

$\Delta H_{formation}$ but for $\Delta H_{\text{Re}action}$.

But on the basis of given data it is the best ans

52. In basic medium CrO_4^{2-} oxidises $S_2O_3^{2-}$ to form SO_4^{2-} and itself changes into $Cr(OH)_4^{-}$.

The volume of 0.154 M CrO_4^{2-} required to react with 40 mL of 0.25 M $S_2O_3^{2-}$ is_______mL (Rounded off to the nearest integer)

Key:173

Solution: $\overset{+6}{C}rO_4^{2-} + \overset{+2}{S}_2O_3^{2-} \rightarrow \overset{+6}{S}O_4^{2-} + \overset{+3}{C}r(OH)_4^{-}$

$gm\ equi.\ of\ CrO_4^{2-} = S_2O_3^{2-}$

$0.14 \times 3 \times v = 0.25 \times 40 \times 8$

$v = 173.16 = 173\ ml$

Hence answer is (173)

53. 0.4 g mixture of $NaOH, Na_2CO_3$ and some inert impurities was first titrated with $\frac{N}{10} HCl$ using phenolphthalein as an indicator, 17.5 mL of HCl was required at the end point. After this methyl orange was added and titrated. 1.5 mL of same HCl was required for the next end point. The weight percentage of Na_2CO_3 in the mixture is _________(Rounded off to the nearest integer)

Key:4

Solution: Upto first end point

$gm\ equi.\ of\ (NaOH + Na_2CO_3) = HCl$

$x + y \times 1 = \frac{1}{10} \times 17.5$

$x + y = 1.75$ ….. (1)

Upto second end point

$NaOH + Na_2CO_3 \equiv HCl$

$x + y + 2 = \frac{1}{10} \times 19$

$x + 2y = 1.9$ …… (2)

$\%Na_2CO_3 = \frac{0.15 \times 10^{-3} \times 106}{0.4} \times 100$

$= 3.975\%$

$= 4\%$

Hence answer is (4)

54. Among the following, the number of halide(s) which is /are inert to hydrolysis is ___

1) BF_3 **2)** $SiCl_4$ **3)** PCl_5 **4)** SF_6

Key:1

Solution: Among carbon group, $cc\ell_4$ doesn't hydrolyse remaining chlorides are tends to hydrolyse.

SF_6 is more stable, due to steric reasons therefore doesn't tend to hydrolyse.

5th group, $PCl_5 + H_2O \rightarrow POCl_3 + 2HCl$

$POCl_3 + 3H_2O \rightarrow H_3PO_4 + 3HCl$

BF_3 also tends to hydrolyse to give arthobasic acid.

55. A car tyre is filled with nitrogen gas at 35 psi at 27^oC. It will burst if pressure exceeds 40 psi. The temperature in ^oC at which the car tyre will burst is _______(Rounded off to the nearest integer)

Key:70

Solution: $P \propto T$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \Rightarrow \frac{40}{35} = \frac{T_2}{300}$$

$T_2 = 342.854\ K$

$= 69.70^0C \simeq 70^0C$

Hence answer is (70)

56. Using the provided information in the following, paper chromatogram :

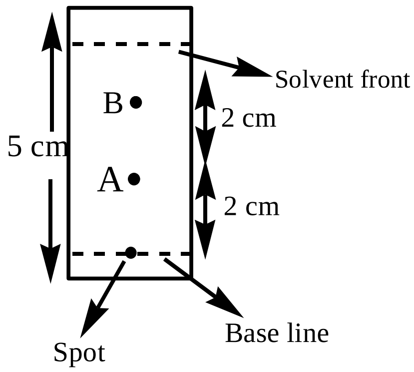


Fig : Paper chromatography for compounds A and B, the calculated R_f value of A_________$\times 10^{-1}$.

Key: 0.40

Solution: $R_f = \frac{Distance\ of\ substance\ from\ Base\ line(x)}{Distance\ of\ solvent\ from\ Base\ line\ (y)}$

$\Rightarrow \quad Far(A) \rightarrow x = 2; y = 5$

$\Rightarrow \quad (R_f)_A = \frac{2}{5} = 0.4 = 4 \times 10^{-1}$

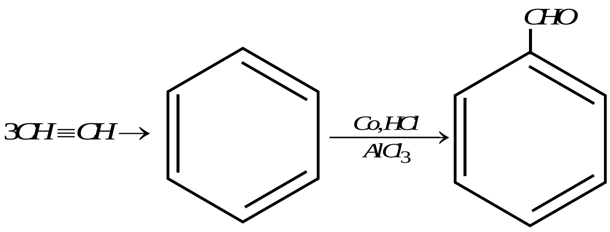
57. Consider the following chemical reaction

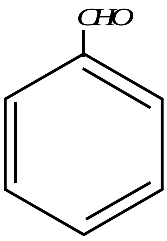
$$CH \equiv CH \xrightarrow[2)\ CO, HCl, AlCl_3]{1)\ Red\ hot\ Fe\ tube, 873\,K} Product$$

The number of sp^2 hybridized carbon atom(s) present in the product is _______

Key: 7

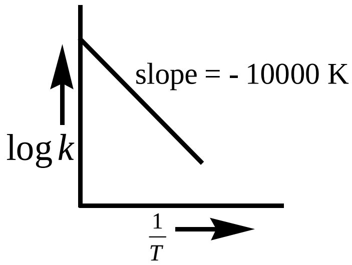
Solution:





contains seven sp^2 carbons

58. For the reaction $aA + bB \rightarrow cC + dD.$ Theplot of log k vs $\frac{1}{1}$ is given below:



The temperature at which the rate constant of the reaction is $10^{-4} s^{-1}$ is ______K (Rounded off to the nearest integer)

[Given : The rate constant of the reaction is $10^{-5} s^{-1}$ at 500 K]

Key: 526

Solution: $\log K = \log A - \frac{Ea}{2.303RT}$

$$|Slope| = \frac{Ea}{2.303R} = 10.000$$

$$\log\left(\frac{K_2}{K_1}\right) = \frac{Ea}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\log\left(\frac{10^{-4}}{10^{-5}}\right) = 10.000\left[\frac{1}{500} - \frac{1}{T_2}\right]$$

$$T_2 = 526.31 \simeq 526K$$

Hence answer is (526)

59. 1 molal aqueous solution of an electrolyte A_2B_3 is 60% ionized. The boiling point of the solution at 1 atm is ________K (Rounded off to the nearest integer)

[Given k_b for $\left(H_2O\right)$=0.52 K kg mol^{-1}

Key: 375

Solution: $i = 1 + (n-1)\alpha$

$= 1 + 4 \times 0 - 6$

$= 1 + 2.4$

$= 3.4$

The expression for the elevation of boiling point is

$\Delta Tb = Kb \times m \times i = 0.52 \times 10 \times 3.4 = 1.768$

The boiling point 1 molar aqueous

Solution is $373.15K + 1.768 = 374.918K \approx 375K$

60. The reaction of cyanamide, $NH_2CN(s)$ with oxygen was run in a bomb calorimeter and ΔU was found to be $-742.24\ kJ\ mol^{-1}$. The magnitude of ΔH_{298} for the reaction

$NH_2CN_{(s)} + \frac{3}{2}O_{2_{(g)}} \rightarrow O_{2_{(g)}} + H_2O_{(1)}$

Is_____kJ. (Rounded off to the nearest integer)

[Assume ideal gases and $R = 8.314\ J\ mol^{-1}K^{-1}$]

Key: 741

Solution: $\Delta H = \Delta U + \Delta n_g RT$

$= -742.24 + \frac{1}{2} \times \frac{8.314}{1000} \times 298$

$= -741\ kJ / mol$

Hence answer is (741)

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2 - 4x + y + 8}{x - 2}$, then this curve also passes through the point:

1) $(5,5)$ **2)** $(4,5)$ **3)** $(4,4)$ **4)** $(5,4)$

Key: 1

Solution: $\frac{dy}{dx} = \frac{x^2 - 4x + y + 8}{x - 2} = \frac{(x-2)^2 + (y+4)}{(x-2)}$

$\frac{dy}{dx} = (x-2) + \frac{(y+4)}{(x-2)}$ **........ (1)**

LET $x - 2 = t \Rightarrow dx = dt$

$y + 4 = u \Rightarrow dy = du, \frac{dy}{dx} = \frac{du}{dt}$

$(1) \Rightarrow \frac{du}{dt} = t + \frac{u}{t} \Rightarrow \frac{du}{dt} - \frac{u}{t} = t$

$I.F = e^{\int -\frac{1}{t} dt} = e^{-\ln(t)} = \frac{1}{t}$

Solution is, $u.\frac{1}{t} = \int t.\frac{1}{t}\, dt \Rightarrow \frac{u}{t} = t + c$

$\frac{y+4}{x-2} = x - 2 + c$

Passing through $(0,0) \Rightarrow c = 0$

$\Rightarrow y + 4 = (x-2)^2$

$(1) \Rightarrow (5,5) \Rightarrow 0 = 0$

By verification option (2) is correct.

62. The statement $A \to (B \to A)$ is equivalent to:

1) $A \to (A \wedge B)$ **2)** $A \to (A \to B)$ **3)** $A \to (A \vee B)$ **4)** $A \to (A \leftrightarrow B)$

Key:3

Solution:

Given statement: $A \longrightarrow (B \to A)$

$\simeq \sim A \vee (B \to A)$

$\simeq \sim A \vee (\sim B \vee A)$

$\simeq(\sim A\vee A)\vee B$

$\simeq t\vee B\simeq t$

$(3)\Rightarrow\ A\to(A\vee B)\simeq\sim A\vee(A\vee B)$

$\simeq(\sim A\vee A)\vee B$

$\simeq t\vee B$

$\simeq t$

63. If the curves, $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$ intersect each other at an angle of 90^0, then which of the following relations is TRUE?

1) $a+b=c+d$ **2)** $a-c=b+d$ **3)** $a-b=c-d$ **4)** $ab=\frac{c+d}{a+b}$

Key:3

Solution: $\frac{x^2}{a}+\frac{y^2}{b}=1, \frac{x^2}{c}+\frac{y^2}{d}=1$

$px^2+qy^2=1,\ p^1x^2+q^1y^2=1$ cuts orthogonally

$\frac{1}{p}-\frac{1}{q}=\frac{1}{p'}-\frac{1}{q'}\Rightarrow a-b=c-d$

64. The integer $'k'$, for which the inequality $x^2-2(3k-1)x+8k^2-7>0$ is valid for every x in R, is:

1) 0 **2)** 2 **3)** 3 **4)** 4

Key:3

Solution: $x^2-2(3k-1)x+\left(8k^2-7\right)>0, \forall x\in R$

$\Rightarrow\ D<0$

$\left(2(3k-1)\right)^2-4.\left(8k^2-7\right)<0$

$\Rightarrow\quad 4\left(9k^2-6k+1\right)-32k^2+28<0$

$\Rightarrow\quad k^2-6k+8<0$

$\Rightarrow\quad (k-4)(k-2)<0$

$\Rightarrow\quad 2<k<4\ \Rightarrow\quad k=3$

65. If $0<\theta,\phi<\frac{\pi}{2}, x=\sum_{n=0}^{\infty}\cos^{2n}\theta, y=\sum_{n=0}^{\infty}\sin^{2n}\phi$ *and* $z=\sum_{n=0}^{\infty}\cos^{2n}\theta.\sin^{2n}\phi$ then:

1) $z=\frac{xy}{xy-1}$ **2)** $xy-z=(x+y)z$

3) $xyz=4$ **4)** $xy+z=(x+y)z$

Key:1

Solution: $x = 1 + \cos^2\theta + \cos^4\theta + ... = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta}$

$y = 1 + \sin^2\theta + \sin^4\theta + ... = \frac{1}{1-\sin^2\theta} = \frac{1}{\cos^2\theta}$

$z = 1 + \sin^2\theta\cos^2\theta + \sin^4\theta\cos^4\theta + ... = \frac{1}{1-\sin^2\theta\cos^2\theta}$

$\Rightarrow \quad z = \frac{1}{\left(1-\frac{1}{x}\times\frac{1}{y}\right)} \Rightarrow z = \frac{xy}{xy-1}$

66. Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is:

1) $\frac{1}{2}$ **2)** $\frac{5}{8}$ **3)** $\frac{3}{4}$ **4)** $\frac{3}{8}$

Key:2

Solution: $\ell + m - n = 0$ **...... (1)**

$\ell^2 + m^2 - n^2 = 0$

$\ell^2 + m^2 - (\ell + m)^2 = 0$

$\ell^2 + m^2 - \left[\ell^2 + m^2 + 2\ell m\right] = 0$

$2\ell m = 0$

$\ell = 0,\ m = 0$

$\ell = 0$

$1.\ell + 0.m + 0.m = 0$ **......... (2)**

$0.\ell + 1.m + 0.n = 0$ **.......... (3)**

Solving (1) & (2)

ℓ	m	n	
1	-1	1	1
0	0	1	0

$\frac{\ell}{0-0} = \frac{m}{-1-0} = \frac{n}{0-1}$

$\frac{\ell}{0} = \frac{m}{-1} = \frac{n}{-1}$ Dr's of first line $(a_1, b_1, c_1) = (0, -1, -1)$

Solving (1) & (3)

ℓ	m	n	
1	-1	1	1
1	0	0	1

$\frac{\ell}{0+1}=\frac{m}{0-0}=\frac{n}{1-0}$ Dr's at second line $(a_2,b_2,c_2)=(1,0,1)$

Dc's at second line $(\ell_2,m_2,n_2)=\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$

$\cos\theta=|\ell_1\ell_2+m_1m_2+n_1n_2|=\left|0+0-\frac{1}{2}\right|$ $=\frac{1}{2}$

$\theta=60^0=\alpha$ $\sin^4\alpha+\cos^4\alpha=\sin^4 60+\cos^4 60=\left(\frac{\sqrt{3}}{2}\right)^4+\left(\frac{1}{2}\right)^4=\frac{9+1}{16}=\frac{10}{16}$ $=\frac{5}{8}$

67. A tangent is drawn to the parabola $y^2=6x$ which is perpendicular to the line $2x+y=1$. Which of the following points does NOT lie on it?

1) $(5,4)$ **2)** $(4,5)$ **3)** $(0,3)$ **4)** $(-6,0)$

Key:1

Solution: Given parabola $y^2=6x\Rightarrow 4a=6$

Given line $2x+y=1$ $a=\frac{3}{2}$ Slope of $\perp^r$ line $m=\frac{1}{2}$

Equation of tangent $y=mx+\frac{a}{m}$ $y=\frac{1}{2}x+\frac{\frac{3}{2}}{\frac{1}{2}}$

$y=\frac{1}{2}x+3$

$2y=x+6$

$x-2y+6=0$

(1) $(5,4)$ lies on $x-2y+6=0$

(2) $(4,5)\Rightarrow 4-10+6=0$

(3) $(9,3)\Rightarrow 0-6+6=0$

$(4)(-6,0)\Rightarrow -6-0+6=0$

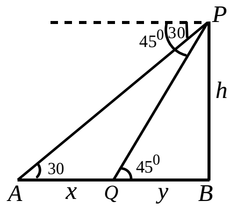
$\therefore$ $(5,4)$ does not lies on tangent

68. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30 (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45^0. Then the time taken (in seconds) by the boat from B to reach the base of the tower is:

1) $10(\sqrt{3}+1)$ **2)** $10\sqrt{3}$ **3)** $10(\sqrt{3}-1)$ **4)** 10

Key:1

Solution:



$v=\frac{d}{t}$ $\quad t=\frac{d}{v}$ $\quad 20=\frac{x}{v}$ $\quad x=20\text{v}$

$ABP\ Tan30^0=\frac{h}{x+y}$

$\frac{1}{\sqrt{3}}=\frac{h}{x+y}\Rightarrow x+y=\sqrt{3}h$

$\Delta PQB\ \ Tan45^0=\frac{h}{y}$ $\quad y=h$

$x+y=\sqrt{3}y$

$x=\left(\sqrt{3}-1\right)y$

$20V=\left(\sqrt{3}-1\right)y$

$\frac{y}{v}=\frac{20}{\sqrt{3}-1}=10\left(\sqrt{3}+1\right)\text{sec}$

69. The coefficients a,b and c of the quadratic equation, $ax^2+bx+c=0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is:

1) $\frac{1}{36}$ **2)** $\frac{5}{216}$ **3)** $\frac{1}{54}$ **4)** $\frac{1}{72}$

Key:2

Solution: Given quadratic equation $ax^2+bx+c=0$ has eqal roots $\Delta=0$ $\quad \frac{b^2}{4}=ac$

$n\left(S\right)=6^3=216,\ a,b,c\in S$

A die is throw them $S=\{1,2,3,4,5,6\}$

$\therefore$ Req probability $=\frac{5}{216}$

70. If Rolle's theorem holds for the function $f\left(x\right)=x^3-ax^2+bx-4,x\in\left[1,2\right]$ with $f'\left(\frac{4}{3}\right)=0$, then ordered pair $\left(a,b\right)$ is equal to:

1) $\left(5,-8\right)$ **2)** $\left(-5,8\right)$ **3)** $\left(5,8\right)$ **4)** $\left(-5,-8\right)$

Key:3

Solution: $a=1,\ \ b=2$

$$f(1)=f(2)$$

$$1-a+b+1=8-4a+2b+1$$

$$3a-b=7 \qquad \text{........ (1)}$$

$$f'(x)=3x^2-2ax+b$$

$$f'\left(\frac{4}{3}\right)=0$$

$$3\times\frac{16}{9}-2a\times\frac{4}{3}+b=0$$

$$\frac{16}{3}-\frac{8a}{3}+b=0$$

$$-8a+3b+16=0$$

$$8a-3b=16 \qquad \text{.......... (2)}$$

$$9a-3b=21$$

$$8a-3b=16$$

Solving (1) & (2) $\dfrac{\begin{matrix}- & + & -\end{matrix}}{a=5,\ \ b=8}$ $\qquad \therefore\ \ (a,b)=(5,8)$

71. $\lim\limits_{n\to\infty}\left(1+\dfrac{1+\frac{1}{2}+........+\frac{1}{n}}{n^2}\right)^n$ is equal to

1) 1 **2)** 0 **3)** $\dfrac{1}{e}$ **4)** $\dfrac{1}{2}$

Key:1

Solution: $\underset{n\to\infty}{Lt}\left(1+\dfrac{1+\frac{1}{2}+......+\frac{1}{n}}{n^2}\right)^n$

$$=\underset{n\to\infty}{Lt}\left(\frac{1+\frac{1}{2}+\frac{1}{3}+.......+\frac{1}{n}}{n}\right)$$

$$=e^{\underset{n\to\infty}{Lt}\left(\frac{1}{n}+\frac{1}{2n}+\frac{1}{3n}+.......+\frac{1}{m}\right)}$$

$$=e^0$$

$$=1$$

72. The value of $\int_{-1}^{1} x^2 e^{\left[x^3\right]} dx,$ where $[t]$ denotes the greatest integer $\le t$, is:

1) $\frac{e+1}{3}$ **2)** $\frac{e+1}{3e}$ **3)** $\frac{e-1}{3e}$ **4)** $\frac{1}{3e}$

Key:2

Solution: $\int_{-1}^{1} x^2 e^{\left[x^3\right]} dx \quad = \int_{-1}^{0} x^2 e^{-1} dx + \int_{0}^{1} x^2 dx$

$$= \frac{1}{e}\left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1} = \frac{1}{e}\left(0-\left(-\frac{1}{3}\right)\right)+\frac{1}{3} \qquad = \frac{1}{3e}+\frac{1}{3}$$

73. Let $f, g: N \to N$ such that $f(n+1) = f(n) + f(1) \forall n \in N$ and g be any arbitrary function. Which of the following statements is NOT true?

1) f is one-one

2) If fog is one-one, then g is one-one

3) If g onto, then fog is one-one

4) If f is onto, then $f(n) = n \forall n \in N$

Key:3

Solution: $f: N \to N$

$g: N \to N$

$f(n+1) = f(n) + f(1) \ \forall n \in N$

$f(2) = 2f(1)$

$f(3) = 3f(1)$

$f(4) = 4f(1)$

$f(n) = nf(1)$

$f(n) = nf(1)$

$f(x)$ is one-one

fog is one-one only if g is $1-1$

$\therefore$ option (3)

74. When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that is it not intercepted, is $\frac{3}{4}$. If three missiles are find independently from the ship, then the probability that all three hit the target, is:

1) $\frac{3}{4}$ **2)** $\frac{1}{27}$ **3)** $\frac{3}{8}$ **4)** $\frac{1}{8}$

Key:4

Solution:

P (missile intercepted) $=\frac{1}{3}$

P (missile not intercepted) $=\frac{2}{3}$ P (hit the target) $=\frac{3}{4}$

Req probability $=\left(\frac{2}{3}.\frac{3}{4}\right)^3=\frac{1}{8}$

75. The value of the integral

$$\int\frac{\sin\theta\ .\sin 2\theta\left(\sin^6\theta+\sin^4\theta+\sin^2\theta\right)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}d\theta \text{ is:}$$

(where c is a constant of integration)

1) $\frac{1}{18}\left[9-2\cos^6\theta-3\cos^4\theta-6\cos^2\theta\right]^{\frac{3}{2}}+c$

2) $\frac{1}{18}\left[11-18\sin^2\theta+9\sin^4\theta-2\sin^6\theta\right]^{\frac{3}{2}}+c$

3) $\frac{1}{18}\left[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta\right]^{\frac{3}{2}}+c$

4) $\frac{1}{18}\left[9-2\sin^6\theta-3\sin^4\theta-6\sin^2\theta\right]^{\frac{3}{2}}+c$

Key:3

Solution:

$$\int\frac{\sin\theta.\sin 2\theta\left(\sin^6\theta+\sin^4\theta+\sin^2\theta\right)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}d\theta$$

$$=\int\frac{2\sin^2\theta\left(\sin^6\theta+\sin^4\theta+\sin^2\theta\right)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{2\sin^2\theta}.\cos\theta\ d\theta$$

Put $\sin\theta=t$

$\cos\theta d\theta=dl$

$=\int\left(t^6+t^4+t^2\right)\sqrt{2t^4+3t^2+6}\ dt \qquad =\int\left(t^5+t^3+t\right)\sqrt{2t^6+3t^4+6t^2}dt$

$2t^6+3t^4+6t^2=y$

$12\left(t^5+t^3+t\right)dt=dy=\frac{1}{12}\Big|\sqrt{y}dy \qquad =\frac{1}{12}\frac{2}{3}y^{\frac{3}{2}}+C \qquad =\frac{1}{18}y^{\frac{3}{2}}+C$

$$=\frac{1}{18}\left(2\sin^6\theta+3\sin^4\theta+6\sin^2\theta\right)^{\frac{3}{2}}+C$$

$$=\frac{1}{18}\left[-2\cos^6\theta+9\cos^4\theta-18\cos^2\theta+11\right]^{\frac{3}{2}}+C$$

76. All possible values of $\theta\in[0,2\pi]$ for which $\sin 2\theta+\tan 2\theta>0$ lie in:

1) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{11\pi}{6}\right)$ **2)** $\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)$

3) $\left(0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$ **4)** $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{5\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$

Key:4

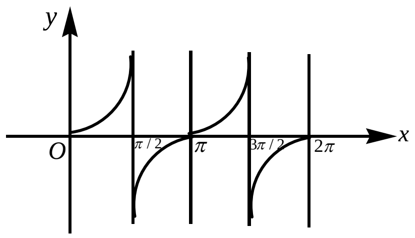
Solution:

$$\sin 2\theta+\frac{\sin 2\theta}{\cos 2\theta}>0\Rightarrow \qquad \frac{\sin 2\theta(\cos 2\theta+1)}{\cos 2\theta}>0$$

$$\Rightarrow \quad Tan 2\theta(1+\cos 2\theta)>0$$

$$\Rightarrow \quad 2\theta\in\left(0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{2}\right)\cup\left(2\pi,\frac{5\pi}{2}\right)\cup\left(3\pi,\frac{7\pi}{2}\right)$$

$$\Rightarrow \quad \theta\in\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{5\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$$



77. Let the lines $(2-i)z=(2+i)\overline{z}$ and $(2+i)z+(i-2)\overline{z}-4i=0$, (here $i^2=-1$) be normal to a circle C. If the line $iz+\overline{z}+1+i=0$ is tangent to this circle C, then its radius is:

1) $3\sqrt{2}$ **2)** $\frac{3}{2\sqrt{2}}$ **3)** $\frac{1}{2\sqrt{2}}$ **4)** $\frac{3}{\sqrt{2}}$

Key:2

Solution:

$L_1=(2-i)z=(2+i)\overline{z}$

$L_2=(2+i)z+(i-2)\overline{z}-4i=0$ be the normals

To the circles

Also $iz+\overline{z}+1+i=0$ is a tangent to the circle. Let $z=x+iy$ then

$L_1\equiv(2-i)(x+iy)=(2+i)(x-iy)$

$\Rightarrow \quad (2x+y)+i(-x+2y)=(2x+y)+i(x-2y)$

$\Rightarrow \quad -x+2y=x-2y\Rightarrow 2x-4y=0$

$\Rightarrow \quad x-2y=0 \qquad (1)$

$L_2 \equiv (2+i)(x+iy)+(i-2)(x-iy)-4i=0$

$\Rightarrow \quad (2x-y)+(y-2x)+i(2y+x+x+2y)=4i$

$\Rightarrow \quad 2x+4y=4 \Rightarrow x+2y=2 \qquad (2)$

(1) + (2) $\Rightarrow 2x=2 \Rightarrow x=1$

Then $2y=2-1=1 \Rightarrow y=\frac{1}{2}$

$\Rightarrow C=\left(1,\frac{1}{2}\right)$

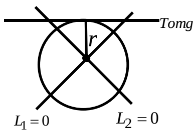
Also, $iz+\overline{z}+i+i=0$

$i(x+iy)+(x-iy)+1+i=0$

$\Rightarrow \quad -y+x+1=0$

$x-y+1=0$

Now, radium $=r=\frac{\left|1-\frac{1}{2}+1\right|}{\sqrt{1+1}}=\frac{\left|\frac{3}{2}\right|}{\sqrt{2}}=\frac{3}{2\sqrt{2}}$



78. The image of the point $(3,5)$ in the line $x-y+1=0$, lies on:

1) $(x-2)^2+(y-2)^2=12$ **2)** $(x-2)^2+(y-4)^2=4$

3) $(x-4)^2+(y+2)^2=16$ **4)** $(x-4)^2+(y-4)^2=8$

Key:2

Solution:

Let $P(x_1 l y_1)=(3,5)$ Image of P is Q (h,k)

$L=x-y+1=0$

$\frac{h-3}{1}=\frac{k-5}{-1}=-2\frac{(3-5+1)}{1+1}$

$\Rightarrow \quad \frac{h-3}{1}=\frac{k-5}{-1}=-2\frac{(-1)}{2}=+1$

$\Rightarrow \quad h=+1+3=4 \quad k-5=-1 \Rightarrow k=4$

$\Rightarrow Q(h,k)=(4,4)$

$(4,4)$ lies on $(x-2)^2+(y-4)^2=4$

(By optimal verification).

79. The total number of positive integral solutions (x, y, z) such that $xyz = 24$ is:

1) 36 **2)** 45 **3)** 24 **4)** 30

Key:4

Solution:

Given $xyz = 24$

$$= 2\times 12$$

$$= 2\times 2\times 2\times 2\times 3 = 2^3 \times 3'$$

No. of positive integral solutions

Case (i) : 2^3 $\underset{x_1+}{\overset{x}{\downarrow}}$ $\underset{x_2+}{\overset{y}{\downarrow}}$ $\underset{x_3=3}{\overset{z}{\downarrow}}$

No. of ways $= (n+r-1)_{r-1} = 5_{c_2} = 10$

Case (ii) : $3'$ $\underset{x_1+}{\overset{x}{\downarrow}}$ $\underset{x_2+}{\overset{y}{\downarrow}}$ $\underset{x_3=1}{\overset{z}{\downarrow}}$

No. of ways $= (n+r-1)_{c_{r-1}} = 3_{c_2} = 3$

No. of +ve integral solutions $= 10\times 3 = 30$.

80. The equation of the line through the point $(0,1,2)$ and perpendicular to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is:

1) $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$ **2)** $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

3) $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$ **4)** $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

Key:1

Solution:

Point $(0,1,2)$

Given line is $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$

Equ. Of required line passing through $(0,1,2)$ is

$$\frac{x-0}{\ell} = \frac{y-1}{m} = \frac{z-2}{n}$$

Here $2\ell + 3m - 2n = 0$ (1)

By verification, eq. of req. line is

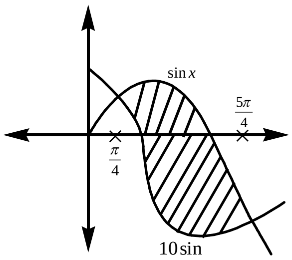
$$\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}.$$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

81. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to__________.

Key:64



Solution:

$$A = \int_{\frac{\pi}{4}}^{5\frac{\pi}{4}} (\sin x - \cos x)\,dx$$

$$= -(\cos x)_{\frac{\pi}{4}}^{5\frac{\pi}{4}} - (\sin x)_{\frac{\pi}{4}}^{5\pi/4} \qquad = -\left[\frac{-1}{\sqrt{2}}\frac{-1}{\sqrt{2}}\right] - \left[\frac{-1}{\sqrt{2}}\frac{-1}{\sqrt{2}}\right]$$

$$= \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$A^4 = 2^4(4) \quad A^4 = 64$$

82. Let $A_1, A_2, A_3, ...$ be squares such that for each $n \ge 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is_______.

Key:9

Solution:

Sides are $12, \frac{12}{(\sqrt{2})}, \frac{12}{(\sqrt{2})^2}........$

$$\left(\frac{12}{(\sqrt{2})^{n-1}}\right)^2 < 1$$

$$144 < 2^{n-1}$$

$$2^{n-1} > 144$$

$$n - 1 \ge 8$$

$$n \ge 9 \qquad n = 9$$

83. If the system of equations

$kx+y+2z=1$

$3x-y-2z=2$

$-2x-2y-4z=3$

Has infinitely many solutions, then k is equal to_________.

Key:21

Solution: $\Delta=\begin{vmatrix} k & 1 & 2 \\ 3 & -1 & -2 \\ -2 & -2 & -4 \end{vmatrix}=0$

$\Delta_3=\begin{vmatrix} k & 1 & 1 \\ 3 & -1 & 2 \\ -2 & -2 & 3 \end{vmatrix}=0 \qquad \Rightarrow \quad k=21$

$\Delta_1=0 \ and \ \Delta_2=0 \qquad \Rightarrow \quad k=21$

84. The locus of the point of intersection of the lines $\left(\sqrt{3}\right)kx+ky-4\sqrt{3}=0$ and $\sqrt{3}x-y-4\left(\sqrt{3}\right)k=0$ is a conic, whose eccentricity is____________.

Key:2

Solution: $k\left(\sqrt{3}x+y\right)=4\sqrt{3},$ $\qquad \left(\dfrac{\sqrt{3}x-y}{4\sqrt{3}}\right)\left(\sqrt{3}x+y\right)=4\sqrt{3}$

$3x^2-y^2=48 \qquad \dfrac{x^2}{16}-\dfrac{y^2}{48}=1 \qquad e=\sqrt{\dfrac{16+48}{16}}=\sqrt{\dfrac{64}{16}}=\sqrt{4}=2$

85. The number of points, at which the function $f\left(x\right)=\left|2x+1\right|-3\left|x+2\right|+\left|x^2+x-2\right|, x\in R$ is not differentiable, is________.

Key:2

Solution:

$f\left(x\right)=\left|2x+1\right|-3\left|x+2\right|+\left|x^2+x-2\right|$

$=\left|2x+1\right|-3\left|x+2\right|+\left|\left(x-1\right)\left(x+2\right)\right|$

$$f\left(x\right)=\begin{cases} x^2+2x+3, & x<-2 \\ -x^2-6x-5, & -2\le x\le -1 \\ -x^2-2x-3, & -\dfrac{1}{2}<x<1 \\ x^2-7 & x>1 \end{cases}$$

$$f'(x)\begin{cases} 2x+2 & x<-2 \\ -2x-6 & -2\le x\le -\frac{1}{2} \\ -2x-2 & -\frac{1}{2}<x<1 \\ 2x & x>1 \end{cases}$$

$f'\left(-2^-\right)=f'\left(-2^+\right) and \ f\left(-2^-\right)=f\left(-2^+\right)$

$f'\left(-\frac{1}{2}^-\right)\pm f\left(-\frac{1}{2}^+\right)$

$f'\left(1^-\right)\equiv f'\left(1^+\right)$

NOT DIFFERENCE AT $-\frac{1}{2},1$

NO OF DIFFERENTIABLE POINTS 2

86. Let $f(x)$ be a polynomial of degree 6 in $x,$ in which the coefficient of x^6 is unity and it has extrema at $x=-1$ and $x=1$. If $\lim\limits_{x\to} \frac{f(x)}{x^3}=1,$ then $5f(2)$ is equal to______.

Key:

Solution:

87. Let $\vec{a}=\hat{i}+2\hat{j}-\hat{k},\vec{b}=\hat{i}-\hat{j}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$ and $\vec{r}.\vec{b}=0$, then $\vec{r}.\vec{a}$ is equal to________.

Key:12

Solution: $\overline{a}=\hat{i}+2\hat{j}+\hat{k},\overline{b}=\hat{i}-\hat{j},\overline{c}=\hat{i}-\hat{j}-\hat{k}$

Since $\overline{r}\times\overline{a}=\overline{c}\times\overline{a}$ and $\overline{r}.\overline{b}=0$

$\Rightarrow \quad (\overline{r}-\overline{c})\times\overline{a}=0\Rightarrow(\overline{r}-\overline{c})\|^{le}\ \overline{a}$

$\Rightarrow \quad \overline{r}-\overline{c}=t\ \overline{a}$

$\Rightarrow \quad \overline{r}.\overline{b}=\overline{c}.\overline{b}=t\left(\overline{b}.\overline{a}\right) \qquad \Rightarrow \quad O-\overline{c}.\overline{b}=t\left(\overline{b}.\overline{a}\right)$

$$t=\frac{-\left(\overline{b}.\overline{c}\right)}{\left(a.\overline{b}\right)}=\frac{-(1+1-0)}{1-2-0}=\frac{-2}{-1}=2$$

Now $\overline{r}=\overline{c}+t\overline{a}$

$=\left(\hat{i}-\hat{j}-\hat{k}\right)+2\left(\hat{i}+2\hat{j}-\hat{k}\right)$

$=3\hat{i}+3\hat{j}-3\hat{k}$

$\overline{r}.\vec{a}=3+6+3=12$

88. Let $A=\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that $x+y+z>0$ and $xyz=2$. If $A^2=I_y$, then the value of $x^3+y^3+z^3$ is________.

Key:7

Solution:

$$A^2=\begin{pmatrix} x & y & z \\ y & z & x \\ z & x & y \end{pmatrix}\begin{pmatrix} x & y & z \\ y & z & x \\ z & x & y \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \quad x^2+y^2+z^2=1;\ xy+yz+zx=0$$

$$\left|A^2\right|=|I|$$

$$|A|^2=1 \Rightarrow |A|=\pm 1 \Rightarrow 3xyz-\left(x^3+y^3+z^3\right)=\pm 1$$

$$3(2)\pm 1=x^3+y^3+z^3$$

$$\Rightarrow \ x^3+y^3+z^3=7 \ or \ 5$$

$$x^3+y^3+z^3=7$$

89. If $A=\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $\left(I_2+A\right)\left(I_2-A\right)^{-1}=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13\left(a^2+b^2\right)$ is equal to________.

Key:

Solution:

$$A=\begin{bmatrix} 0 & -Tan\frac{\theta}{2} \\ +Tan\frac{\theta}{2} & 0 \end{bmatrix} \text{ and } I=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$(I+A)=\begin{pmatrix} 1 & -Tan\frac{\theta}{2} \\ Tan\frac{\theta}{2} & 1 \end{pmatrix}; I-A=\begin{pmatrix} 1 & Tan\frac{\theta}{2} \\ -Tan\frac{\theta}{2} & 1 \end{pmatrix}$$

$$(I-A)^T=\begin{pmatrix} 1 & -Tan\frac{\theta}{2} \\ Tan\frac{\theta}{2} & 1 \end{pmatrix}$$

Now $(I+A)(I-A)^T = \begin{pmatrix} 1 & -Tan\frac{\theta}{2} \\ Tan\frac{\theta}{2} & 1 \end{pmatrix}\begin{pmatrix} 1 & -Tan\frac{\theta}{2} \\ Tan\frac{\theta}{2} & 1 \end{pmatrix}$

$$= \begin{pmatrix} 1-Tan^2\frac{\theta}{2} & -2Tan\frac{\theta}{2} \\ 2Tan\frac{\theta}{2} & 1-Tan^2\frac{\theta}{2} \end{pmatrix} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

$a = 1 - Tan^2\frac{\theta}{2} \quad b = 2Tan\frac{\theta}{2}$

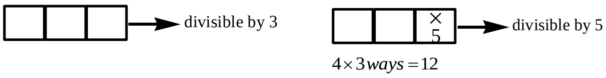
$13\left(a^2+b^2\right) = 13\left\{\left(1-Tan^2\frac{\theta}{2}\right)^2 + 4Tan^2\frac{\theta}{2}\right\} = 13\left\{\left(1+Tan^2\frac{\theta}{2}\right)^2\right\} = 13\sec^4\frac{\theta}{2}$

90. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1,2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is__________.

Key:32

Solution:

Sum of digits $= 1+2+3+4+5 = 15$



$3,4,5 \to (sum\,12) \to 3! = 6$

$2,3,4 \to (sum 9) \to 3! = 6$

$1,3,5 \to (sum 9) \to 3! = 6$

$1,2,3 \to (sum 6) \to 3! = 6$

Repaired no. of ways $= 24 + 12 - 4 \quad = 32$.